COPPE UFRJ

Instituto Alberto Luiz Coimbra de Pós-Graduação e Pesquisa de Engenharia

Adaptive sliding mode control

Liu Hsu

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1. Exact tracking using HOSM differentiators I

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- Robust Exact Differentiator (RED) (Levant 2003)
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- Conclusions

1.2 Introduction: Main objective and idea

- Main Objective:
  - Propose an SMC scheme in order to guarantee global stability and asymptotic exact tracking
  - Output feedback is considered
  - Uncertain linear systems are considered
- Main Idea:
  - Implement a VS-MRAC with a combination of an standard lead filter and the RED.

1.2.1 Problem statement

- Uncertain SISO LTI systems;
- Arbitrary relative degree;
- Plant:

$$y_p(t) = K_p \frac{N_p(s)}{D_p(s)} u(t)$$

- Reference Model:

$$y_m(t) = \frac{K_m}{D_m(s)} r(t)$$

- Objective: design $u(t)$ such that

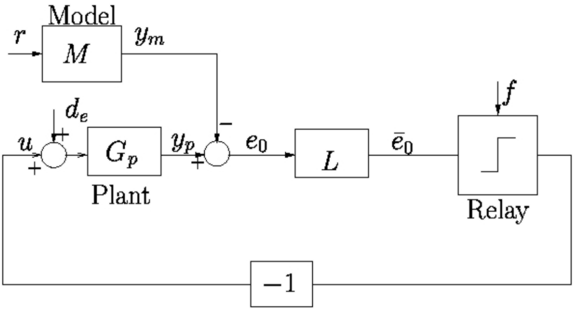
$$e_0 = y_p - y_m \to 0$$

- Standard MRAC assumptions are considered

1.2.2 Ideal VS-MRAC ($n^* > 1$)

- Main idea: consider operator $L(s)$ which reduce to $n^* = 1$.

$$\bar{e}_0 = L(s)e_0 \rightarrow \bar{e}_0 = k^* ML(s)[u + \bar{U}]$$

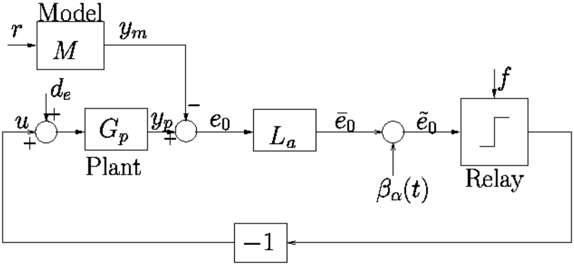


- However, the noncausal operator $L(s)$ cannot be implemented

1.3 LF/VS-MRAC ($n^* > 1$)

- Use a lead filter to obtain a causal realization of $L(s)$

$$\hat{e}_l = L_a(s) e_0 = \left[\frac{L(s)}{F(\tau s)} \right] e_0; \qquad F(\tau s) = (\tau s + 1)^l$$



- If $|\beta_\alpha(t)| \leq \tau K_R$ then $\|e(t)\| \leq K e^{-at} \|e(0)\| + O(\tau)$

1.3.1 Robust Exact Differentiator (RED) I

- Based on second order sliding modes
- The scheme is given by:

$$\begin{cases} \dot{z}_i & = & v_i, \\ v_i & = & -\lambda_i \left| z_i - v_{i-1} \right|^{(n-i)/(n-i+1)} sgn(z_i - v_{i-1}) + z_{i+1}, \\ i & = & 0, \dots, n-1 \\ \dot{z}_n & = & -\lambda_n sgn(z_n - v_{n-1}) \end{cases}$$

- Input: $v_{-1} = e_0(t)$ Outputs: z_i and $v_{i-1} \to e_0^{(i)}(t), \; i = 0, \dots, n$
- Differentiator Parameters: $\lambda_i, \; i = 0, \dots, n$

1.3.1 Robust Exact Differentiator (RED) II

- Convergence in finite time
  - Necessary Condition: $e_0^{(n)}(t) \mapsto$ Lipschitz constant C_{n+1}
  - If λ_i are properly chosen, then:

$$z_0 = e_0(t), \qquad z_i = v_{i-1} = e_0^{(i)}(t), \quad i = 1, \dots, n$$

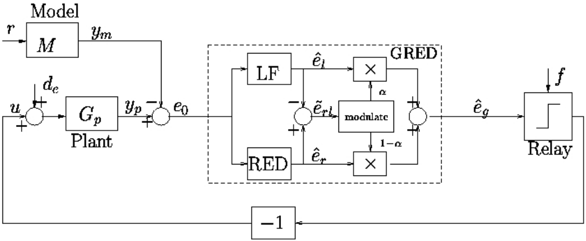
- RED realization of $L(s)$:
  - $L(s) = \gamma_0 s^{(n^*-1)} + \gamma_1 s^{(n^*-2)} + \cdots + \gamma_{(n^*-2)} s + \gamma_{(n^*-1)}$
  - $\bar{e}_0 = L(s) e_0 = \gamma_0 e_0^{(n^*-1)} + \gamma_1 e_0^{(n^*-2)} + \cdots + \gamma_{n^*-2} \dot{e}_0 + \gamma_{n^*-1} e_0$
  - $\hat{e}_r = \gamma_0 z_{n^*-1} + \gamma_1 z_{(n^*-2)} + \cdots + \gamma_{n^*-2} z_1 + \gamma_{n^*-1} z_0$
  - Exact realization after a finite time ($\hat{e}_r = \bar{e}_0$)

1.4 Global output feedback exact tracking

- VS-MRAC with lead filter compensation:
  - Approximated estimate of $\bar{e}_0$ (estimation error of order τ)
  - Global stability
  - Residual tracking error
- RED compensation
  - Exact estimate of $\bar{e}_0$
  - Local stability
  - Asymptotic convergence of the tracking error

1.4.1 GRED/VS-MRAC Control Scheme

- The idea is to combine both compensators



- Convex combination

$$\hat{e}_g = \alpha(\tilde{e}_{rl})\hat{e}_l(t) + [1 - \alpha(\tilde{e}_{rl})]\,\hat{e}_r(t)$$

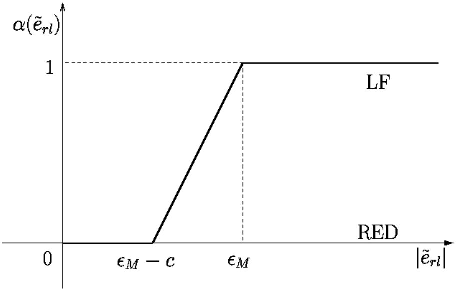
1.4.2 Switching Function

- Objective:
  - Preserve global stability property
  - Ensure the full error converges to zero

- what is proposed:
  - Select the compensator using the difference between their estimations
  - Make the control structure similar to the LF/VS-MRAC structure
  - Ensure that after a transient process only the RED will remain active

The switching function is defined as a boundary layer based on $\tilde{e}_{rl}$

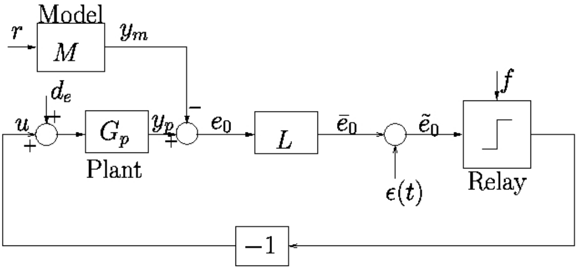
1.4.2 Switching Function II



- where:
  - $\tilde{e}_{rl} = \hat{e}_r - \hat{e}_l = \epsilon_r - \epsilon_l$
  - $\hat{e}_r$ is the estimation of $\bar{e}_0$ given by the RED
  - $\hat{e}_l$ is the estimation of $\bar{e}_0$ given by the lead filter
  - $\epsilon_M = \tau K_R$

1.4.3 GRED/VS-MRAC analysis representation

- Equivalent Representation



$$
\begin{aligned}
\text{Error Equation:} \quad \bar{e}_0 &= k^* M(s) L(s)[u + \bar{U}] \\
\text{Control Signal:} \quad u &= -f(t) sgn(\bar{e}_0 + \epsilon) \\
\text{Output Estimation Error:} \quad \epsilon &= \alpha(\tilde{e}_{rl})\epsilon_l + [1 - \alpha(\tilde{e}_{rl})]\,\epsilon_r
\end{aligned}
$$

1.5 Convergence of the GRED/VS-MRAC I

- After a finite time $\hat{e}_r(t) = \bar{e}_0(t)$
- K_R is chosen such that $\epsilon_M > \epsilon_l(t) + c, \quad \forall t \geq T_1$
- Since $\alpha(\tilde{e}_{rl}) = \begin{cases} 0, & \text{for } |\tilde{e}_{rl}| < \epsilon_M - c \\ \frac{|\tilde{e}_{rl}| - \epsilon_M + c}{c}, & \text{for } \epsilon_M - c \leq |\tilde{e}_{rl}| < \epsilon_M \\ 1, & \text{for } |\tilde{e}_{rl}| \geq \epsilon_M \end{cases}$
- Hence $\alpha(\tilde{e}_{rl}) = 0, \forall t \geq T_2$
- Therefore $\epsilon = 0$ and the error state converges to zero.

1.5 Convergence of the GRED/VS-MRAC II

The ISpS property

- The key property to demonstrate formally the stability and convergence of the hybrid scheme is as follows
- The system with the causal lead compensator $L_a(s)$ is Input to State practical Stability (ISpS) w.r.t. the a small bounded disturbance $\beta_\alpha = O(\tau)$ at the output of the compensator (see Sec. 1.4)
- Then, with $L_a(s)$ alone, the system state ultimately tends to a small compact set of order $O(\tau)$.

1.6 Simulation Results

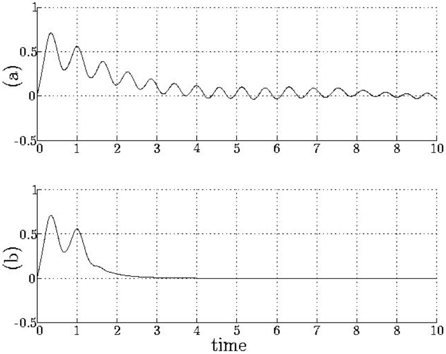
- Plant: $G_p(s) = \frac{1}{(\mu s+1)}\left[\frac{1}{(s+2)(s+1)(s-1)}\right]$, with $\mu = 0.125$
- Model: $M(s) = \frac{4}{(s+2)^3}$
- Input Signal: $r(t) = sin(0.5t)$
- Input disturbance: $d_e(t) = sqw(5t)$
- $L(s)$ Operator: $L(s) = (s+2)^2$

1.6 Simulation Results II

- RED: $\lambda_0 = 3C_3^{1/3},\ \lambda_1 = 1.5C_3^{1/2},\ \lambda_2 = 1.1C_3,\ C_3 = 250$
- Lead Filter: $F(\tau s) = (0.01s + 1)^2$
- Initial Conditions: $y_p(0) = 0, \dot{y}_p(0) = 2,\ \ddot{y}_p(0) = 4$

1.6.1 Plant $n^* = 3$ + unmodeled dynamics

- Tracking error $e_0(t)$

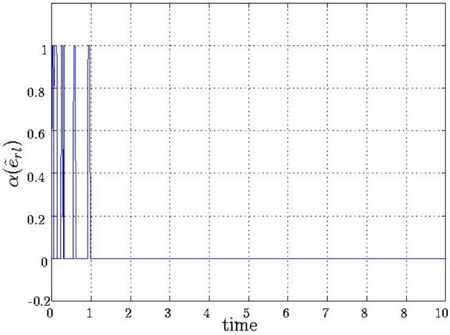


(a) LF/VS-MRAC: (Lead Filter Compensator only)

(b) GRED/VS-MRAC ($\epsilon_M = 400\tau$ e $c = 50\tau$)

1.6.1 Plant $n^* = 3$ + unmodeled dynamics II

- Weighted Switching Function



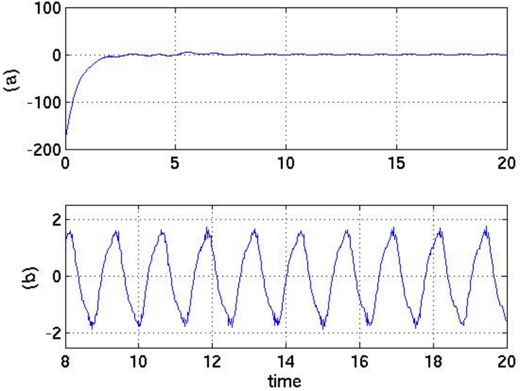
1.7 Experimental Results I

- GRED/VS-MRAC applied to a servomechanism (SRV-02) of angular positioning built by Quanser Consulting.
- Objective: show that the arm can follow a reference signal without significant chattering
- Servomechanism Nominal Model:
  $G_p(s) = \frac{\theta(s)}{V(s)} = \frac{80}{s(\frac{1}{38}s+1)}$
- Control Signal:
  - Modulation Function: $f = 5$ (Maximum input voltage)
  - Boundary Layer (Δ)

1.7.1 Experimental Results: Case 1

- Objective: illustrate the transient behaviour of the GRED/VS-MRAC
- Reference signal: $r(t) = 50sin(5t)$
- Initial Conditions: $y_p(0) = 180; \quad \dot{y}_p(0) = 0$
- Model: $M(s) = \frac{400}{(s+20)^2}$
- $L(s)$ Operator: $L(s) = (s+2)$
- Lead Filter: $F(\tau s) = (0.04s+1)^2$
- RED: $C_2 = 20, \lambda_0 = 1.5\sqrt{C_2}, \lambda_1 = 1.1C_2$
- Boundary Layer: $\Delta = 20;$
- weighted switching function: $\epsilon_M = 30, \; c = 20.$

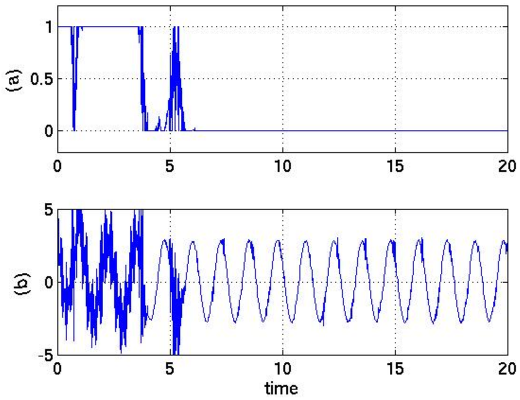
1.7.1 Experimental Results: Case 1 II



(a) Tracking error $e_0(t)$ in degrees

(b) Zoom of $e_0(t)$

1.7.1 Experimental Results: Case 1 III



(a) Time behavior of $\alpha(\tilde{e}_{rl})$

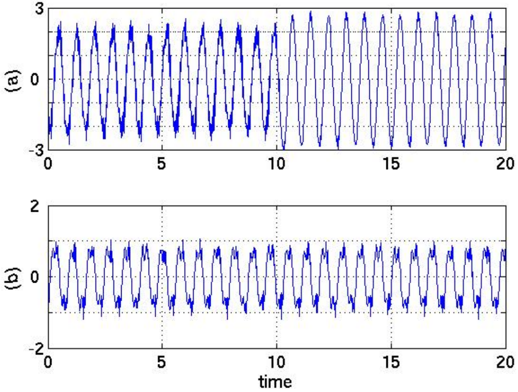
(b) Control signal $u(t)$

1.7.1 Experimental Results: Case 1 IV

- Objective: Comparison between the LF/VS-MRAC and GRED/VS-MRAC
- Methodology:
  - Compare both controllers using the same $\Delta = 15$ $(t \in [0, 10])$
  - Compare both controllers for the same level of chattering
    - the boundary layer of the LF/VS-MRAC is adjusted
    - LF/VS-MRAC $\rightarrow \Delta = 25 \quad (t \in (10, 20])$
- Design Parameters:
  - Reference signal: $r(t) = 70sin(8t)$
  - RED: $\lambda_0 = 5, \lambda_1 = 50$
  - Other design parameters are as in Case 1

1.7.2 Experimental Results: Case 2

- Tracking error $e_0(t)$



(a) LF/VS-MRAC ($t \in [0, 10] \to \Delta = 15$, $t \in (10, 20] \to \Delta = 25$)

(b) GRED/VS-MRAC ($\Delta = 15$)

1.8 Conclusions

- Theoretical analysis for uncertain plants of arbitrary relative degree was developed [Nunes et al.(2009)Nunes, Hsu, and Lizarralde]
- The full error system is globally exponential stable with respect to a residual set;
- After a finite time the estimation of compensated output error $\bar{e}_0$ is given exclusively by the RED;
- The full error state converges asymptoticly to zero;
- Simulation and Experimental results validate the analysis and illustrate the applicability of the hybrid scheme in real conditions

The players

(Nunes, Lizarralde, Cunha, Oliveira and Jacoud (circa 2010))

2.1 Introduction

- This section: motivated by the stronger robustness of HGOs w.r.t. unmodeled dynamics, as compared to the ordinary linear lead filters used in a previous work (GRED) $\Rightarrow$ HGO
  - Uncertain MIMO linear systems with non-uniform arbitrary relative degree
  - Combine through a switching law:
    - MIMO HGO
    - Locally exact nonlinear MIMO differentiator (MIMO RED)
- Main Objectives:
  - Exact tracking using only output feedback
  - Global stability and convergence properties
- Main idea: Switching Adaptation to select between a MIMO HGO and the MIMO RED

2.2 Uncertain MIMO LTI plants

- Uncertain MIMO LTI plant with m inputs and m outputs

$$\dot{x}_p = A_p x_p + B_p[u + d]\,, \qquad y_p = H_p x_p$$

- Assumptions:
  - Known non-uniform arbitrary relative degree $\{\rho_1, \ldots, \rho_m\}$
  - The high frequency gain (HFG) matrix K_p is nonsingular
  - A matrix S_p is known such that $-K_p S_p$ is diagonally stable: $\exists D > 0$ (diagonal) such that $D(K_p S_p) + (K_p S_p)^T D > 0$
  - the uncertain disturbance $d(x,t)$ is assumed bounded by $|d(x,t)| \leq k_x|x| + k_d$, where $k_x, k_d \geq 0$ are *known* scalars.

2.2.1 Reference Model

- Reference Model: $y_M = W_M(s)r$
  $W_M(s) = \text{diag}\left\{(s+\gamma_1)^{-1}, \ldots, (s+\gamma_m)^{-1}\right\} L^{-1}(s), \;\; \gamma_j > 0$
- $L(s) = \text{diag}\{L_1(s), \ldots, L_m(s)\}$
- $L_i(s) = s^{(\rho_i - 1)} + l^{[i]}_{\rho_i - 2} s^{(\rho_i - 2)} + \cdots + l^{[i]}_1 s + l^{[i]}_0$

Arbitrary *Vector* Relative Degree

- Systems with non-uniform arbitrary vector relative degree $\{\rho_1, \ldots, \rho_m\}$
- Main Idea: use an operator $L(s)$ to generate an ideal sliding variable σ with uniform vector relative degree one

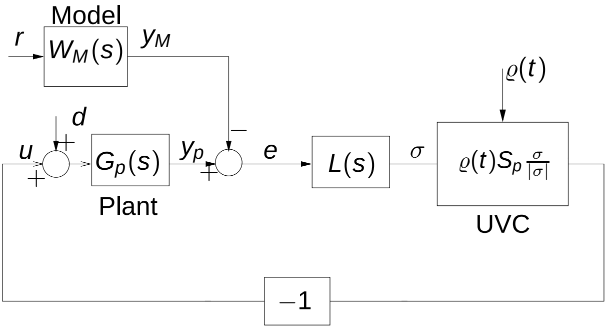
$$\sigma = L(s)e = \begin{bmatrix} e_1^{(\rho_1-1)} + \cdots + l_1^{[1]}\dot{e}_1 + l_0^{[1]}\, e_1 \\ \vdots \\ e_m^{(\rho_m-1)} + \cdots + l_1^{[m]}\dot{e}_m + l_0^{[m]} e_m \end{bmatrix} .$$

$$\sigma = L(s)W_M(s)K_p\left[u - \bar{U}\right] .$$

$$L(s)W_M(s) = \operatorname{diag}\left\{(s+\gamma_1)^{-1}, \ldots, (s+\gamma_m)^{-1}\right\}, \ \gamma_j > 0 .$$

Ideal Lead compensator for UVC

- Close the control loop with a modulated UVC based on the ideal sliding variable σ: $u = -\varrho(t) S_p \frac{\sigma}{|\sigma|}$,



- However, σ is not directly available to implement u

UVC with GRED based on MIMO lead filter

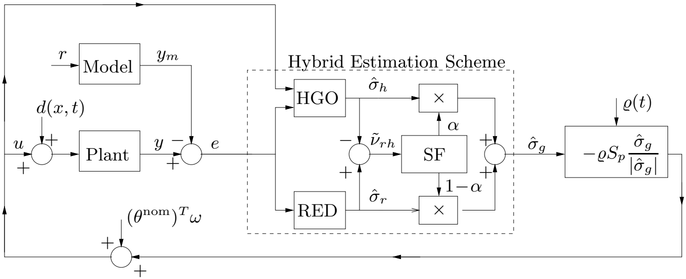
- Use a MIMO lead filter

  $$\hat{\sigma}_l = L_a(s)e, \quad L_a(s) = L(s)F^{-1}(\tau s), \quad F(\tau s) = \mathrm{diag}\{(\tau s+1)^{\rho_j}$$

  combined with MIMO RED to obtain an estimate of σ [Nunes et al.(2014)Nunes, Peixoto, Oliveira, and Hsu]
- The error system is globally practically stable and is ISpS w.r.t a bounded disturbance at the output of the lead filter
- Global exponential stability and finite time convergence of the sliding vector can be proved.
- Caveat: Unmodeled dynamics of the plant destroy ideal sliding mode loop: chattering prone

2.3 Unit Vector Control with HGO based GRED

- Use MIMO HGO/RED to obtain an estimate of σ (Hsu, Nunes, Oliveira and Peixoto 2015 (RASM))



- An ideal sliding mode loop can be *preserved* with unmodeled dynamics $\rightarrow$ chattering avoidance

UVC using a MIMO HGO: Advantages

- Ideal Sliding Loop around the discontinuous function preserved in spite of unmodelled dynamics. Chattering alleviation is thus expected.
- HGO structure allows a more natural extension to nonlinear plants (nonlinearities depending on unmeasured states)
- The error system is globally practically stable and is ISpS w.r.t a bounded disturbance in the *output* $\hat{\sigma}_h$
- In spite of the high-gain observer, global stability is guaranteed with a *peaking free control signal*.

2.3.1 Switching Law for HGO-GRED-UVC

Very similar to the SISO case. The key property is the HGO compesator leadinding to ISpS property w.r.t. to HGO output disturbance. The properties and strategies below ensue.

- Main Idea:
  - Globally drive the error state into a compact set D_R where the convergence of the MIMO RED can be guaranteed
  - Ensure that ultimately only the MIMO RED is used
- Problem: it is not possible to know when the error state enters the set D_R (the error state is not available)
- Solution:
  - Select the estimator using the difference between their estimations
  - Make the Hybrid Estimator equivalent to the MIMO HGO plus a bounded disturbance ϵ_M

2.4 Simulation Results I

- Consider a nonlinear plant with non-uniform relative degree ($\rho_1 = 2$, $\rho_2 = 3$) described by
$$G(s) = \begin{bmatrix} \frac{\kappa(s+2)}{(s-1)(s+1)(s+3)} & \frac{\kappa}{(s+1)(s+2)} \\ \frac{1}{(s-1)(s+1)(s+3)^2} & \frac{1}{(s+1)(s+2)(s+3)} \end{bmatrix},$$
where constant $\kappa \in [4, 10]$ is uncertain and $K_p = \begin{bmatrix} \kappa & \kappa \\ 0 & 1 \end{bmatrix}$ is the HFG matrix.
- disturbance
$d(x) = \left[0.2\cos(t)\sin(x_2\, x_3)|x_4| \quad \frac{1}{2\pi}\left(e^{-|x_5|}|x_1| + |x_2|\right)\right]^T$.
[Emelyanov et. al. 1992].

Plant and Reference Model

- The reference signal and model are chosen as $r = [\sin(t) \;\; \sin(0.5t)]$ and $W_m(s) = \text{diag}\left\{\frac{1}{(s+1)^2}, \frac{1}{(s+1)^2(s+2)}\right\}$
- Relative Degree: $\rho_1 = 2, \;\; \rho_2 = 3$
- HFG: $K_p = \begin{bmatrix} \kappa & \kappa \\ 0 & 1 \end{bmatrix}$
- Model:
  $$y_M = W_M(s) r, \quad W_M(s) = \begin{bmatrix} \frac{1}{(s+1)^2} & 0 \\ 0 & \frac{1}{(s+1)^2(s+2)} \end{bmatrix}$$
- Reference Signal $r(t) = [\sin(t) \;\; \sin(0.5t)]$

Tracking Performance and Switching Function

Results

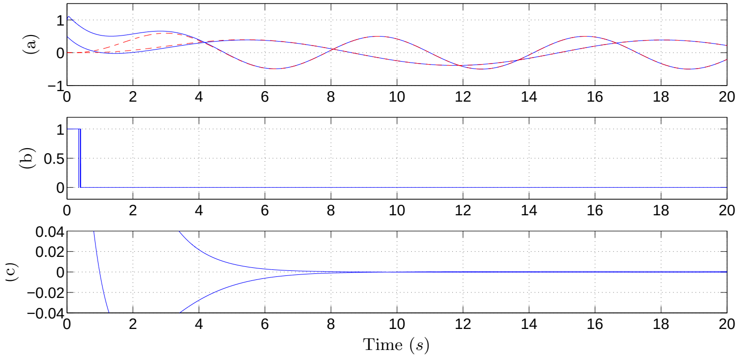


Fig. 2. (a) Tracking performance: y (—) and y_m (- -); (b) Time behavior of switching function (SF) $\alpha(\cdot)$; (c) Zoom of tracking errors $e(t)$

Unmodeled dynamics and high frequency noise effects

- Unmodeled dynamics of actuator with transfer matrix given by

$$G_a(s) = \begin{bmatrix} \frac{1}{\mu s+1} & 0 \\ 0 & \frac{1}{\mu s+1} \end{bmatrix}, \qquad \mu = 0.1 \tag{1}$$

- Noise: output is corrupted by a high frequency measurement noise, i.e.,

$$y_{noise}(t) = \begin{bmatrix} y_1(t) + 0.01\sin(200t) \\ y_2(t) + 0.01\cos(200t) \end{bmatrix}$$

Results with unmodeled dynamics and output noise

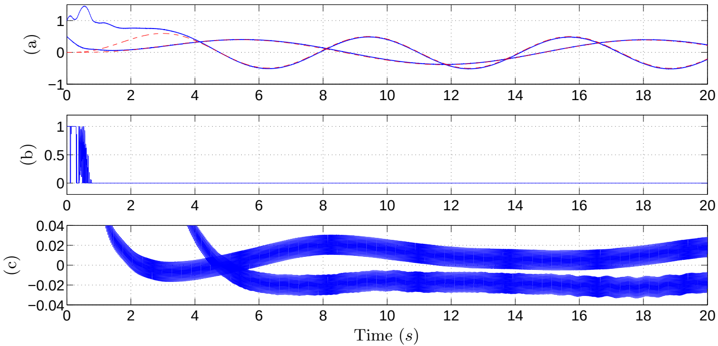


Fig. 3. (a) Tracking with unmodeled dynamics and noise: y (—) and y_m (- -); (b) Time behavior of switching function (SF) $\alpha(\cdot)$; (c) Zoom of tracking errors $e(t)$

2.5 Conclusions

- An output-feedback sliding mode exact tracking controller based on a hybrid estimator was described for uncertain MIMO systems with non-uniform arbitrary relative degree
- A class of nonlinear state-dependent disturbance was allowed.
- The MIMO hybrid estimation scheme combines:
  - Locally exact differentiator MIMO RED
  - MIMO HGO that provides global practical stability properties
- MIMO GRED based UVC:
  - Asymptotic exact tracking is proved
  - Robustness to unmodeled dynamics and output noise is verified.

The players

(Nunes, Lizarralde, Cunha, Oliveira and Jacoud (circa 2010)) hideallsubsections

3.1 Introduction

- Model Reference Robust Control (MRRC):
  - Nonlinear systems / Relative degree one
    L. Yan and J. Xu, 2004
- Sliding Mode Control (SMC):
  - Uncertain nonlinear systems / state feedback
    S. Drakunov, 1993
  - First order nonlinear systems
    G. Bartolini, A. Ferrara and L. Giacomini 2003
  - Uncertain linear systems / relative degree one
    L. Yan, L. Hsu, R.R. Costa and F. Lizarralde, 2003

Problem Statement I

- Uncertain nonlinear SISO plant:

$$\boxed{y = G_p(s)[u + d_e(y,t)] = k_p \frac{N_p(s)}{D_p(s)}[u + d_e(y,t)]}$$

- Assumptions on the plant:

(A1) Standard MRAC Assumptions for $G_p(s)$

(A2) $G_p(s) \to$ known relative degree n^*

(A3) The sign of $k_p \neq 0$ (HFG) is unknown

Problem Statement II

(A4) d_e: *locally Lipschitz* (y) & piecewise continuous (t)

$$\boxed{|d_e(y,t)| \leq \bar{d}_e(y,t) \leq \Psi(|y|) + k_\Psi\,, \quad \forall (y,t)\,,}$$

where $\Psi \in \mathcal{K}_\infty$ and $k_\Psi > 0$ is a constant

- Finite-time escape is not precluded $\rightarrow t \in [0, t_M)$
- Reference Model (of order n^*)

$$\boxed{y_m = M(s)r = \frac{k_m}{D_m(s)} r}$$

  - $r(t)$: piecewise continuous & uniformly bounded
- Control objective:
  - Global or semi-global stability

Problem Statement III

- Asymptotic convergence of output error

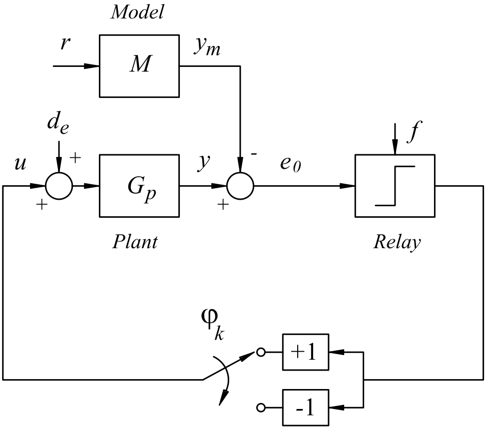
  $$e_0(t) := y(t) - y_m(t)$$

  to zero as $t \to \infty$ *(exact tracking)*

- Output error equation: $e_0 = k^* M(s)[u - u^*]$
  - $k^* = k_p / k_m$
  - $u^*(t) := {\theta^*}^T \omega(t) - W_d(s) * d_e(t)$

3.2 Relative degree one

- Control Scheme ($n^* = 1$)



3.2 Relative degree one II

- Control Law ($n^* = 1$)

$$u = \begin{cases} u^+ = -f(t)\,\mathrm{sgn}(e_0) & , \quad t \in T^+ , \\ u^- = f(t)\,\mathrm{sgn}(e_0) & , \quad t \in T^- \end{cases}$$

  - $f \rightarrow$ Modulation function
  - T^+ and T^- have the form $[t_k, t_{k+1}) \cup \cdots \cup [t_j, t_{j+1})$
  - t_k or t_j denotes the switching time for u
  - For simplicity $M(s) = \frac{k_m}{(s+a_m)}$ $(a_m, k_m > 0)$

3.2 Relative degree one III

- An upper bound for the output error e_0
  - *Comparison Theorem*

$$\boxed{|e_0(t)| \leq e^{-a_m(t-\bar{t}_0)}|e_0(\bar{t}_0)| + c_0 e^{-\delta_0 t}}$$

where $c_0, \delta_0 > 0$ are unknown and $\bar{t}_0 \rightarrow$ initial time

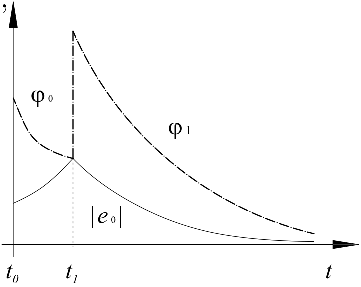
- *Monitoring Function* $(n^* = 1)$

$$\boxed{\varphi_k(t) = e^{-a_m(t-t_k)}|e_0(t_k)| + (k+1)e^{-\frac{t}{k+1}}}$$

$$t \in [t_k, t_{k+1}),\ t_0 := 0,\ (k = 0, 1, \ldots)$$

- The *switching time* t_k for u

$$t_{k+1} = \begin{cases} \min\{t > t_k : |e_0(t)| = \varphi_k(t)\}, & \text{if it exists}\,, \\ t_M, & \text{otherwise} \end{cases}$$



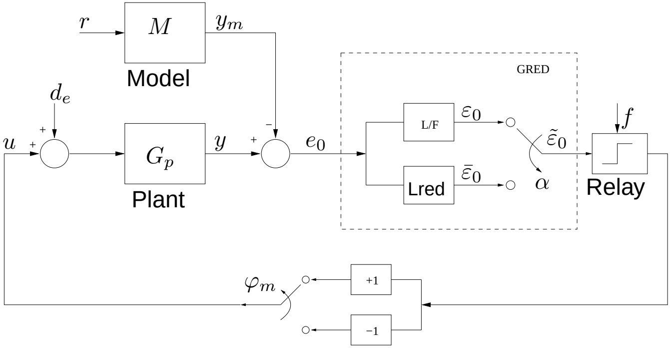
3.2 Relative degree one V

- Stability Results
  - **Theorem 1** and **Corollary 1**
    1. *The complete error state and the tracking error e_0 will converge to zero at least exponentially*
    2. *The control direction switching stops at a correct sign*

    [*Reverse Dynamics Argument*]

3.3 Arbitrary relative degree

- Control Scheme ($n^* > 1$)



3.3 Arbitrary relative degree II

- Control Law ($n^* > 1$)

$$u = \begin{cases} u^+ = -f(t)\,\mathrm{sgn}(\tilde{\varepsilon}_0) & , \quad t \in T^+ , \\ u^- = f(t)\,\mathrm{sgn}(\tilde{\varepsilon}_0) & , \quad t \in T^- \end{cases}$$

3.3 Arbitrary relative degree III

- GRED $\boxed{\tilde{\varepsilon}_0 = (1-\alpha)\bar{\varepsilon}_0 + \alpha\varepsilon_0}$ [Convex Combination]

$$\alpha(\tilde{e}) = \begin{cases} 0, & |\bar{\varepsilon}_0 - \varepsilon_0| < \tau k_R \quad (RED), \\ 1, & |\bar{\varepsilon}_0 - \varepsilon_0| \geq \tau k_R \quad (Lead), \end{cases}$$

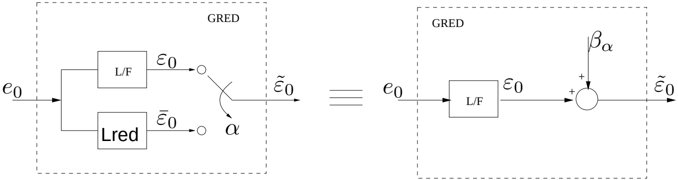
where $k_R > 0$ is a design constant

3.3 Arbitrary relative degree IV

- Equivalent structure for the nonlinear hybrid filter

$$\boxed{\tilde{\varepsilon}_0 = \varepsilon_0 + \beta_\alpha}$$

where $\beta_\alpha = \tau k_R$ is $\mathcal{O}(\tau)$



- In order to simplify the analysis: $\tilde{\varepsilon}_0 = \varepsilon_0$ $(\alpha = 1)$

3.3 Arbitrary relative degree V

- Auxiliary error $\boxed{\varepsilon_0 = k^* ML\,[u - u^*] + \beta_{\mathcal{U}} + e_F^0}$
- Difficulties:
  1. Disturbance $\beta_{\mathcal{U}} := k^* ML(s)\,[1 - F(\tau s)]\, F^{-1}(\tau s) * (u - u^*)$
  2. Decaying *peaking term*: $|e_F^0| \leq \frac{R_a}{\tau^N} e^{-\lambda_a t}$
     R_a depends on IC's only (τ-independent)
- Solutions:
  1. $|\beta_{\mathcal{U}}(t)| \leq \bar{\beta}_{\mathcal{U}}(t) = \tau W_\beta(s) * f(t) \Rightarrow$ (τW_β is a FOAF)
  2. Decaying peaking term: $|e_F^0| \leq R_a e^{-\lambda_a(t - t_e(\tau))}$
     $\Rightarrow$ *$t_e(\tau)$ is the peak extinction time*

- *Lemma 1*: An upper bound for ε_0 ($\mathrm{sgn}(k_p)$ correct)

$$\boxed{\begin{aligned} |\varepsilon_0(t)| \; &\leq (|\varepsilon_0(t_k)| + |\beta_{\mathcal{U}}(t_k)|)e^{-a_m(t-t_k)} \\ &+ (2R_a e^{\bar{\lambda}_a \bar{t}_e})e^{-\bar{\lambda}_a t} + 2\|(\beta_{\mathcal{U}})_{t,\bar{t}_e}\| \end{aligned}}$$

- *Monitoring Function* $(n^* > 1)$

$$\boxed{\begin{aligned} \varphi_k(t) \; &:= (|\varepsilon_0(t_k)| + |\bar{\beta}_{\mathcal{U}}(t_k)|)e^{-a_m(t-t_k)} \\ &+ a(k)e^{-\lambda_c t} + 2\|(\bar{\beta}_{\mathcal{U}})_t\| \end{aligned}}$$

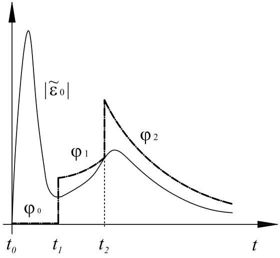
$a(k) > 0$ is any unbounded monotonically increasing sequence

3.3 Arbitrary relative degree VII

- The *switching time* t_k for u

$$t_{k+1} = \begin{cases} \min\{t > t_k : |\tilde{\varepsilon}_0(t)| = \varphi_k(t)\}, & \text{if it exists}, \\ t_M, & \text{otherwise} \end{cases}$$

where $k \geq 1$, $t_0 := 0$, $t_1 := \bar{t}_e$ and $\varphi_0(t) = 0$, $\forall t \in [t_0, t_1)$



- Stability Results $z^T := [X_e^T, x_f]$
  - **Proposition 2** *The complete error system is bounded by*

    $$|z(t)| \leq k_{z0}|z(0)| + k_a \sum_{i=1}^{k} a(i) + \mathcal{O}(\tau)$$

    $\forall t \in [0, t_M)$ *where* k_{z0}, k_a *are positive constants.*

- **Theorem 2**
  1. *The control sign switching stops*
  2. *Semi-GAS (or GAS) - compact set*
  3. *Ultimately exponential convergence* $\mathcal{O}(\tau)$

- Chattering Avoidance and Exact Tracking
  - Levant's RED algorithm $(\eta_i(t) \rightarrow e_0^{(i)}(t),\ i=0,1,2)$:

$$
\begin{aligned}
\dot{\eta}_0 &= v_0, \\
v_0 &= -\lambda_0 \left|\eta_0 - e_0\right|^{\frac{2}{3}} \operatorname{sgn}(\eta_0 - e_0) + \eta_1, \\
\dot{\eta}_1 &= v_1, \\
v_1 &= -\lambda_1 \left|\eta_1 - v_0\right|^{\frac{1}{2}} \operatorname{sgn}(\eta_1 - v_0) + \eta_2, \\
\dot{\eta}_2 &= -\lambda_2 \operatorname{sgn}(\eta_2 - v_1)
\end{aligned}
$$

- **Corollary 2** *With the nonlinear hybrid filter:*
  1. *Theorem 2 holds*
  2. *Exact tracking is achieved (finite time or exp.)*
  3. *Switchings stops at the correct sign*

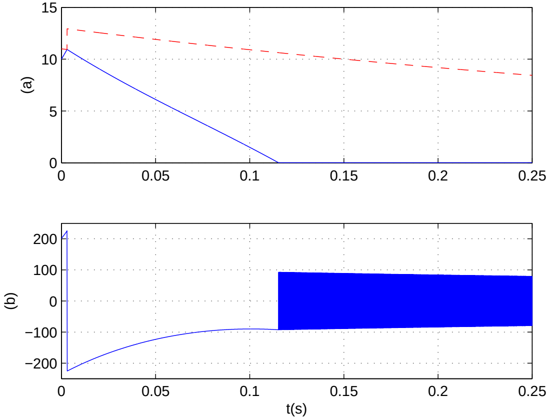
- $n^* = 1$ case: $G_p(s) = \frac{s+1}{(s+2)(s-1)}$

Table: Controller Parameters

IC's	$y(0) = 10,\ \dot{y}(0) = 2$
Model	$M(s) = \frac{2}{(s+2)},\ r(t) = \sin(t)$
Disturbance	$d_e(y,t) = y^2 + sqw(5t)$
Monitor	$a(k) = k+1,\ a_m = 2,\ b(k) = \frac{1}{k+1}$

3.4 Simulation and experiments II

- $n^* = 1$ case: tracking error e_0, monitoring function φ_k and control signal u



3.4 Simulation and experiments III

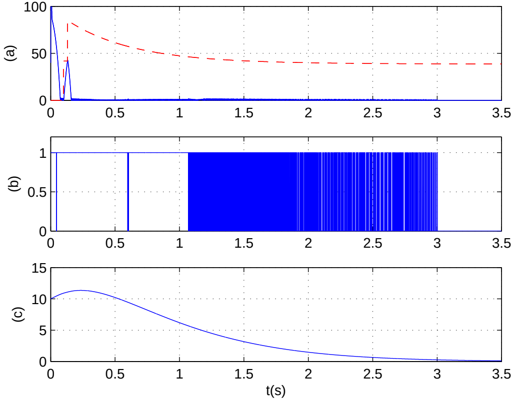
- $n^* > 1$ case: $G_p(s) = \frac{1}{(s+2)(s+1)(s-1)}$ $(n^* = 3)$

Table: Controller Parameters

IC's	$y(0)=10,\ \dot{y}(0)=10,\ \ddot{y}(0)=10$
Model	$M(s) = \frac{4}{(s+2)^3},\ r(t) = \sin(t)$
Disturbance	$d_e(y,t) = y^2 + sqw(5t)$
Monitor	$a(k) = k+1,\ a_m = 2,\ \lambda_c = 1,\ t_1 = \bar{t}_e = 0.1s$
Lead	$\frac{L}{F} = \frac{(s+2)^2}{(\tau s+1)^2},\ \tau = 10^{-3}$
RED	$\lambda_0 = 3C_3^{\frac{1}{3}},\ \lambda_1 = 1.5C_3^{\frac{1}{2}},\ \lambda_2 = 1.1C_3,\ C_3 = 250,\ k_R = 600$

3.4 Simulation and experiments IV

- $n^* > 1$ case: auxiliary error ε_0, monitoring function φ_k, switching law α and tracking error e_0



Experiments

- Experimental results were made with a DC motor position control ($n^* = 2$) (Peixoto et al CDC2006)
- They confirm the efficacy of the monitoring function approach
- The ultimate RED compensator was verified to RED-lead to less chattering in the control signal than the linear-Lead.

3.5 Multivariable and Nonlinear systems I

- The monitoring function can be extended to classes of multivariable nonlinear systems.
  - (ACC2007, NY): nonlinear systems-linearly bounded in unmeasured states, MIMO, decaying monitoring function, unknown control direction(UCD);
  - (Peixoto, Leite, Oliveira and Hsu ACC2009, St Louis): MIMO nonlinear systems general normal form, norm observer, experimental results with Zebra-0 visual servoing, UCD;
  - MIMO experimental results, visual servoing [Oliveira, Leite, Peixoto and Hsu(2014)].

3.5 Multivariable and Nonlinear systems II

Classes of NL and MIMO systems considered

- Class 1 - NL-SISO (Oliveira-Peixoto-Hsu ACC2007)

$$\dot{x}_p = A_p + \phi(x_p, t) + B_p u \,, \quad y = C_p x_p \,, \tag{2}$$

where,

- $f_p(x_p, t) = A_p x_p + \phi(x_p, t)$, $x_p \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the measured output
- $\phi : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is regarded as uncertain state dependent, possibly unmatched.
- ϕ satisfies $|\phi(x_p, t)| \leq k_x |x_p| + \varphi(y, t)$, $\forall x_p, t$ and
- $\varphi(y, t) \leq \Psi_\varphi(|y|) + k_\varphi$, where $\Psi_\varphi \in \mathcal{K}_\infty$ and $k_\varphi > 0$ is constant.

3.5 Multivariable and Nonlinear systems III

- Class 2 - NL MIMO (Oliveira-Peixoto-Leite-Hsu ACC2009)

$$\dot{\eta} = \phi_0(\eta, y, t)\,, \tag{3}$$

$$\dot{y} = K_p u + \phi_1(\eta, y, t)\,, \tag{4}$$

where $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^m$ is considered as the measured output and the states $\eta \in \mathbb{R}^{n-m}$ of the η-subsystem, referred to as an "inverse system", are not assumed to be measurable.

3.5 Multivariable and Nonlinear systems IV

Techiques for dealing with nonlinear uncertainties

- The use of norm-observers base on FOAF's is instrumental.
- The unmeasured state can be norm bounded by the norm-observers in Class 1 and in Class 2.

- Example: in Class 1 $|x_p(t)|$ can be bounded as follows. One has $|x_p(t)| \leq \hat{x}_p(t) + \hat{\pi}(t)$, where

$$\hat{x}_p(t) = \frac{1}{s+\lambda_x}\left[c_1 \varphi(y,t) + c_2|\omega(t)|\right], \tag{5}$$

with $c_1, c_2, \lambda_x > 0$ The exponentially decaying term $\hat{\pi}$ accounts for initial conditions, see [24].

3.5 Multivariable and Nonlinear systems VI

- Class 2, such norm observer requires an ISS property for the inverse system (zero-dynamics) (3) w.r.t. y.

3.5 Multivariable and Nonlinear systems VII

Switching Strategy for MIMO systems

- In the scalar case the switching is simply betweem $+1$ and -1. Very simple!
- What if we have a matrix gain K_p?
- Strategy: K_p (nonsingular) should be transformed to a anti-Hurwitz matrix by premultiplying the control vector by some matrix $\bar{S}$ such that $-K_p\bar{S}$ becomes Hurwitz

3.5 Multivariable and Nonlinear systems VIII

- We then assume there exists a finite index set $\mathcal{Q}$ of *known* matrices $S_q \in \mathbb{R}^{m\times m}$ such that $-K_pS_q$ is Hurwitz for some $q \in \mathcal{Q}$.
- The multivariable switching scheme is realized by cycling through the elements of the finite index set $\mathcal{Q}$ [?] so that stability and the tracking objective are achieved

3.5 Multivariable and Nonlinear systems IX

Example: Robotics uncalibrated visual servoing

(MIMO) Consider a visual servoing system ($n^* = 1$) for a robot manipulator [?]. Neglecting the robot dynamics (kinematic robot), the end-effector (target) motion in the camera image coordinate system is modelled by:

$$\dot{y}(t) = K_p[u + \phi]\,, \;\; K_p = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix} \begin{bmatrix} \cos(\psi) & \sin(\psi) \\ -\sin(\psi) & cos(\psi) \end{bmatrix}$$

where $y(t) \in \mathbb{R}^2$ denotes the end-effector position vector in the camera space, ψ represents the rotation angle of the camera framework with respect to the task-space framework and $h_i > 0$ ($i = 1, 2$) are uncertain scaling factors, belonging to a known compact set.

3.5 Multivariable and Nonlinear systems X

Note that K_p is nonsingular. We have included an input disturbance $\phi(y) = y^2$ satisfying (A3) (with $k_x = 0$), in order to illustrate the disturbance rejection property of the proposed scheme.

3.5 Multivariable and Nonlinear systems XI

The desired dynamics and target position trajectory in the camera space is defined by the reference model:

$$\dot{y}_m(t) = -y_m(t) + r\,, \;\; y_m, r \in \mathbb{R}^2\,.$$

The error dynamics $e_0 = y - y_m$ is given by

$$\dot{e}_0(t) = -e_0(t) + K_p(u - u^*)\,, \tag{6}$$

where $u^* = -\phi(y) - K_p^{-1}(y - r)$.
The finite set of matrices S_q, $q \in \mathcal{Q} = \{0, 1, 2, 3\}$ is chosen as:

$$S_0 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, S_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

3.5 Multivariable and Nonlinear systems XII

For any ψ, $-K_pS_q$ is Hurwitz for some S_q. So, the usual restriction $|\psi|<90^o$ [?] can be removed. Here, the plant initial conditions and reference signals are $y_1(0)=y_2(0)=0$, $r_1(t)=2\cos(t)$ and $r_2(t)=2\sin(t)$, $h_1=h_2=1$ and $\psi=90^o$.

3.5 Multivariable and Nonlinear systems XIII

- Experimental setup

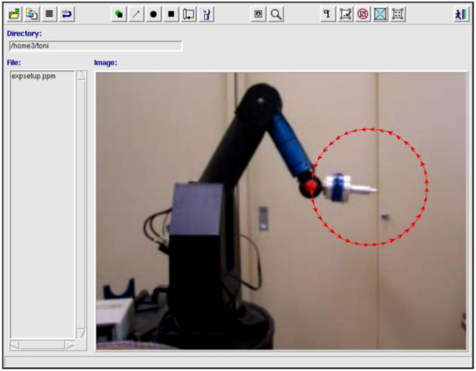


Fig. 4. Experimental station.

3.5 Multivariable and Nonlinear systems XIV

- Monitoring function, error and switchings

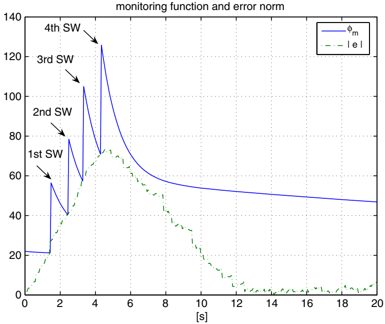


Fig. 5. Monitoring function (φ_m) and error norm ($|e|$).

The monitoring function, $|e_0|$ and switchings are shown. Note that, at the 3rd switching ($k = k^* = 3$), the correct S_3 matrix is selected ($-K_p S_3$ is Hurwitz).

3.5 Multivariable and Nonlinear systems XV

- Experimental visual tracking results

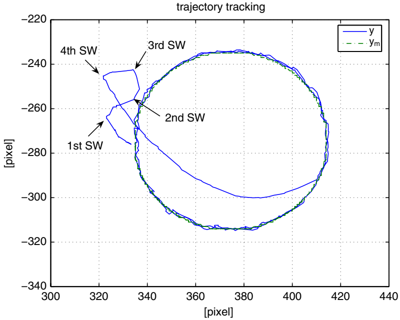


Fig. 7. Trajectory tracking in the image frame.

The target trajectory is illustrated above, where one notices that the tracking is achieved even for $\psi = 90^o$.

3.6 Conclusions

- A sliding mode output-feedback model-reference controller for a class of uncertain nonlinear systems was proposed
- Multivariable plants with **unknown control direction** and **arbitrary relative degree** were covered:
  - A monitoring function $\rightarrow$ **unknown control direction**
  - Nonlinear hybrid filter $\left\{ \begin{array}{c} \textbf{arbitrary relative degree} \\ \textbf{exact tracking} \end{array} \right.$
- GAS or semi-GAS with respect to a compact set
- Ideal sliding mode is reached in finite time.

The players

(Cunha, Oliveira, Jacoud, Leite (circa 2010))

4. Adaptive Sliding Mode Control via X-equivalent control

4.1 Introduction

- Sliding mode control (SMC) is robust to disturbances.
- Caveats:
  - Bounds for disturbances must be known;
  - Overestimating disturbances increases chattering.
- Solution:
  - SMC with adaptive modulation function.

4.1 Introduction II

- Adaptive SMC strategies review:

1. Monotonically increasing gains
   [Yan, Hsu, and Xiuxia(2006), Oliveira, Leite, Peixoto and Hsu(2014), Moreno, Negrete, Torres-González, and Fridman(2016)]
   Problem: Disturbances may be overestimated.
2. Increasing and decreasing gains
   [Plestan, Shtessel, Brégeault, and Poznyak(2010), Bartolini, Levant, Plestan, Taleb, and Punta(2013), Estrada, Plestan, and Allouche(2013)]
   Problem: Sliding modes may fail temporarily.
3. Equivalent control
   [Bartoszewicz(1989), Utkin and Poznyak(2013), Edwards and Shtessel(2016)]

4.1 Introduction III

Benefits of equivalent control approaches

- Provides an estimate of the disturbance once sliding motion is established;
- Control gain is updated according to disturbance amplitude;
- Less conservative control amplitude reduces chattering and power loss.

A new adaptive SMC approach

extended equivalent control.

4.2 Extended Equivalent Control

- Equivalent control defined on the sliding mode

$$\sigma(x(t), t) = 0$$

- Extended equivalent control valid on and outside the sliding mode [9].

4.2 Extended Equivalent Control II

- Consider the nonlinear system:

$$\dot{x} = f(x,t) + B(x,t)u$$

- Extended equivalent control:

$$u_{\mathrm{eq}}(t) = -[GB(x(t),t)]^{-1}\left[Gf(x(t),t) + \frac{d}{dt}\sigma(x(t),t)\right]$$

where $G = \frac{\partial}{\partial x}\sigma(x(t),t)$.

- On the sliding mode $\sigma(x(t),t)=0 \implies \frac{d}{dt}\sigma(x(t),t)=0$, then, the *extended equivalent control* is the usual *equivalent control*:

$$u_{\mathrm{eq}}(t) = -[GB(x(t),t)]^{-1}Gf(x(t),t)\,.$$

- Outside the sliding mode the *extended equivalent control* is a piecewise-continuous signal given by the *control law*:

$$u_{\mathrm{eq}}(t) = u(t)\,.$$

- Average Control
  - Problem: *extended equivalent control* not available.
  - Alternative: *average control* given by the low-pass filter:

$$\tau \dot{u}_{\mathrm{av}} = -u_{\mathrm{av}} + u \,, \qquad \tau > 0 \,.$$

  - If τ is small enough, then

$$u_{\mathrm{eq}}(t) \approx u_{\mathrm{av}}(t) \,, \quad \forall t \geq 0 \,.$$

4.3 Plant description (nonlinear)

$$\begin{aligned}
\dot{\eta} &= f_0(\eta, \xi, t)\,, \\
\dot{\xi}_1 &= \xi_2\,, \\
&\vdots \\
\dot{\xi}_{r-1} &= \xi_r\,, \\
\dot{\xi}_r &= f(x,t) + g(x,t)\,[u + d(t)]\,,
\end{aligned}$$

- $x = [\eta^T,\ \xi^T]^T \in \mathbb{R}^n$ is the state; $\eta \in \mathbb{R}^{n-r}$; $\xi \in \mathbb{R}^r$; $\xi = [\xi_1\,, \ldots\,, \xi_r]^T \in \mathbb{R}^r$;
- $u \in \mathbb{R}$ is the control input;
- $d(t) \in \mathbb{R}$ is an exogenous input disturbance.

Assumptions on the plant

- Usual assumptions:
  - The input gain is positive: $g(x,t) > \underline{g} > 0$,
  - The internal dynamics $\dot{\eta} = f_0(\eta, \xi, t)$ is *input-to-state stable (ISS)* with respect to ξ.

4.4 Assumptions on the disturbance I

(A1) The input disturbance $d(t)$ is:

- unknown,
- locally integrable and
- norm bounded by $\boxed{|d(t)| \leq \bar{d},\ \forall t,}$ where $\boxed{\bar{d} \geq 0}$ is an *unknown* scalar.
- Moreover, there exist *known* constants $\boxed{c_f > \gamma_f > 0}$ and $\boxed{\tau > 0}$ such that

$$|d(t)| \leq c_f e^{-\gamma_f t} * \left| \tau^{-1} e^{-\frac{t}{\tau}} * d(t) \right| , \quad \forall t \,.$$

4.4 Assumptions on the disturbance II

- Notes on (A1) :
  - Discontinuous disturbances are allowed;
  - For $\tau \to +0$ the averaging filter can be neglected;
  - The average control gives an estimate of the input disturbance during the sliding mode;
  - The *modulation function* must dominate the disturbance:

$$\boxed{\varrho(t) = c_f e^{-\gamma_f t} * |u_{\rm av}(t)| \approx c_f e^{-\gamma_f t} * |d(t)| > |d(t)|\,.}$$

  - Unbounded disturbances are allowed, such as:

$$\boxed{\frac{d\,[\ln|d(t)|]}{dt} \leq c_f - \gamma_f \quad (> 0)\,,}$$

$$\boxed{|d(t)| < d_0\, e^{(c_f - \gamma_f)t}\,, \quad d_0 > 0\,.}$$

4.5 Sliding Mode Control

- Sliding variable:

$$\boxed{\sigma = S\xi\,,}$$

where $S = [s_0\,, \ldots\,, s_{r-1}]$ is such that $s_{r-1}\lambda^{r-1} + \cdots + s_1\lambda + s_0$ is a Hurwitz polynomial.

4.5 Sliding Mode Control II

- Therefore: $$\boxed{\dot{\sigma} = f_\sigma(x,t) + g(x,t)\left[u + d(t)\right] .}$$

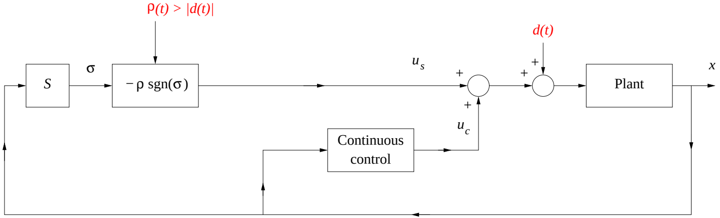
- Control law:

$$u = u_c + u_s \,,$$

$$u_c = -\frac{f_\sigma(x,t)}{g(x,t)} \,,$$

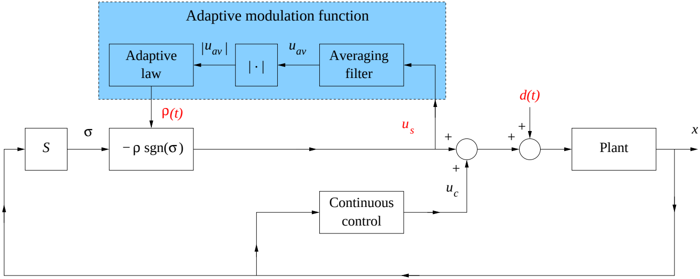
$$u_s = -\varrho(t)\mathrm{sgn}(\sigma) \,.$$

4.5.1 Conventional SMC



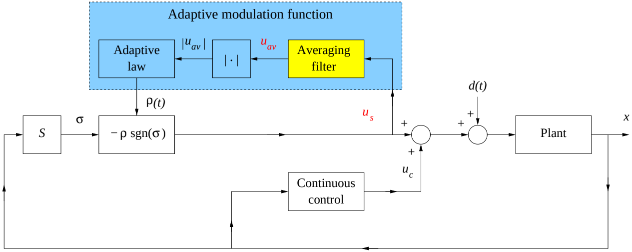
- **Problem:** synthesis of modulation function $\varrho(t)$.
- If $\varrho(t) > |d(t)|, \quad \forall t \geq 0,$ then, the sliding surface $\sigma = 0$ will be reached in finite time.

4.5.2 Adaptive SMC



- Adaptive synthesis of modulation function $\varrho(t)$.

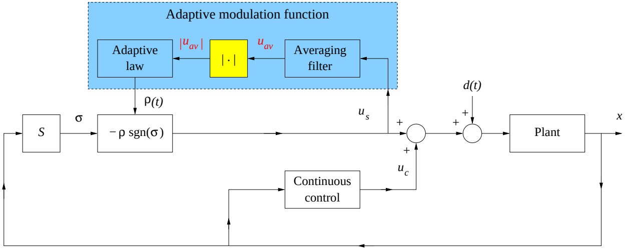
4.5.3 Adaptive SMC



- Averaging filter estimates the extended equivalent control:

$$\tau\dot{u}_{\mathrm{av}} = -u_{\mathrm{av}} + u_{\mathrm{s}}\,.$$

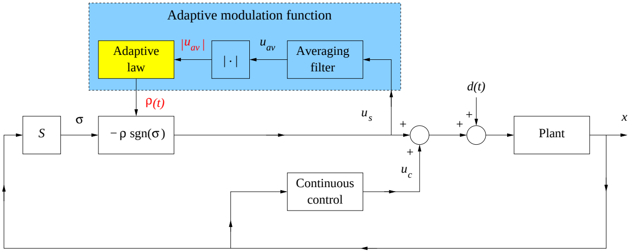
4.5.4 Adaptive SMC



- Estimate the absolute value of the extended equivalent control:

$$|u_{\mathrm{eq}}| \approx |u_{\mathrm{av}}| \ .$$

4.5.5 Adaptive SMC

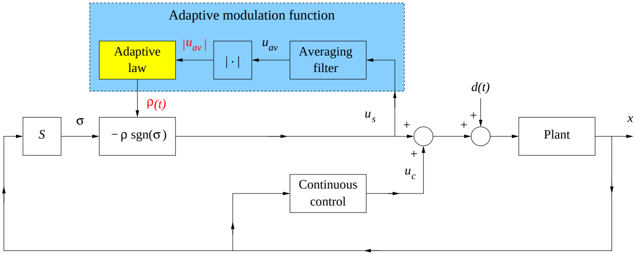


- Adaptive modulation function:

$$\dot{\varrho} = -\gamma_f \varrho + c_f \left(|u_{\rm av}| + \delta\right) .$$

- $\delta > 0$ guarantees a desired minimum excitation for start-up.

4.5.6 Adaptive SMC

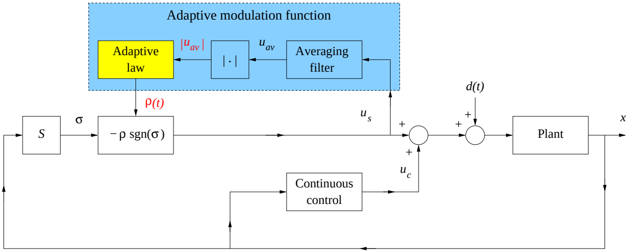


- Adaptive modulation function:

$$\dot{\varrho} = -\gamma_f \varrho + c_f \left(|u_{\text{av}}| + \delta \right) \, .$$

- $\delta > 0$ guarantees a desired minimum excitation for start-up.

4.5.7 While sliding is not reached

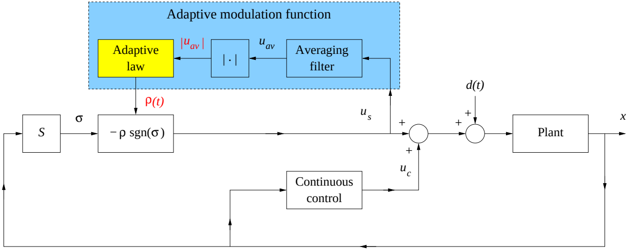


- Adaptive modulation function:

$$\dot{\varrho} = -\gamma_f \varrho + c_f \left(|u_{\rm av}| + \delta\right) .$$

- Before sliding mode $|u_{\rm av}| \approx |u_{\rm eq}| = \varrho$, then ...

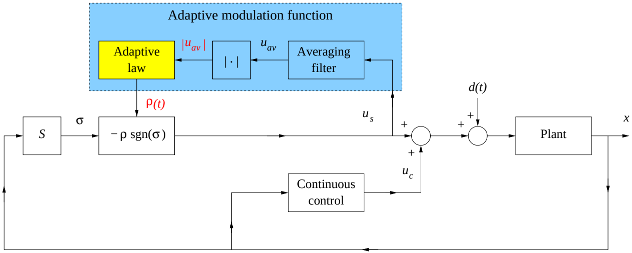
4.5.8 Adaptive modulation law



- Adaptive modulation function:

$$\dot{\varrho} = -\gamma_f \varrho + c_f \left(\varrho + \delta\right) \, .$$

4.5.9 Adaptive modulation law (cont.)

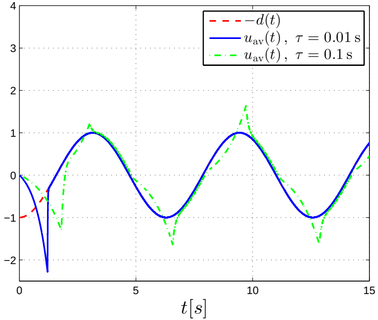


- Adaptive modulation function:

$$\dot{\varrho} = -\gamma_f \varrho + c_f\left(\varrho + \delta\right) .$$

- ... then, $c_f > \gamma_f > 0$ guarantees the loop is unstable, thus, $\varrho(t)$ grows exponentially, faster than $|d(t)|$.

4.5.10 Effect of averaging quality



- Average control signal for different time constants;
- Sinusoidal disturbance.

4.5.11 Theorem 1

Theorem 1 *The following stability properties hold for the proposed control system with the adaptive modulation function* ϱ *:*

1. *the sliding surface* $\sigma = 0$ *is reached in finite time;*
2. *the closed-loop system is uniformly globally exponentially stable in the sense that the state* $x = [\eta^T, \xi^T]^T$ *converges exponentially to the origin and*
3. *all remaining signals are uniformly bounded.*

4.6 Conclusions

- New adaptive SMC to circumvent non smooth disturbances with unknown bounds has been addressed.
- Based on the extended equivalent control and average control.
- The control gain ϱ is adapted to dominate disturbances:
  - ϱ decreases/increases as the disturbance does;
  - Discontinuous and unbounded disturbances are allowed;
  - The precision of the stabilization is improved;
  - Preserves sliding mode;
  - Reduces chattering.
- Main steps of the proof of uniform and global stability as well as perfect disturbance rejection were introduced.

5. Monitoring Function Approach (MFA) for adaptive SMC

The context

- UVC for uncertain MIMO systems
- Disturbance are bounded but with unknown bounds
- Quite general class of nonsmooth disturbances
- Global stabilization/tracking by state or *output feedback*
- Adaptive gain modulates according to the size of disturbances: reduce chattering
- Application to a surface vessel subject to ocean currents, wind and waves is discussed.

Novel properties of the MFA

- Prespecified transient time
- Maximum overshoot
- Guaranteed steady steady

5.2 Problem statement I

Consider (MIMO) systems in *regular form* [43, 30]

$$\dot{\eta} = A_{11}\eta + A_{12}\sigma + d_1(x,t)\,, \tag{7}$$

$$\dot{\sigma} = A_{21}\eta + A_{22}\sigma + d_2(x,t) + B_2 u\,, \tag{8}$$

$u \in \mathbb{R}^m$ is the input, $\sigma \in \mathbb{R}^m$ the output, $\eta \in \mathbb{R}^{n-m}$, $x := [\eta^T, \sigma^T]^T$ the state, $d_1 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n-m}$ is an unmatched disturbance, and $d_2 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^m$ is a matched disturbance. A_{ij} and B_2 are constant and uncertain.

5.2 Problem statement II

- **Assumptions**

(A1) Minimum phase from u to σ: A_{11} is Hurwitz.

(A2) $S_p \in \mathbb{R}^{m\times m}$ is known so that $-K_p$ is Hurwitz, where

$$K_p := B_2 S_p \tag{9}$$

is the *effective high-frequency gain* (HFG).

(A3) $d_1(x,t)$ and $d_2(x,t)$ are locally Lipschitz in x, p.w.c. in t, and satisfy

$$\|d_1(x,t)\| \leq \bar{d}_1 < +\infty\,, \quad \|d_2(x,t)\| \leq \bar{d}_2 < +\infty\,, \quad \forall x \in \mathbb{R}^n\,, \tag{10}$$

(A4) $\bar{d}_1 \geq 0$ and $\bar{d}_2 \geq 0$ are *unknown*!.

5.2 Problem statement III

- **Control Objective**
  - Achieve global stability and convergence of the output signal ($\sigma(t)$) to a small neighborhood of the origin
  - σ may also be the tracking error w.r.t. some desired trajectory
  - Then, stabilization of σ implies tracking of a reference signal.

5.3 Non-adaptive UVC

For known disturbance norm bounds $\bar{d}_1$ and $\bar{d}_2$, the control signal is pre-compensated by $u = S_p u$. Then,

$$\dot{\sigma} = A_{22}\sigma + K_p U + d(t), \tag{11}$$

$$d(t) = A_{21}\eta(t) + d_2(x(t), t)\,, \tag{12}$$

The disturbance $d(t)$ can be rejected by the UVC law

$$U = -\rho(t)\,\frac{\sigma}{\|\sigma\|}\,, \tag{13}$$

if

$$\rho(t) \geq \delta + c_\sigma\|\sigma(t)\| + c_d\|d(t)\|\,, \quad \forall t \geq 0\,, \tag{14}$$

for appropriate $c_\sigma \geq 0$ and $c_d > 0$. The arbitrary constant $\delta > 0$ is required to guarantee $\sigma = 0$ in finite time.

5.3.1 Modulation for UVC I

- State feedback
  If the state $x(t)$ is available, then

$$\|d(t)\| \leq c_1\|\eta(t)\| + \bar{d}_2\,, \tag{15}$$

where $c_1 \geq \|A_{21}\|$ A *modulation function* is thus given by

$$\begin{aligned} \rho(t) &= \delta + c_\sigma\|\sigma(t)\| + c_d c_1\|\eta(t)\| + \bar{d}\,, && (16) \\ \bar{d} &= c_d\bar{d}_2\,. && (17) \end{aligned}$$

- Output feedback

  If η is *unavailable* for feedback, then a bound for norm $\|\eta\|$ is found using a FOAF as follows. The solution $\sigma(t)$ is

$$\eta(t) = \exp(A_{11}t)\eta(0)+\exp(A_{11}t)*[A_{12}\sigma(t)+d_1(x(t),t)]\,, \quad t \tag{18}$$

  Here, $\sigma(t)$ dependent term is norm bounded by the FOAF output.

The resulting modulation function is given by (modulo irrelevant π terms) :

$$\begin{aligned} \rho(t) &= \delta + c_\sigma \|\sigma(t)\| + c_d \bar{\sigma}_{1\sigma}(t) + \bar{d}\,, && (19) \\ \bar{d} &= c_d \left(\frac{c_{\eta d_1} \bar{d}_1}{\gamma_1} + \bar{d}_2 \right) . && (20) \end{aligned}$$

$$\bar{\sigma}_{1\sigma}(t) = \frac{c_{\eta\sigma}}{s + \gamma_1} \|\sigma(t)\|$$

5.4 Adaptive Unit Vector Control I

Now the upper bounds $\bar{d}$ are assumed UNKNOWN! Then, adapt...

The following adaptive law is proposed

$$\hat{d}(t) = \beta(k, t - \bar{t}) = \begin{cases} \beta_1(k)\,, & \text{if} \quad t < \bar{t}\,, \\ \beta_2(k, t - \bar{t})\,, & \text{if} \quad t \geq \bar{t}\,, \end{cases} \quad \forall t \in [t_k\,, t_{k+1}]\,, \quad k = 1, 2, \ldots\,, \tag{21}$$

5.4 Adaptive Unit Vector Control II

Notation:

- t_{k+1} denotes the *new switching time*
- The unknown $\bar{d} \to$ class $\mathcal{K}_\infty$ function $\beta_1(k)$ in Phase 1 $(t < \bar{t})$
- or a class $\mathcal{K}_\infty\mathcal{L}$ function $\beta_2(k, t - \bar{t})$ in the Phase 2 $(t \geq \bar{t})$ of the algorithm
- $\bar{t} \geq 0$ is the phase transition time , and $k \in \mathbb{N}$ is the switching number of a monitoring function to be defined
- $\beta(k, t - \bar{t})$ grows monotonically with k and decreases with $(t - \bar{t})$ (for $t \geq \bar{t}$) after each switching,

5.4 Adaptive Unit Vector Control III

- After each new switching, the function β will give a norm bound for the disturbance $d(t)$ in (12) at least during a sufficiently large interval of time.
- The adopted UVC law is $u = S_b U$ with U given as (13).
- the *modulation function* $\rho(t)$ is designed to overcome the net input disturbance in (11).
- the following adaptive modulation functions have been obtained

SF

$$\rho(t) = \delta + c_\sigma \|\sigma(t)\| + c_d c_1 \|\eta(t)\| + \hat{d}(t)\,, \tag{22}$$

OF

$$\rho(t) = \delta + c_\sigma \|\sigma(t)\| + c_d \bar{\sigma}_{1\sigma}(t) + \hat{d}(t)\,. \tag{23}$$

5.4 Adaptive Unit Vector Control IV

By the UVC Lemma 1,the output signal $\sigma(t)$ tends to zero being bounded by

$$\|\sigma(t)\| \leq k_1 \, \mathrm{e}^{-\lambda_1(t-\bar{t}_0)} \|\sigma(\bar{t}_0)\| \, , \quad \forall t \in [\bar{t}_0, +\infty) \, , \tag{24}$$

where $\bar{t}_0$ denotes any initial time, and $k_1, \lambda_1 [\, , \lambda_2 \,] > 0$ are known constants satisfying some inequalities related to K_p, provided that the estimated disturbance (21) upper bounds the true disturbance $\forall t \geq \bar{t}_0$.

5.4.1 New monitoring switching scheme I

- Preliminary results (Oliveira, Melo, Hsu and Cunha 2017)[42], did not guarantee *pre-specified transient and steady-state performance*.
- A new monitoring switching strategy is now introduced to "fill the gap".

The performance specifications are illustrated by the figure below.

5.4.1 New monitoring switching scheme II

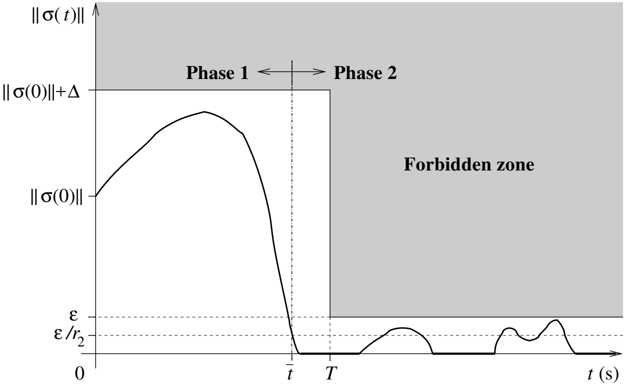


FIGURE 1 Performance specifications on $\|\sigma(t)\|$.

5.4.1 New monitoring switching scheme III

Definition

The stabilization/tracking error σ is said to satisfy the transient and steady-state performance specifications, if:

1. $\|\sigma(t)\| \leq \|\sigma(0)\| + \Delta$, $\forall t \in [0\,,\, T)$, and
2. $\|\sigma(t)\| \leq \varepsilon$, $\forall t \geq T$,

where Δ is the allowed maximum overshoot, $T > 0$ is the maximum transient time, and $\varepsilon \in (0, \Delta]$ is the allowed maximum steady-state error, that can be freely specified.

5.4.1 New monitoring switching scheme IV

- The proposed algorithm has two phases described below.
- This is similar to Yan et al.[Yan, Hsu, and Xiuxia(2006)], the switching times of $\beta(k, t - \bar{t})$ are according to the two phases below.
- In the algorithm, $r_1, r_2 > 1$ are arbitrarily chosen design constants, which can adjust the frequency of switchings.

5.4.1 New monitoring switching scheme V

Phase 1: *Force* $\|\sigma(t)\| \leq \varepsilon/r_2$ before T while avoiding the violation of the allowed maximum overshoot.

If $\|\sigma(t_1)\| = \|\sigma(0)\| \leq \varepsilon/r_2$, then the algorithm passes to Phase 2.

Otherwise, for every k for which $\|\sigma(t_k)\| > \varepsilon/r_2$, the switching time t_{k+1} is defined by

$$t_{k+1} := \min \left\{ t > t_k \ : \ \begin{array}{c} \|\sigma(t)\| = \|\sigma(0)\| + \Delta\left(1 - 1/r_1^k\right) \\ \text{or} \\ t = T\left(1 - 1/r_1^k\right) \quad \text{and} \quad \|\sigma(t)\| > \varepsilon/r_2 \end{array} \right\},$$

If for some time $t = \bar{t} < T$, the condition

$$\|\sigma(\bar{t})\| = \varepsilon/r_2 \tag{26}$$

is reached, then the algorithm proceeds to Phase 2.

5.4.1 New monitoring switching scheme VI

Phase 2: Maintain $\|\sigma(t)\| < \varepsilon\,,\ \forall t > T$.
Let t_{j+1} be the final switching time in Phase 1. The switching time in Phase 2 is defined by

$$t_{k+1} := \begin{cases} \min\left\{\ t > t_k : \|\sigma(t)\| = \varepsilon\left(1 - 1/r_2^{k-j+1}\right)\right\}\,, & \text{if it exists}\,,\\ +\infty\,, & \text{otherwise}\,, \end{cases} \tag{27}$$

where if $k = j + 1$, then t_k of the right-hand side of (27) should be replaced by $\bar{t}$. It can be checked from (27) that

$$\|\sigma(t_{k+1})\| > \|\sigma(t_k)\|\,. \tag{28}$$

Hence, $\beta(k, t - \bar{t})$ switches in Phase 2 only when $\|\sigma\|$ increases and becomes too close to ε.

hideallsubsections

5.4.1 New monitoring switching scheme VII

For each k in (21), the function $\beta(k, t-\bar{t})$ must satisfy

$$\beta(k+1, t-\bar{t}) > \beta(k, t-\bar{t}) \quad > \quad 0\,, \tag{29}$$

$$\lim_{k\to+\infty} \frac{\beta(k, t-\bar{t})}{[\max(r_1, r_2)]^k} \quad = \quad +\infty\,. \tag{30}$$

5.4.1 New monitoring switching scheme VIII

Table: Adaptive sliding mode controller for the system (7)–(8).

Unit vector control law	$u(t) = S_p U(t)\,, \qquad U(t) = -\rho(t)\frac{\sigma(t)}{\\|\sigma(t)\\|}$
ρ for state feedback	$\rho(t) = \delta + c_\sigma \\|\sigma(t)\\| + c_d c_1 \\|\eta(t)\\| + \hat{d}(t)$
ρ for output feedback	$\rho(t) = \delta + c_\sigma \\|\sigma(t)\\| + c_d \bar{\sigma}_{1\sigma}(t) + \hat{d}(t)$
FOAF for output feedback	$\dot{\bar{\sigma}}_{1\sigma}(t) = -\gamma_1 \bar{\sigma}_{1\sigma}(t) + c_{\eta\sigma}\\|\sigma(t)\\|\,, \quad \bar{\sigma}_{1\sigma}(0) \geq 0$
Adaptive law	$\hat{d}(t) = \beta(k, t-\bar{t}) = \begin{cases} \beta_1(k)\,, & \text{if } t < \bar{t}\,, \\ \beta_2(k, t-\bar{t})\,, & \text{if } t \geq \bar{t}\,. \end{cases}$
From Phase 1 to 2	$\bar{t} = \begin{cases} 0\,, & \text{if } \\|\sigma(\bar{0})\\| \leq \varepsilon/r_2\,, \\ t < T \;:\; \\|\sigma(t)\\| = \varepsilon/r_2\,, & \text{otherwise}\,. \end{cases}$
MF during Phase 1	$t_{k+1} := \min \left\{ t > t_k \;:\; \begin{array}{c} \\|\sigma(t)\\| = \\|\sigma(0)\\| + \Delta\left(1 - 1/r_1^k\right) \\ \text{or} \\ t = T\left(1 - 1/r_1^k\right) \quad \text{and} \quad \\|\sigma(t)\\| > \varepsilon/r_2 \end{array} \right\}$
MF during Phase 2	$t_{k+1} := \begin{cases} \min\left\{ t > t_k : \\|\sigma(t)\\| = \varepsilon\left(1 - 1/r_2^{k-j+1}\right)\right\}\,, & \text{if it exists}\,, \\ +\infty\,, & \text{otherwise}\,. \end{cases}$

5.5 Stability Analysis

Specifications and Pratical stability theorem:

Theorem

Assume that **(A1)–(A3)** *hold. Then, with the described monitoring function adaptive UVC, practical stabilization/tracking is achieved, with the output signal or tracking error $\sigma(t)$ converging ultimately close to an ε–neighborhood of the origin. Moreover, all the closed-loop signals are uniformly bounded and all pre-specified transient and steady-state in Definition 1 are guaranteed as well.*

5.5 Stability Analysis II

Remark

In some practical applications, despite the fact the disturbance $d(t)$ has unknown norm bound, it may tend to some specific value or has a minimal upper bound after some finite time (for example, when the disturbance has a large transient and then goes to a small steady state). For these situations, the following corollary can be stated.

Corollary

In Theorem 1, if $d(t)$ has the additional property:

$$\|\eta_{d_1}(t) + d_2(x(t), t)\| < d_l, \quad \forall t \in [t_l, +\infty) \tag{31}$$

$d_l \geq 0$ known constant, then replacing (22) for SF or (23) for OF by the new modulation functions

$$SF: \qquad \rho(t) = \delta + c_\sigma\|\sigma(t)\| + c_d c_1\|\eta(t)\| + \hat{d}(t) + d_l\,, \tag{32}$$

$$OF: \qquad \rho(t) = \delta + c_\sigma\|\sigma(t)\| + c_d\bar{\sigma}_{1\sigma}(t) + \hat{d}(t) + d_l\,, \tag{33}$$

then exact stabilization/tracking is achieved and $\sigma(t)$ is kept in the origin after some finite time. Moreover, $\hat{d}(t) \to 0$, $\forall t > t_l$, which decreases the amplitude of the control signal $u(t)$ needed to keep the sliding mode.

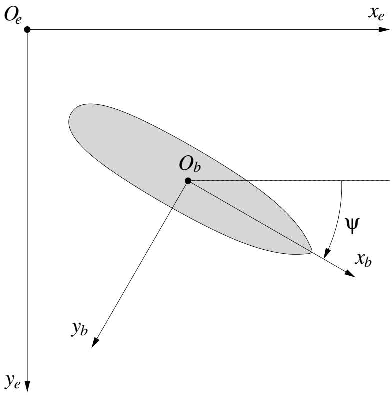


FIGURE 2 Top view of the vessel and coordinate systems.

5.5 Tracking Control of a Surface Vessel II

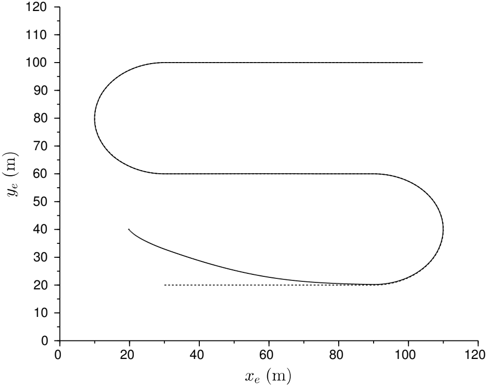


FIGURE 3 Trajectory of the vessel on the water surface (solid line), and reference trajectory (doted line).

5.5 Tracking Control of a Surface Vessel III

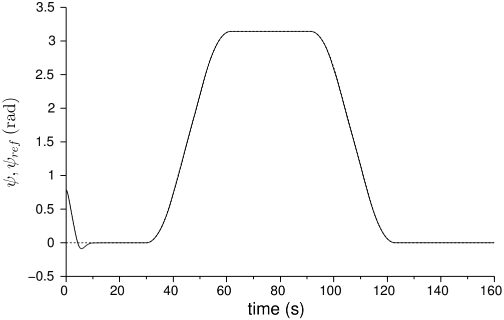


FIGURE 4 Heading angle of the vessel (solid line), and reference heading angle (doted line).

5.5 Tracking Control of a Surface Vessel IV

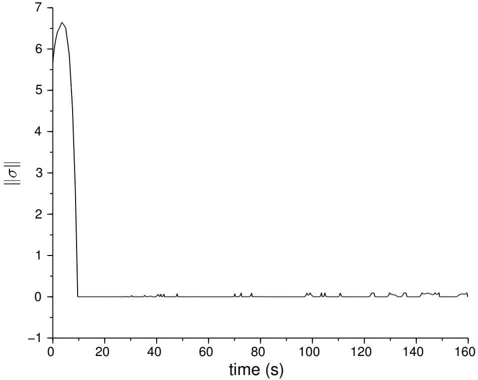


FIGURE 5 Norm of the sliding variable with $\varepsilon = 0.1$.

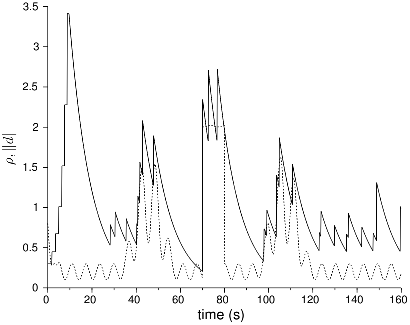


FIGURE 6 Modulation signal (ρ — solid line), and the norm of the disturbance ($\|d\|$ — doted line) with $\varepsilon = 0.1$.

5.5 Tracking Control of a Surface Vessel VI

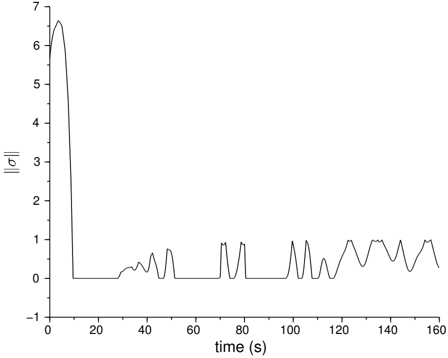


FIGURE 8 Norm of the sliding variable with $\varepsilon = 1$.

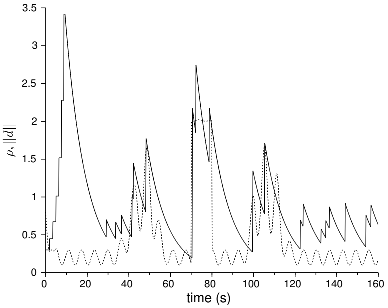
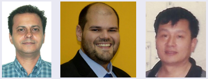


FIGURE 9 Modulation signal (ρ — solid line), and the norm of the disturbance ($\|d\|$ — doted line) with $\varepsilon = 1$.

The players



(Cunha, Oliveira, Yan (circa 2010))

The Champion

Tiago Roux de Oliveira, Best Thesis Award (CAPES 2012) as a 2018 Associate Member of the Brazilian Academy of Sciences

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Questions?

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