Global Tracking for Uncertain Systems by Output Feedback

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Introduction

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A brief history
A 1965 survey

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Adaptive laws

hideallsubsections

2. Control Signal Synthesis

2.1 Control Signal Synthesis: brief history

- Back to the 60's, e.g., Lyapunov control synthesis was exploited.
- Sliding modes or Variable Structure Systems not well acknowledged. However, the need of discontinuous control appeared.
- Lowe & Rowlands (1974) used "signal synthesis" for designing Model Reference Adaptive Control (MRAC).
- Devaud & Caron (1975) pioneered use discontinuous SMC (Sliding Mode Control) in the context Model Reference Control.
- Ambrosino, Celentano & Garofalo (1984) introduced the term Variable Structure MRAC using only input and output measurements.

2.2 A 1965 survey

(L.P.Grayson, Automatica, vol.3, pp. 91-121, 1965)

Technique	Plant	Procedure	Resulting System	Literature
1	$\dot{x}=A(\alpha)x$ is asymptotically stable for all fixed α allowed.	Choose α to minimize $\varphi(\alpha)=\int_0^\infty x'Qxdt$	Linear, time-invariant. Over-all system is optimal.	KRASOVSKII [3], MEEROV [4], ALEX [5]
2	$\dot{x}=Ax+Bu$ where $\dot{x}=Ax$ is asymptotically stable.	Choose u, such that $\|u_i\|\leqslant 1$ to minimize $\varphi(u)=\int_0^\infty x'Qxdt$	Nonlinear, time-invariant. The u_is result in relays or *saturation* elements. A regulator. System is optimal.	BASS [7] KALMAN and BERTRAM [8] GIESEKING [9]
3	$\dot{x}=Ax+bf(\sigma)$ where $\dot{x}=Ax$ is asymptotically stable and $\sigma f(\sigma)\neq 0$ for $\sigma\neq 0$.	Choose $\sigma=a'x$ where $a=-Pb$ and $V=x'Px$.	Nonlinear, time-invariant plant; a linear time-invariant controller. Overall it is a regulator.	BASS [7]
4	$\dot{x}=Ax+bf(\sigma)$ where $\dot{x}=Ax$ is asymptotically stable and $\sigma f(\sigma)\neq 0$ for $\sigma\neq 0$.	Choose σ to satisfy $\dot{\sigma}+k\sigma=ax-lf(\sigma)$ where $a=-Pb$ and $V=x'Px$, $k\geqslant 0$, $l\geqslant 0$, $k^2+l^2\neq 0$.	Nonlinear, time-invariant plant; a nonlinear, time-invariant controller. Overall it is a regulator.	BASS [7]
5	$\dot{x}=Ax+bf(\sigma)$ where $\dot{x}=Ax$ is arbitrary and $f(\sigma)=\text{sgn}\,\sigma$.	Choose σ to satisfy $\dot{\sigma}+k\sigma=a'x-[l-x'Qx]f(\sigma)$ where $a=-Pb$, $V=x'Px$ and $A'P+PA=-Q$.	Time-invariant plant with relays. Controller is nonlinear, time-invariant. A regulator.	BASS [7]
6	$\dot{x}=f(x)+u$ where $\dot{x}=f(x)$ is stable, but not asymptotically stable.	Choose u to make system asymptotically stable, such that the $\|u_i\|$ are bounded, or the time to reach $x=0$ is minimized, or the time con- is a minimum.	Linear or nonlinear controllers.	LEE [10] GEISS [11]

3. Model Reference Adaptive Control (MRAC)

3.1 Simple example: Adaptive roll control of an aircraft (Lavrestky 2008)

3.1.1 System equations

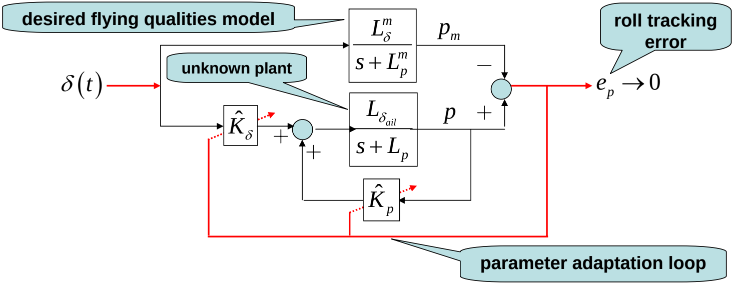
- ***Uncertain* Roll dynamics**: → $\dot{p} = L_p\, p + L_{\delta_{ail}}\, \delta_{ail}$
  - p is roll rate,
  - δ_{ail} is aileron position
  - $\left(L_p,\, L_{\delta_{ail}}\right)$ are *unknown* damping, aileron effectiveness
- **Flying Qualities Model**: → $\dot{p}_m = L_p^m\, p_m + L_\delta^m\, \delta(t)$
  - $\left(L_p^m,\, L_\delta^m\right)$ are *desired* damping, control effectiveness
  - $\delta(t)$ is a reference input, (pilot stick, guidance command)
  - roll rate tracking error: → $e_p(t) = \left(p(t) - p_m(t)\right) \to 0$
- **Adaptive Roll Control**: → $\delta_{ail} = \hat{K}_p\, p + \hat{K}_\delta\, \delta$

$$\begin{cases} \dot{\hat{K}}_p = -\gamma_p\, p\,(p - p_m) \\ \dot{\hat{K}}_\delta = -\gamma_{\delta_{ail}}\, \delta(t)\,(p - p_m) \end{cases}, \quad \left(\gamma_p, \gamma_{\delta_{ail}}\right) > 0$$

parameter adaptation laws

E. Lavretsky

3.1.2 Block diagram of adaptive roll rate control



3.1.3 Adaptive control law

The roll control problem is a particular case of the following system:

- Plant: $\dot{x} = ax + bu$
- Model reference: $\dot{x}_m = a_m x_m + b_m r$
- Regressor vector: $\omega^T = [x \;\; r]$
- Model matching control (unknown):

$$u^* = -k^* x + l^* r; \quad l^* = b_m/b; \;\; k^* = (a_m + a)/b$$

- Adaptive parameter vector: $\theta^T = [l \;\; k]$
- Control parameterization: $u := \theta^T \omega$
- Output (tracking) error: $e = x - x_m$
- Adaptation gain matrix: $\Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$
- Adaptation law: $\dot{\theta} = -sign(b)\Gamma\omega e$

3.2 Lyapunov based MRAC design

Lyapunov based design for adaptive control (Parks, 1966)

362   IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-11, NO. 3, JULY, 1966

Liapunov Redesign of Model Reference Adaptive Control Systems

PATRICK C. PARKS

A landmark in modern adaptive control theory.

3.2.1 MRAC — System equations

For model following a necessary assumption is that the plant be minimum-phase!

- Plant: $G(s) = K_p \frac{N(s)}{D(s)}; \quad y = W(s)u$
- Reference Model (SPR): $W_m(s) = K_m \frac{Z(s)}{R(s)}; \quad y_M = W_M(s)r$
- Output error: $e_1 = y - y_M$
- State variable filters ($\omega_1, \ \omega_2 \in \mathbb{R}^{n-1}$)

$$\begin{aligned} \dot{\omega}_1 &= \Lambda\omega_1 + gu \\ \dot{\omega}_2 &= \Lambda\omega_2 + gy \end{aligned}$$

- Regressor vector: $\omega^T = [\omega_1{}^T \ \omega_2{}^T \ y \ r]$
- Adaptive parameter vector: $\theta^T = [\theta_1^T \theta_2^T \theta_3 \theta_4]$

3.2.2 Error equations

- The output error is denoted $e_1 = y - y_M$
- The parameter error is $\tilde{\theta} := \theta - \theta^*$
- Error dynamic equations (including filters)

$$\dot{e} = Ae + \rho^* b \tilde{\theta}^T \omega, \quad \rho^* = (\theta_4^*)^{-1} = K_p/K_m, \quad e \in \mathbb{R}^{3n-2}, \quad e_1 = h^T e$$

- We arrive at a similar error equation but $e \in \mathbb{R}^1 \rightarrow e \in \mathbb{R}^{3n-2}$
- Why $(3n-2)$? ... to include the state variable filters
- $e_1 = h^T e$ for some $h \in \mathbb{R}^{3n-2}$
- $\{A, b, h\}$ is a *nonminimal* realization of model $W_M(s)$

3.4 Lyapunov design, $n^* = 1$

The (simplified) Kalman-Yakubovitch-Popov Lemma (*)

Let $G(s) = C((sI - A)^{-1}B$ be a $p \times p$ transfer function, where (A, B) is controllable and (A, C) is observable. Then $G(s)$ is strictly positive real iff $\exists P = P^T > 0, \;\; Q > 0$ such that

$$\begin{aligned} PA + A^T P &= -Q \\ PB &= C \end{aligned}$$

Choose candidate Lyapunov function V and adaptive law for $\dot{V} \leq 0$

3.4 Lyapunov design, $n^* = 1$ II

Remarks

- Generalization of the KYP to noncontrollable systems was made by Meyer. We need it because (A, b, h) is nonminimal.
- **Fact**: $\exists \theta^*$ s.t. plant matches reference model with $u^* = {\theta^*}^T \omega$ with regressor vector ω.
- **Assumptions**: known n, known sign of K_p.

3.4.1 Adaptive laws, $n^* = 1$

- The Lyapunov function:

$$V = \frac{1}{2}e^T Pe + \frac{1}{2}\tilde{\theta}^T |\rho^*|\Gamma^{-1}\tilde{\theta} > 0$$

- $\dot{V} = e^T P\dot{e} + |\rho^*|\tilde{\theta}^T \Gamma^{-1}(\dot{\theta})$

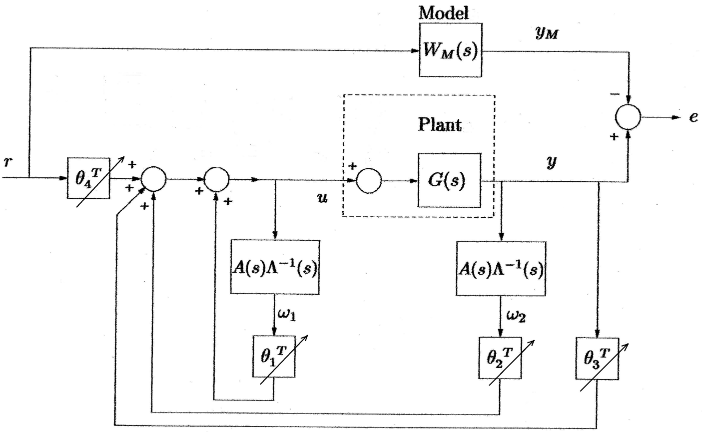
Adaptive control law – SISO, $n^* = 1$

- Control law: $u = \theta^T \omega$
- Adaptation law: $\dot{\theta} = -sign(K_p)\Gamma\omega e; \quad \Gamma = \Gamma^T > 0$

- $\dot{V} = e^T P(Ae + b\rho^*[\tilde{\theta}^T\omega] + |\rho^*|\theta^T\Gamma^{-1}(-sign(\rho^*))\Gamma\omega e_1$
- or $\dot{V} = -e^T Qe + e_1\rho^*[\tilde{\theta}^T\omega] - \rho^*\tilde{\theta}^T\omega e_1;$
- Thanks to the KYP Lemma:

$$\dot{V} = -e^T Qe \leq 0 \quad \text{(semidefinite negative)}$$

3.4.2 MRAC block diagram



3.4.3 Hidden difficulties of semi-definite $\dot{V}$

With $V(e, \tilde{\theta}) > 0$ but $\dot{V} = -e^T Qe \leq 0$ (semi-definite) one can conclude or unconclude:

- $e(t) \in \mathcal{L}_\infty \bigcup \mathcal{L}_2$ and $\tilde{\theta}(t) \in \mathcal{L}_\infty$
- $\dot{e}(t) \in \mathcal{L}_\infty$
- $e(t) \to 0$
- The parameteric error $\tilde{\theta}(t) := (\theta - \theta^*)$ may not converge to zero. It requires *Persistency of Excitation* or $r(t)$ sufficiently rich.

In fact,

The adaptation transient can be extremely slow or oscillatory.
Still a rather open problem in adaptive control!

3.5 MRAC general case of $n^* \geq 1$

Limitation

SPR implies relative degree 1.

- Major difficulty of the general case: relative degree ≥ 1.
- The Reference Model can not be SPR.
- Solution for adaptive control:
  Monopoli's augmented error
- Adaptive algorithm analysis and synthesis much more complicated!

4. Transforming MRAC to VS-MRAC I

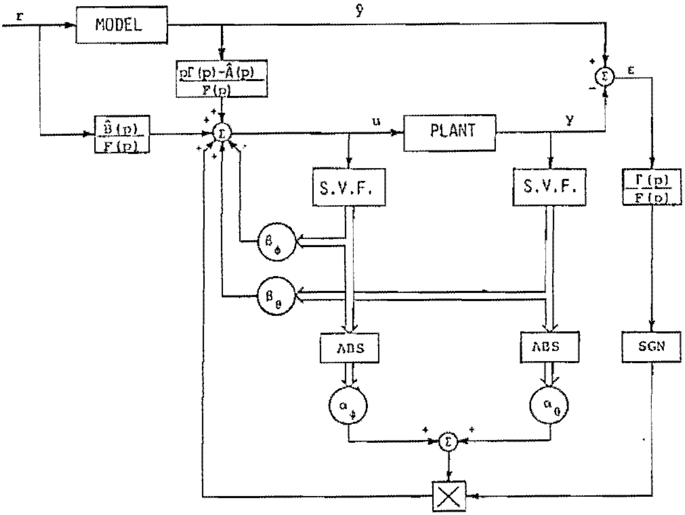
4.1 Brief history

STATE FEEDBACK: (Devaud & Caron 1975), (Zinober, El-Ghezawi & Billings, 1982) and references therein.

OUTPUT FEEDBACK:

1. Ambrosino, Celentano & Garofalo (1984): "Variable structure model reference adaptive control systems" (VS-MRAC) first named this technique. *However, the control was ill-defined...*
2. Bartolini & Zolezzi (1988): "The V.S.S. Approach to the Model Reference Control of Nonminimum Phase Linear Plants", *a very ambitious objective –Problem: requires a stringent a priori signal boundedness condition to assure stability.*

4.2 ACG VS-MRAC (1984)



From MRAC to VS-MRAC with $n^* = 1$

Underlying ideas (Hsu & Costa 1989)

- What if the adaptation gain tends to ∞ and the parameters are defined memoryless?
- Then $V(e) = \frac{1}{2}(e^T P e)$
- ...Back to Lyapunov Synthesis Approach!
- ...But using only output feedback.

4.3 Lyapunov design of VS-MRAC II

Recall MRAC error equations and KYP lemma.

- Error equations (including I/O filters)

$$\dot{e} = Ae + \rho^* b\tilde{\theta}^T\omega, \;\; \rho^* = (\theta_4^*)^{-1} = K_p/K_m, \;\; e \in \mathbb{R}^{3n-2}, \;\; e_1 =$$

- We arrive at a similar error equation but $e \in \mathbb{R}^1 \to e \in \mathbb{R}^{3n-2}$
- $e_1 = h^T e$ for some $h \in \mathbb{R}^{3n-2}$
- $\{A, b, h\}$ is a *nonminimal* realization of model $W_M(s)$
- Chose an SPR model: $\exists P, \;\; Q > 0$ such that $A^T P + PA = -Q < 0, \;\; Pb = h$ (KYP Lemma)

4.3.1 Lyapunov control synthesis

VS control

Similar to adaptive control law: $u = \sum_{i=1}^{2n} \psi_i \omega_i$.

Now, instead of adapting the parameters psi_i with an integral law, we let them switch.
The switching functions ψ_i is designed from the Lyapunov function

$$V(e) = \frac{1}{2} e^T P e \,,$$

where $P = P^T > 0$ satisfies the KYP lemma.

4.3.1 Lyapunov control synthesis II

Calculating dV/dt with respect to error dynamic equations one has (recall $\theta^*_{2n} > 0$):

$$\begin{aligned}\dot{V} &= -e^T Q e + (\theta^*_{2n})^{-1}\left(u - \theta^{*T}\omega\right) e_1 \\ &= -e^T Q e + (\theta^*_{2n})^{-1}\sum_{i=1}^{2n}(\psi_i - \theta^*_i)\,\omega_i e_1 \,.\end{aligned}$$

4.3.1 Lyapunov control synthesis III

Now, choosing

$$\psi_i = -\bar{\theta}_i \operatorname{sign}(\omega_i e_1)\,,$$

where $\bar{\theta}_i > |\theta_i^*|$, $\forall i$, then

$$\dot{V} = -e^T Q e + (\theta_{2n}^*)^{-1} \sum_{i=1}^{2n} \left(-\bar{\theta}_i |\omega_i e_1| + \theta_i^* \omega_i e_1\right)\ .$$

Since summation above is non-positive, then

$\dot{V}$ is negative definite!
Exponential stability guaranteed!

$$\dot{V} < -e^T Q e < 0\,.$$

4.3.1 Lyapunov control synthesis IV

Summarizing:

- Lyapunov function candidate: $V(e) = \frac{1}{2}e^T Pe$
- SPR allows: $e_1 = (Pb)^{-1}e$
- Upper bounds $\bar{\theta}_i > \theta^*{}_i$ are known
- Choose $\psi_i = -\bar{\theta}_i sign(\omega_i e_1)$
- Conclude $\dot{V} < -e^T Qe < 0$

Remark:

SPR made the "magic" of sign-indefinite terms being cancellable!

4.3.2 Link between MRAC and VS-MRAC

Consider the adaptation law:

$$\mu\dot{\theta} = -\sigma\theta - \Gamma\omega e_1, \quad \mu > 0$$

with forgetting factor $\sigma/\mu > 0$ and singular perturbation $\mu \to 0^+$ and "normalized gain"

$$\Gamma = diag\left[\frac{(\sigma/\mu)\bar{\theta}_i}{|e_1\omega_i|}\right]$$

Type	σ/μ	μ
MRAC	0	1
transition	> 0	small
VS-MRAC	∞	0

This is in agreement with the "fast forgetting and high adaptation gain" interpretation of the VS-law.

4.4 The (output feedback) VS-MRAC, $n^* = 1$

Compact form

(Hsu & Araújo 1990)[?]

$$u = -\rho(\omega)\mathrm{sign}(e_1)$$
$$\rho = \left[\sum_{1}^{2n} \bar{\theta}_i |\omega_i| + \delta\right]$$

ρ is called “gain” or “modulation” function of the relay function sign(.), with arbitrary $\delta > 0$.

4.4.1 Main result

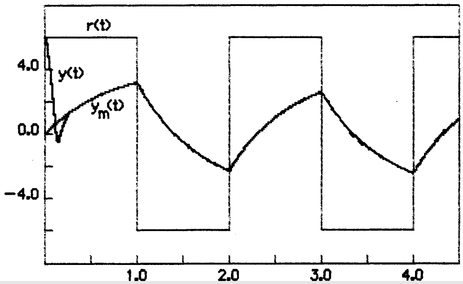
Theorem (Global Stability): For every initial condition,

- $\|e(t\| \to\ 0$ with at least an exponential rate, independent of the excitation $r(t)$;
- The output error $e_1(t) = h^T e$ becomes zero after finite time $t_1 \geq t_0$, in sliding mode.

4.4.2 Simulation results

Uncertain nonlinear time-varying plant
(Hsu and Costa 1989)

$$
\begin{aligned}
\dot{x}_1 &= [1 + a(t)]x_2 \\
\dot{x}_2 &= \sin x_1 - 2\sin x_2 + d(t) + u \\
\dot{y}_m &= -2y_m + r(t); \\
y &= 6x_1 + x_2
\end{aligned}
$$

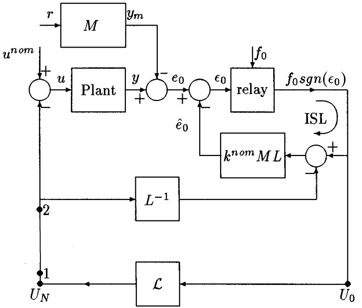


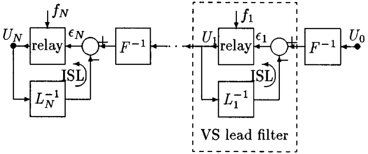
4.5 VS-MRAC, $n^* \geq 1$

As for MRAC, an augmented error was also proposed by (Hsu 1990) for the VS-MRAC, inspired by:

- (Monopoli, 1974)
- predicted error and prediction error (Goodwin and Mayne 1987)

4.4.1 Block diagram, $n^* \geq 1$





$k^{nom} = k^* \rightarrow \varepsilon_0 = k^* ML(-U_0 - L^{-1}u^*)$

$\mathcal{L}$

-$\mathcal{L}$ is an approximation of $L = L_1 \ldots L_N$;

-$L_i = (s + \alpha_i)$; $F^{-1} = 1/(\tau s + 1)$ is an averaging filter.

-ISL: is an "Ideal Sliding Loop" if $ML \in$ SPR

4.4.2 Global stability/tracking

- Partial proof ($n^* = 2$) was presented in (Hsu, Araújo, Costa, 1994) [Hsu, Araújo, and Costa 1994]
- The complete stability proof was published in (Hsu, Lizarralde and Araújo 1997)[Hsu, Lizarralde, and Araújo 1997]
- Two fundamental lemmas were developed to this end:

4.4.3 Fundamental Lemmas

Lemma 1

Consider the I/O relationship

$$\varepsilon_0(t) = M(s)[u + d(t) + \pi(t)], \quad u = -f(t)sign(\varepsilon_0)$$

where $M(s)$ is SPR, $d(t), /\pi(t)$ are LI (locally integrable), $|\pi(t)| \leq Re^{-at}$, $a > 0$. Let x be the state of a stable realization of $M(s)$. If $f(t)$ is LI and $f(t) \leq |d(t)|$, $\forall t \geq 0$, then the inequality

$$\|\varepsilon_0(t)\| \text{and} \|x(t)\| \leq [c_1\|x(0)\| + c_2 R]e^{-\lambda_1 t}$$

holds $\forall t \geq 0$ and for positive constants c_1, c_2, λ_1. Moreover, if $f(t) \leq |d(t)| + \epsilon$, $\forall t \geq 0$, for arbitrary $\epsilon > 0$, then $\varepsilon_0(t)$ tends to zero in finite time.

Proof: [Hsu and Costa 1989], (Hsu and Lizarralde 1992).

4.4.3 Fundamental Lemmas II

Lemma 2

Consider the I/O relationship

$$\varepsilon(t) = \frac{1}{s+\alpha}\left[u + d(t)\right] + \pi(t) + \beta(t), \quad u = -f(t) sign(\varepsilon_0)$$

where $\pi(t)$ is as in Lemma 1 and $\beta \in L_{\infty e}$, are both absolutely continuous. If $f(t) \geq |d(t)|$, $\forall t$, then with $\hat{e}(t) := \varepsilon(t) - \beta(t)$:

$$|\hat{e}(t)| \text{ and } |\varepsilon(t)| \leq |\hat{e}(0)|e^{\alpha t} + 2\left[R\, e^{-min(\alpha,\lambda)t} + sup_t|\beta|\right]$$

.

Proof: Nontrivial! [Hsu, Lizarralde, and Araújo 1997]

4.4.3 Fundamental Lemmas III

Lemma 3 (FOAF (*) Lemma)

Consider the stable strictly proper input/output relationship $z = W(s)d$. Let γ_0 be a positive constant satisfying $0 < \gamma_0 < \min_j \left|Re(p_j)\right|$ (p_j are the poles of $W(s)$), and $\bar{d}(t)$ be an instantaneous upper bound of $d(t)$, i.e., $|d(t)| \leq \bar{d}(t)\ \forall t$. Then, there exists a positive constant c_1 such that the impulse response $w(t)$ satisfies $|w(t)| \leq c_1 \gamma_0 e^{-\gamma_0 t}$ and the following inequalities hold

$$|W * d(t)| \leq c_1 \frac{\gamma_0}{s+\gamma_0} * \bar{d}(t); \quad (1)$$

$$\left|z(t) - z^0(t)\right| \leq c_1 \left|\hat{d}(t) - \hat{d}(t)^0\right|; \quad \hat{d} = (\frac{\gamma_0}{s+\gamma_0})\bar{d} \quad (2)$$

$$|z(t)| \leq c_1 \hat{d}(t) + \exp \quad (3)$$

where $z^0, \hat{d}^0$ and "exp" depend on the initial conditions and decay exponentially to zero with rate γ_0 (for a proof see [?]).

(*) First Order Approximation Filter

4.4.3 Fundamental Lemmas IV

Corollary

Consider $z = G_F(\tau s)G_L(s)d = G_F(\tau s)\frac{1}{s+\alpha}\bar{G}_L(s)d$ where G_F, G_L are rational, stable, strictly proper, $\bar{G}_L$ has *positive impulse response* (p.i.r.), $\alpha > 0$. If $\tau \in [0, \bar{\tau}]$ and $\bar{\tau}$ is sufficiently small, there exists $k > 0$ such that (2) and (3) hold with

$$\hat{d}(t) = kG_L\bar{d}(t)$$

4.4.4 Stability Theorems I

Theorem 1

Consider the auxiliary errors ε_i, $i = 0, \ldots, N$ $(N = n^* - 1)$. Then, with the relay modulation functions satisfying $(i = 0, \ldots, N-1)$

$$f_i \geq \left|(F_{1,i}^{-1} L_{i+1,N}^{-1}) * (\bar{U})\right| \quad \text{and} \quad f_N \geq \left|F_{1,N}^{-1} * U_d\right| ; \tag{4}$$

the auxiliary errors e_i' $(i = 0, \ldots, N-1)$ tend to zero, at least exp. Moreover,

$$\left|e_i'(t)\right|, \|x_e(t)\| \leq \Pi^0; \quad \left|e_N'(t)\right| \leq 2\tau\kappa K_{eN} C(t) + \Pi ;$$

$$|\pi_{ei}(t)|, |\pi_{0i}(t)| \leq \Pi^0; \quad i = 0, \ldots, N; \quad |\beta_{uN}(t)| \leq \tau K_{\beta N} C(t) + \Pi^0$$

where, $\Pi^0(t)$ and $\Pi(t)$ are exp. decaying terms depending on the initial conditions, and

$$C_1(t) = \sup_t \|\omega(t)\| ; \quad C(t) = M_\theta C_1(t) + M_{red}$$

with some positive constants M_θ, M_{red} and $\tau := \max_i \tau_i$.

4.4.4 Stability Theorems II

Interpretation of Theorem 1

Basically, Theorem 1 says that all auxiliary errors decay exponentially to zero, except the last one ε_N which tends exponentially to a "small" residual value of order $\tau C(t)$. But $C(t)$ depends on the states of the system, so in order to conclude stability, a further step is Theorem 2.

4.4.4 Stability Theorems III

Theorem 2: Global stability/tracking for $n^* \geq 1$

Assume that the modulation functions satisfy Theorem 1. Then, for sufficiently small $\tau > 0$, the full error system with state z is globally exponentially stable with respect to a residual set of order τ, i.e., there exist positive constants K and δ such that $\forall z(0)$, $\forall t \geq 0$, $\|z(t)\| \leq K e^{-\delta t} \|z(0)\| + O(\tau)$.

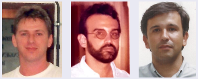
Proof:
Based on

- a small gain argument
- a recurrence relation relating the full error state z from time t to $t + T$ where T is some large enough period

4.4.4 Stability Theorems IV

This proves stability and convergence to a residual set, the size being independent of the initial conditions.

The players



(Costa, Araújo, Lizarralde (circa 1995))

5. From theory to practice I

hideallsubsections

5. From theory to practice II

The VS-MRAC was successfully applied to a number of practical problems.

5.1 Dynamic Positioning of an ROV I

Dynamic positioning of an ROV is perfect for SMC application due to model uncertainties and environmental disturbances

Two main publications report the application of the VS-MRAC to ROV Dynamic Positioning Control:

- (da Cunha, Costa and Hsu 1995) – IEEE J. of Ocean Engineering
- (Hsu, Costa, Lizarralde and da Cunha J. 2000) – IEEE Robotics and Automation Magazine

5.1 Dynamic Positioning of an ROV II

The Passive Arm

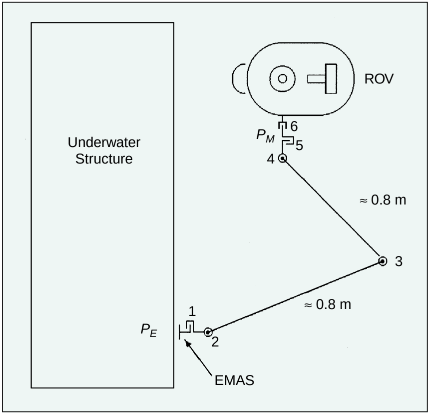


Figure 1. *Passive arm.*

The ROV-Passive Arm system

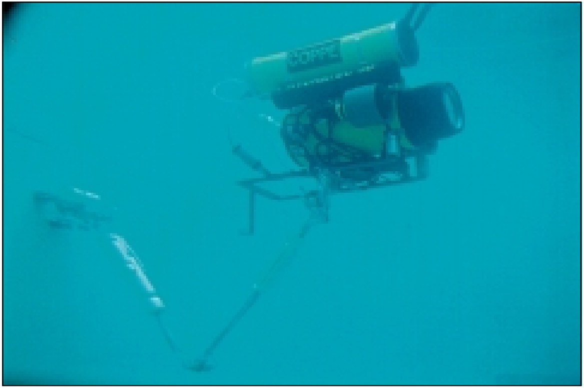


Figure 3. *The passive arm installed on the MKII ROV.*

5.1 Dynamic Positioning of an ROV IV

ROV Coordinate system

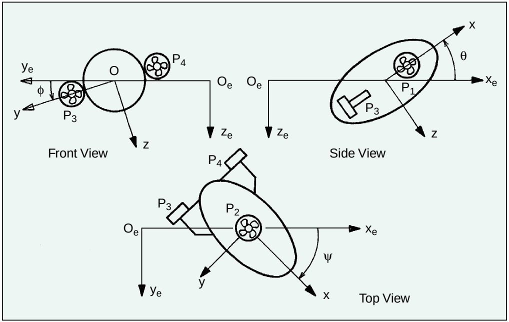


Figure 7. *Schematic ROV views and coordinate systems, where $\mathcal{O}$ and $\mathcal{O}_e$ are the origins of the body and inertial frames, respectively; $P = [x_e \quad y_e \quad z_e]^T$ is the ROV position given by the inertial coordinates of $\mathcal{O}$; x, y, and z are the body coordinate axes; x_e, y_e, and z_e are the inertial coordinate axes (also the inertial coordinates of $\mathcal{O}$); ϕ, θ, and ψ are the roll, pitch, and heading Euler angles, respectively; $Q = [\phi \quad \theta \quad \psi]^T$ is the ROV attitude.*

P-PI (*Proportional-Proportional Integral*) linear Control

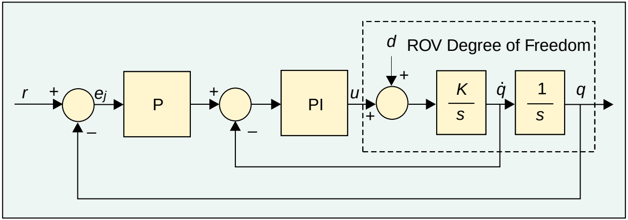


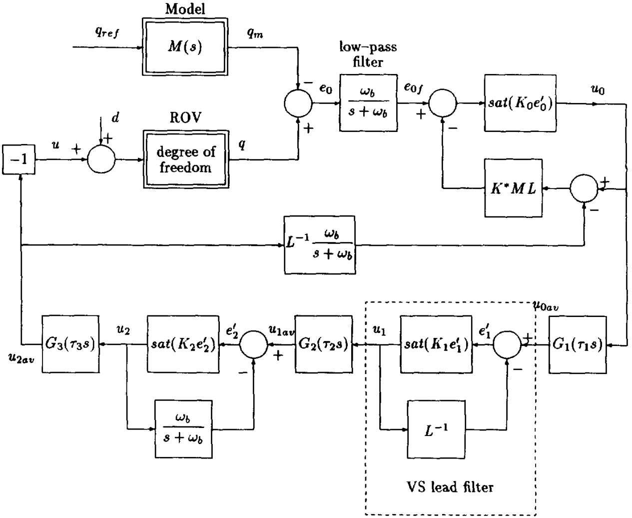
Figure 8. *Block diagram of the P-PI.*

(IEEE RAM 2000)

5.1.1 Linear vs nonlinear control algorithms II

VS-MRAC ($n^* = 3$) as applied for ROV DP (Note the noise filter)

5.1.1 Linear vs nonlinear control algorithms III



5.1.2 Experimental results with 350Kg ROV (Tatuí-I) P-PI

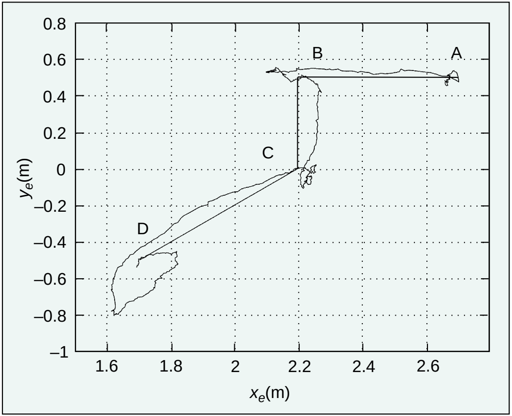


Figure 10. *Trajectory tracking tests with the P-PI control algorithm applied to a large ROV. Horizontal $x_e y_e$ plane view.*

(IEEE RAM 2000)

5.1.3 Experimental result of ROV (Tatuí-I) VS-MRAC DP

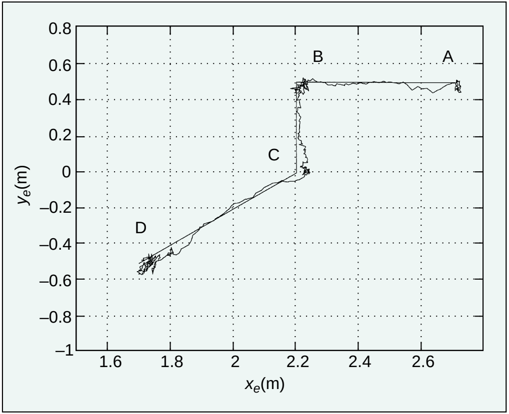


Figure 11. *Trajectory tracking tests with the VS-MRAC control algorithm applied to a large ROV. Horizontal $x_e y_e$ plane view.*

Movie

5.2 Robot manipulator applications I

- VS-MRAC extended to the tracking control of robot manipulators without joint velocity measurements (Hsu and Lizarralde 1995)
- A decentralized VS-MRAC was implemented on a PUMA 560 manipulator
- Better results than in the literature
- R. Guenther developed the VS-MRAC for Flexible Link and Rigid Link Electrically Driven manipulators using cascade control

5.2 Robot manipulator applications II

Equations of n-link rigid manipulator in joint space

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \Gamma \tag{5}$$

- $q \in \Re^n$ is the vector of joints;
- $\Gamma \in \Re^n$ is the vector of torques;
- $H(q) \in \Re^{n \times n}$ is the inertia matrix;
- $C(q,\dot{q})\dot{q}$ represents the centrifugal and Coriolis torques/forces;
- $g(q) \in \Re^n$ is the vector of gravitational torques/forces

OOPS! a nonlinear system!!

We wish to design a suitable control to ensure that the joint tracking error

$$\tilde{q} = q - q_d \tag{6}$$

remains *small*.
The desired trajectory and derivatives $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are given.

Lagrangian systems are nonlinear.

We need to bring our system to a linear form with nonlinear disturbances.
The proposed strategy is based on the following ideas:

- Using *Computed Torque*, linearize and decouple into n subsystems, with the available (nominal) parameter information;
- Regard imperfect compensation as an input disturbance to each subsystem;
- Control each subsystem by means of the I/O VS-MRAC. This circumvents the problem of velocity measurement.

Linearization and decoupling is trivial in the case of perfect parameter knowledge and with position and velocity measurements. Indeed, using

$$\Gamma = H(q)u + U_{ff} \tag{7}$$

$$U_{ff} = H(q)\ddot{q}_d + C(q,\dot{q})\dot{q} + g(q) \tag{8}$$

we obtain from the dynamic equation of the manipulator (5), the following system

$$\ddot{\tilde{q}} = u \tag{9}$$

where u is the control vector to be designed so as to achieve asymptotic tracking ($\tilde{q} \to 0$). We can thus control separately each joint, reduced to simple double integrators.

5.2 Robot manipulator applications VI

Now, when the system parameters are known only *nominally* and $\dot{q}$ is not measured, the feedforward terms of U_{ff} (8) can be replaced by an approximation, using nominal parameter values and desired trajectory quantities, i.e.,

$$U_{ff}^{o} = H^{o}(q_d)\ddot{q}_d + C^{o}(q_d, \dot{q}_d)\dot{q}_d + g^{o}(q_d) \tag{10}$$

where the superscript ^{o} in H, C and g indicates that nominal parameters are being used.
Now, the subsytems is reduced disturbed and coupled double integrators
with the control law given by

$$\Gamma = H^{o}(q)u + U_{ff}^{o} \tag{11}$$

5.2 Robot manipulator applications VII

one obtains from (5), the following system

$$\ddot{\tilde{q}} = u + d \tag{12}$$

where

$$d = d_\alpha + d_\beta + d_\gamma + d_\delta \tag{13}$$

with disturbances terms bounded (elementwise) by:

$$|d_{\alpha i}| \leq K_{1i}^{\alpha} \left\| \dot{\tilde{q}} \right\| + K_{2i}^{\alpha} \left\| \dot{\tilde{q}} \right\|^2 \tag{14}$$

$$|d_{\beta i}| \leq \delta_{1i} \| u \| \tag{15}$$

$$|d_{\gamma i}| \leq \delta_{2i} \| \ddot{q}_d \| + \delta_{3i} \| \dot{q}_d \|^2 + \delta_{4i} \tag{16}$$

$$|d_{\delta i}| \leq K_{1i}^{\delta} \| \ddot{q}_d \| + K_{2i}^{\delta} \| \dot{q}_d \|^2 + K_{3i}^{\delta} \tag{17}$$

where δ_{ki}, K_{ki}^{δ} and K_{ki}^{α} are nonnegative constants.
Note that the constants δ_{ki} tend to zero when nominal values approach the true values, i.e., $\tilde{H} \to 0$, $\tilde{C} \to 0$ and $\tilde{g} \to 0$.

5.2 Robot manipulator applications VIII

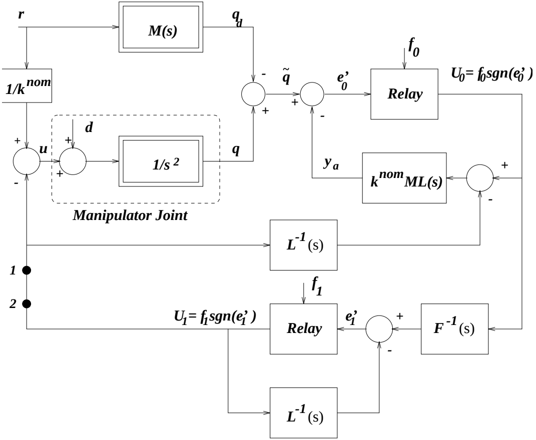
The controller is designed to generate the control signal u_i for each of the subsystems of (12), namely,

$$\ddot{\tilde{q}}_i = u_i + d_i \tag{18}$$

where $\tilde{q}_i$, u_i and d_i are the i-th component of $\tilde{q}$, u and d, respectively.
As can be observed, the plant (18) has relative degree $n^* = 2$.

Thus, the VS-MRAC for $n^* = 2$ can be applied to each generic degree of freedom.

5.2 Robot manipulator applications X



Theorem

Consider system MRAC error system with $u(t)$ given by the VS-MRAC of Fig. 56. Let $z(t)$ be the complete state of the error system as defined above and let $C(t)$ be defined as

$$C(t) = M_\theta C_\omega(t) + \|W_d\| \, C_d(t) \tag{19}$$

where $C_\omega(t) = \sup_t |\omega(t)|$; $C_d(t) = \sup_t |d(t)|$ and $\|\theta^*\| \leq M_\theta$. Then one has

$$\|e\| \leq \tau K_e C(t) + EXP \tag{20}$$

Moreover, if the following stability condition

$$C(t) \leq K_1 \|z(0)\| + K_2 \tag{21}$$

holds $\forall z(0)$, then $z(t)$ is globally exponentially stable with respect to some small residual set with magnitude of order τ.

5.2 Robot manipulator applications XII

The proof of **(B)** invokes the Frobenius-Perron's Theorem, due to the residual coupling of the control of each subsystem which is fortunately of order $O(\tau)$.

5.2 Robot manipulator applications XIII

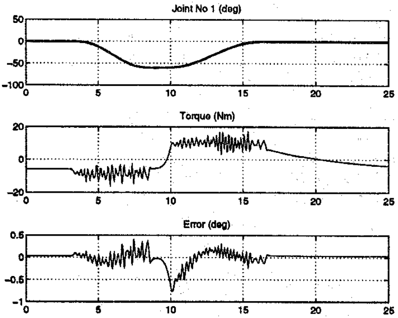


Figure 2: Joint $N°1$, a) q and q_d in degrees, b) control signal Γ in Nm and c) tracking errors in degrees.

5.2 Robot manipulator applications XIV

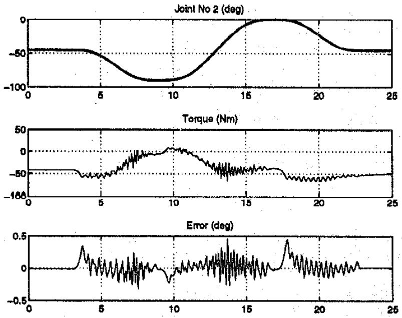


Figure 3: Joint $N°2$, a) q and q_d in degrees, b) control signal Γ in Nm and c) tracking errors in degrees.

5.2 Robot manipulator applications XV

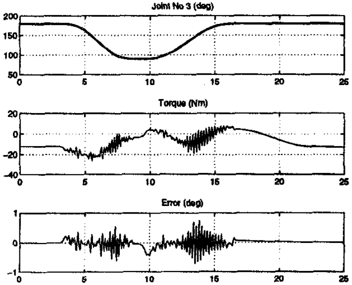


Figure 4: Joint $N°3$, a) q and q_d in degrees, b) control signal Γ in Nm and c) tracking errors in degrees.

5.3 Other Applications

- A.D. de Araújo (UFRN, Natal) developed several successful applications with DC and Induction motor control (a CHESF project 2009).
- He proposed several variations of the VS-MRAC, including adaptive pole placement control with variable structure (VS-APPC).
- Sahjendra N. Singh (UNLV, Las Vegas) and A. D. Araújo: applications of the VS-MRAC to aerospace and aircraft problems. One example (2012) is in satellite formation control.

The players




(Costa, Lizarralde, Cunha, Araújo (circa 2000))

6. MIMO VS-MRAC I

6.1 Multivariable VS-MRAC

References: (Cunha, Hsu, Costa and Lizarralde 2002, 2003, 2006, 2008-FOAF)

Other approaches: (Spurgeon and Edwards 1998), (Emelyanov et al 1992), (Chien et al 1996), (Bandhiopadhyai 2002 (dicrete-time))

A powerful approach is the High Gain Observer approach for output feedback SMC (Oh and Khalil 1995, 1997). However, peaking and noise sensitivity are of concern.

6.1.1 UV-MRAC $n* = 1$

- Designs for linear and nonlinear multivariable plants;
- Unit vector control;
- $-K_p$ Hurwitz uncertain High Frequency Gain (N & S for sliding!);
- Global stability with output feedback.

Remark: First Order Approximation Filters (**FOAF**) are instrumental to extend VS-MRAC to nonlinear systems.

6.1.1 UV-MRAC $n* = 1$ II

Problem statement

- Plant

$$\dot{x}_p = A_p x_p + \phi(x_p, t) + B_p u \,, \qquad y = C_p x_p$$

$$x_p, \phi \in \mathbb{R}^n, \quad y, u \in \mathbb{R}^m$$

- Linear subsystem transfer function matrix:

$$G(s) = C_p(sI - A_p)^{-1} B_p$$

- High frequency gain matrix:

$$K_p = C_p B_p$$

6.1.1 UV-MRAC $n* = 1$ III

Special assumptions

(A1) S_p is known such that $-K_p S_p$ is Hurwitz (relaxes the positive definiteness condition)

(A2) $\phi(x_p, t)$: piecewise continuous in t and locally Lipschitz in x_p

(A3) $\|\phi(x_p, t)\| \leq k_x \|x_p\| + \varphi(y, t)\,, \quad k_x, \varphi \geq 0$ are known

(A1) relaxes a positive definiteness condition. All uncertainty is expressed as a Hurwitz condition.
It is less conservative than allowing $\phi = B\xi(t, x_p, u)$ and requiring

$$\|\xi(t, x_p, u)\| \leq k_1 \|u\| + \alpha(t, x_p)$$

and bounding the gain k_1 as made in several other published works (e.g. (Edwards and Surgeon 1998)).

6.1.1 UV-MRAC $n* = 1$ IV

Unit Vector control law

$$u = u^{nom} - S_p\,\rho \frac{e}{\|e\|}$$

Modulation (or variable gain) function:

$$\rho = \delta + c_1\|\omega\| + c_2\|r\| + c_3\|e\| + \hat{\phi}(t)$$

output feedback!.

6.1.1 UV-MRAC $n* = 1$ V

Lemma

Consider the MIMO system

$$\dot{e}(t) = A_M e(t) + K\left[u + d(t) + \pi(t)\right] , \tag{22}$$

$$u = -\rho(e,t)\frac{e}{\|e\|} , \tag{23}$$

where $A_M, K \in \mathbb{R}^{m\times m}$; $d(t)$, $\pi(t)$ and ρ are LI. If $-K$ is Hurwitz and

$$\rho(e,t) \geq \delta + c_e\|e(t)\| + (1 + c_d)\|d(t)\| , \tag{24}$$

where $c_e, c_d \geq 0$ are appropriate constants, and $\delta \geq 0$ is an arbitrary constant, then $\exists k_1, k_2, \lambda_1 > 0$ such that

$$\|e(t)\| \leq (k_1\|e(0)\| + k_2 R)\exp(-\lambda_1 t) . \tag{25}$$

Therefore, for $\pi(t) \equiv 0$ the system is globally exponentially stable. Moreover, if $\delta > 0$, then the sliding mode at $e = 0$ is reached after some finite time $t_s \geq 0$.

Proof: see (Cunha et al, 2003) [Cunha, Hsu, Costa, and Lizarralde 2003].

6.1.1 UV-MRAC $n* = 1$ VI

Theorem

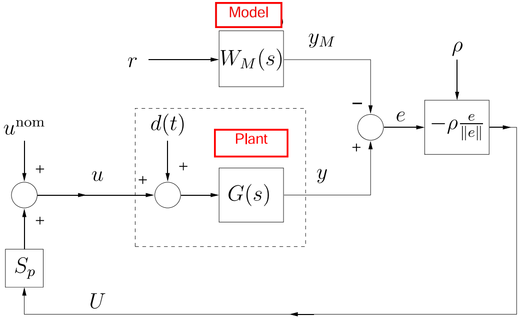
Theorem 1 If certain assumptions including (A1)–(A3) are verified, then the The UV-MRAC system is globally exponentially stable. Moreover, if $\delta > 0$, the output error $e(t)$ becomes zero after some finite time.

Proof: Application of a Lemma 1 to the nonminimal realization of error equation and the equations for the transient state of W_d and of the filter that generates $\hat{d}$. The transient state is incorporated to the π term of Lemma 2 of Sec. 4 . ■

6.1.1 UV-MRAC $n* = 1$ VII

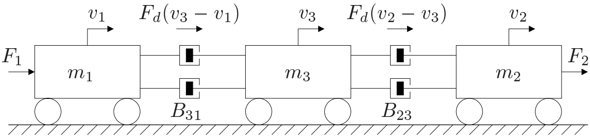
Remark: The Hurwitz condition is necessary and sufficient for UVC.

UV-MRAC (n*=1) Block diagram



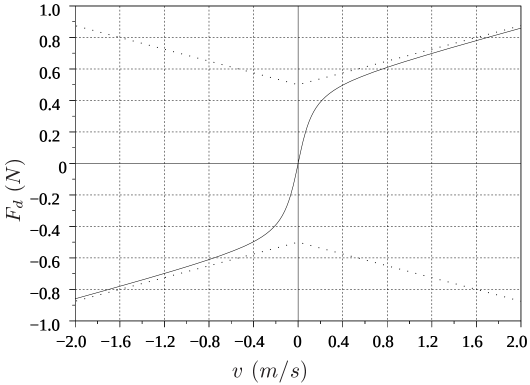
6.1.1 UV-MRAC $n* = 1$ IX

Simulation Example: Three car chain

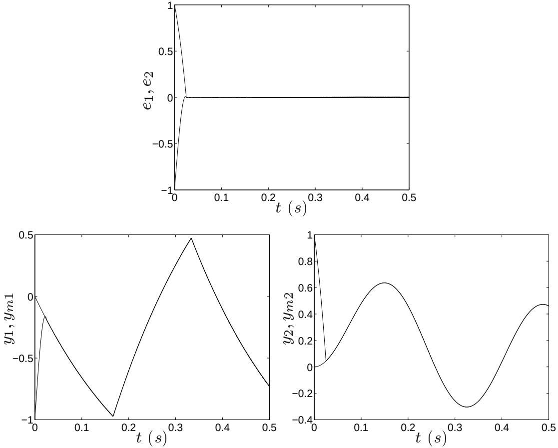


$$y = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad x_p = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Nonlinear damper



Position control of carts 1 and 2



6.2 UV-MRAC, $n^* \geq 1$

Two options have been proposed:

- Generalize the VS-MRAC SISO by using unit vectors instead of relays
- Use High Gain Observers (HGO) to get the necessary (error) state estimation of uncertain plants.

6.2.1 UV-MRAC properties, $n^* \geq 1$]

Summary of properties

- Applicable for a class of uncertain multivariable nonlinear systems;
- Assumes uniform relative degree $n^* \geq 1$;
- Includes nonlinear state dependent and unmatched disturbances;
- Peaking free (in contrast to well known HGO based design);
- Global or semi-global stability with respect to some residual set.

Reference:(Hsu et al IFAC2005)
Related literature: (Edwards and Spurgeon 1998), (Oh and Khalil 1995)

6.2.2 Problem statement

- Plant (square system: $y,\ u \in \mathbb{R}^m$

$$
\begin{aligned}
\dot{x}_p &= A_p x_p + \phi(x_p, t) + B_p u \\
y &= C_p x_p
\end{aligned}
$$

- Linear subsystem transfer function matrix:

$$G(s) = C_p(sI - A_p)^{-1} B_p$$

- High frequency gain matrix:

$$K_p = C_p A_p^{n^*-1} B_p$$

6.2.3 Assumptions

(A1) Standard MRAC assumptions for $G(s)$

(A2) $G(s) \rightarrow$ known relative degree n^*

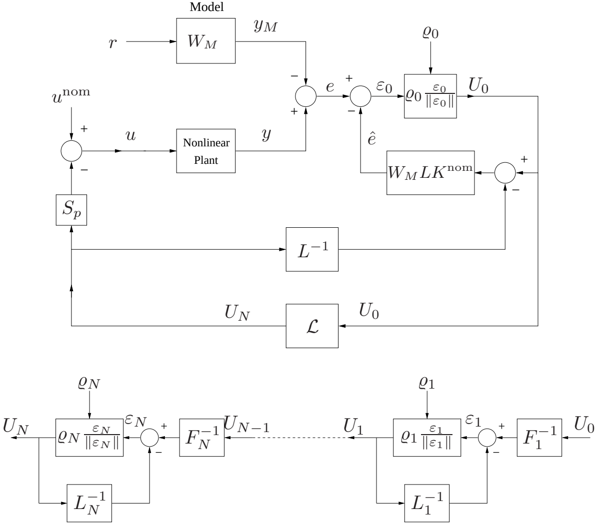
(A3) Matrix S_p known such that $-K_pS_p$ Hurwitz $\rightarrow$ *reduce prior knowledge of K_p*

(A4) ϕ is **locally Lipschitz** in x_p and piecewise continuous in t

$$\phi(x_p,t)\| \leq k_x\|x_p\| + \varphi(y,t)\,,\ \forall (x_p,t)\,,\ \text{with } k_x,\varphi \ \textbf{known}$$

Note: $\varphi = \|y\|^2 \rightarrow$ finite-time escape is not precluded, *a priori*

6.2.3 UV-MRAC Block Diagram, $n^* \geq 1$



6.3 VS-MRAC with HGO

- Instead of the VS-lead filters of VS-MRAC, it is possible to use High Gain Observers.
- Its is different from using lead compensators.
- The distinctive difference is that, observers may form an Ideal Sliding Loop, even if the plant has unmodeled dynamics.
- Therefore the controller is expected to be less prone to chattering.

6.3.1 VS-MRAC with HGO, SISO

First consider the SISO case.
Model Reference: $\{A_M, \ B_M, \ C_M\}$, in observer canonical form.
The Model Following error state equation:

$$\begin{aligned} \dot{x}_e &= A_M x_e + B_M \frac{K_p}{K_M} \left[u - \theta^{*T} \omega + W_d(s) * d(t) + \pi_e \right] \\ e &= C_M x_e \end{aligned}$$

The high gain observer (smart placement!):

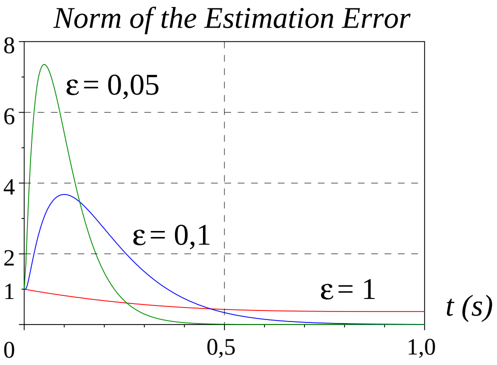
$$\begin{aligned} \dot{\hat{x}}_e &= A_M \hat{x}_e + B_M k^{nom} U - [\alpha(\varepsilon^{-1}) - a_M]\tilde{e} \\ \tilde{e} &= C_M \hat{x}_e - e, \quad e = y - y_M \end{aligned}$$

The OF SMC law

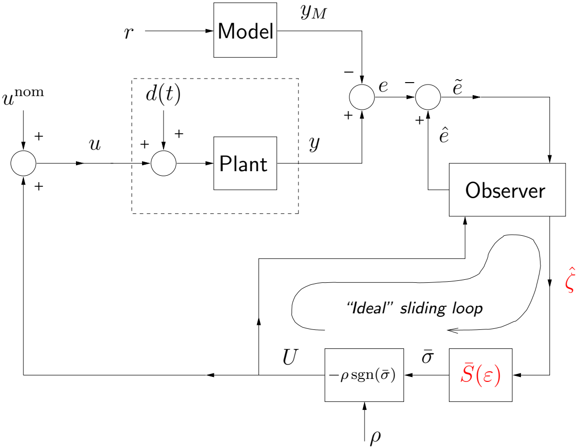
- Control law: $u = u^{nom} - \rho\,\mathrm{sign}(S\hat{x}_e)$
- Modulation function: $\rho(t) \geq \left\| (\theta^{\mathrm{nom}} - \theta^*)^T \omega + W_d(s) * d(t) \right\|$
- $\hat{x}_e$ is the estimate of x_e (from a HGO).
- S is s.t. $S(sI - A_M)^{-1} B_M = W_M(s) L(s)$ is SPR

6.3.2 Caveat: HGO has peaking

Problem: Peaking in the HGO as $\varepsilon \to 0+$.



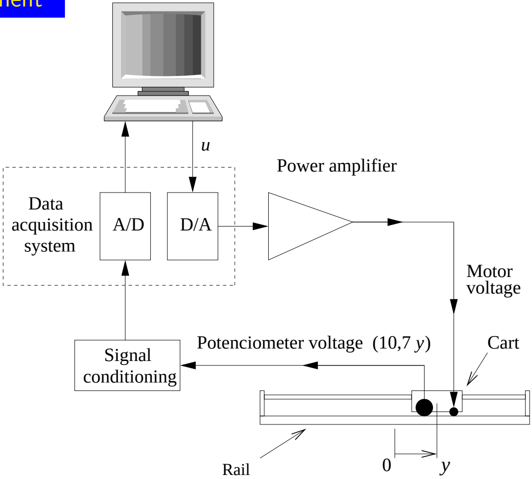
6.3.3 Peaking-free control with ISM via HGO



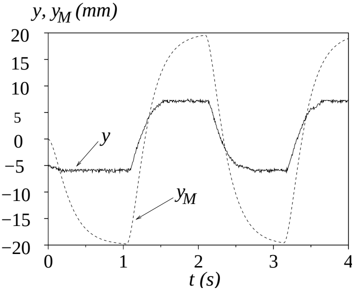
Better robustness than using lead compensators using differentiators? (to be confirmed theoretically...)

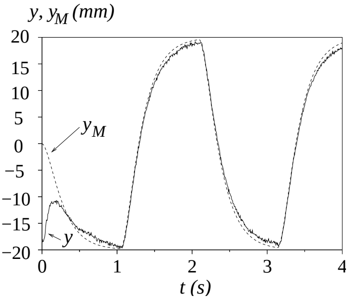
6.3.4 Experimetal setup

Experiment



6.3.5 HGO VS-MRAC cart position control





Nominal linear control	HGO + VSC + SVF
Nominal cart mass	Augmented cart mass

6.3.6 Conclusion for VS-MRAC with HGO

Conclusion

- New algorithm VSC + HGO + State Variable Filters
- The sliding surface is generated using the HGO state.
- Modulation function based on the filters state
- Main result: global exponential stability without *peaking*

The players

(Costa, Lizarralde, Cunha, Peixoto (circa 2000))

7. Binary MIMO MRAC and Passivation

7.1 Motivation

- MRAC leads to continuous control signal but lacks robustness and can present bad adaptation transient.
- UV-MRAC exhibits invariance properties, robustness and good convergence. Needs infinite switching frequency and is chattering prone.
- B-MRAC acts as a bridge between them and combines their desirable properties and avoiding their drawbacks.
- The B-MRAC consists basically of the conventional MRAC modified by parameter projection combined with high adaptation gain.

7.2 Passivity framework

- The Lyapunov based design of MIMO MRAC requires the SPR passivity condition for the error equation.
- This implies a stringent symmetry condition on the high frequency gain matrix K_p.
- A new generalized passivity requires the weaker WSPR condition.
- WSPR does not require K_p to be positive definite symmetric. It only requires it to have Positive Diagonal Jordan form (PDJ).

SPR condition

The system

$$\dot{x} = Ax + Bu\,, \tag{26}$$

$$y = Cx\,, \tag{27}$$

is Strictly Passive (SPR) if and only if there exist symmetric and positive definite (SPD) matrices P and Q satisfying

$$A^T P + PA = -Q, \tag{28}$$

$$PB = C^T. \tag{29}$$

Then the symmetry condition is easy to verify:

$$K_p = CB = B^T C^T > 0$$

where the matrix K_p is the high frequency gain matrix, deemed to be SPD.

WSPR condition

The system satisfies the WSPR condition if besides P, Q, there exists W SPD, such that

$$A^T P + PA = -Q, \tag{30}$$

$$PB = C^T W. \tag{31}$$

Note that W is not used for the control design. Only its existence is required!

PDJ condition

- From $PB = C^T W$, it can be noted that

$$B^T PB = B^T C^T W = (CB)^T W$$

is symmetric and positive definite (SPD).

- Given a matrix $CB \in \mathbb{R}^{m\times m}$, then exist a matrix $\bar{W} = \bar{W}^T > 0$, $\bar{W} \in \mathbb{R}^{m\times m}$ such that

$$\bar{W}(CB) = (CB)^T \bar{W} > 0, \tag{32}$$

if and only if CB has real and positive eigenvalues and its Jordan form is diagonal (PDJ).

Application of the concept of passivity on MRAC MIMO

- Control objective: To find $u(t)$ such that

$$e(t) = y_p(t) - y_M(t).$$

tends to zero asymptotically for arbitrary CIs.

- The concepts of WSPR and WASPR can be applied.
- Consider the modified error equation.

$$\dot{x}_e = A_K x_e + B_c K_p [u - u^*]\,,$$
$$e_L = Le\,, \qquad (e = H_o x_e\,),$$

where $A_K = A_c - B_c K_p K L H_o$

- L is chosen so that $\{A_K, B_c K_p, LH_o\}$ is PDJ.

Determination of the passifying multiplier L

- Consider the factorization $K_p = L_p D_p U_p$.
- The diagonal matrix D_0 is chosen.
- A lower triangular multiplier matrix $L = D_0 (L_p D_p)^{-1}$ can be obtained so that

$$\bar{K}_p = L K_p = D_0 (L_p D_p)^{-1} (L_p D_p) U_p = D_0 U_p,$$

- Then the modified error system

$$e_L = W_M(s) L K_p \tilde{u}, \quad \tilde{u} = u - u^*, \text{ is WSPR.}$$

7.2 MIMO B-MRAC

The B-MRAC was proposed by Hsu and Costa in the early 90's for SISO systems. Here we extend it to the MIMO case. To this end, a passivity framework is helpful.

- In the MIMO case, the control law can be parametrized in the followig forms
- The projection of a vector is more natural than the projection of a matrix, then consider.

- Instead of a matrix $\Theta \in \mathbb{R}^{N \times m}$, a modified vector $\theta \in \mathbb{R}^{Nm}$.

- Instead of a vector $\omega \in \mathbb{R}^{N}$, a modified matrix $\Omega \in \mathbb{R}^{Nm \times m}$.

7.2 MIMO B-MRAC II

- Given by

$$\Omega = I_m \otimes \omega = \begin{bmatrix} \omega & & \\ & \ddots & \\ & & \omega \end{bmatrix}, \qquad \theta = \mathrm{vec}(\Theta) = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix},$$

where θ_i corresponds to the i-th column of the parameter matrix Θ.

B-MRAC MIMO

- Thus, the adaptation law B-MRAC MIMO is given by

$$\dot{\theta} = -\sigma\theta - \gamma\Omega e_L,$$

$$\sigma = \begin{cases} 0, & \text{if } \|\theta\| < M_\theta \text{ or } \sigma_{eq} < 0, \\ \sigma_{eq}, & \text{if } \|\theta\| \geq M_\theta \text{ and } \sigma_{eq} \geq 0, \end{cases}$$

$$\sigma_{eq} = \frac{-\gamma\theta^T\Omega e_L}{\|\theta\|^2},$$

where

$$M_\theta > \|\theta^*\|$$

$$u(t) = \Theta^T(t)\omega(t) = \Omega^T(t)\theta(t).$$

Connection between B-MRAC and Unit Control Vector

Consider the B-MRAC adaptive law

$$\gamma^{-1}\dot{\theta} = -\gamma^{-1}\theta\sigma_{eq} - \Omega e_L$$

when $\gamma \to \infty$, it can be verified that θ is collinear with Ωe_L, hence θ can be express by

$$\theta = -M_\theta \frac{\Omega e_L}{\|\Omega e_L\|}.$$

Then, the form of the UVC law can be obtained

$$u = -M_\theta \|\omega\| \frac{e_L}{\|e_L\|}.$$

7.3 Direct adaptive visual tracking

- Direct adaptive visual tracking of planar manipulators.
- Fixed camera (plant) with optical axis orthogonal to the robot workspace.
- The camera orientation angle is uncertain with respect to the coordinates of the robot workspace.

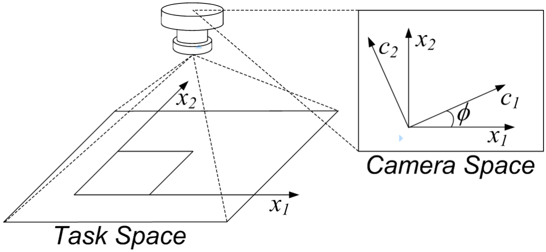


Figure: Representation of the camera-robot system

Equations of the visual tracking system

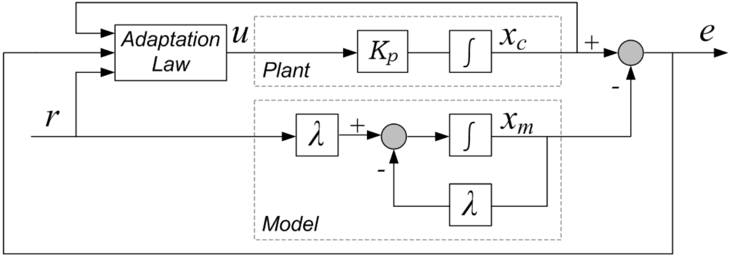


Figure: Representation of the camera-robot system

$$\dot{x}_c = K_p u, \qquad K_p = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix},$$

$x_c \in \mathbb{R}^2$ Coordinates of the end-effector of the image plane.

$K_p \in \mathbb{R}^{2\times 2}$ Rotation matrix.

$u \in \mathbb{R}^2$ Cartesian control law.

Equations of the visual tracking system

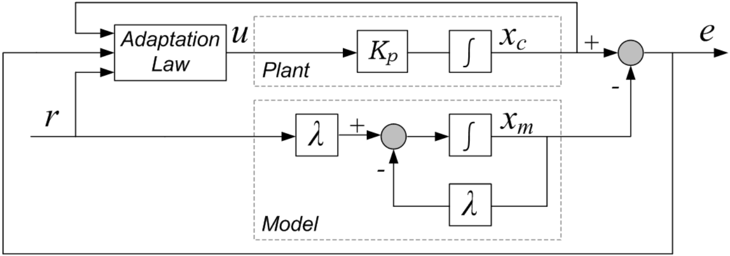


Figure: Representation of the camera-robot system

$$\dot{x}_m = -\lambda x_m + \lambda r(t),$$

$x_m \in \mathbb{R}^2$ Desired image-plane trajectory.
$\lambda \in \mathbb{R}$ A positive constant.
$r \in \mathbb{R}^2$ An arbitrary reference signal piece-wise and

Equations of the visual tracking system

Control objective: Find a control law u such that

$$e = x_c - x_m \to 0 \text{ for arbitrary CIs.}$$

Tracking error equation:

$$\dot{e} = -\lambda e + K_p u - \lambda\omega, \qquad \omega = r(t) - x_c.$$

Ideal control law:

$$u^* = {\Theta^*}^T \omega = \Omega^T \theta^*, \qquad {\Theta^*}^T = \lambda {K_p}^{-1}.$$

Determining the passifying matrix L

To turn the error system WASPR is necessary to find a constant matrix L such that LK_p is PDJ.

$$K_p \quad = \quad L_p \quad D_p \quad U_p,$$

$$K_p = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s/c & 1 \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & 1/c \end{bmatrix} \begin{bmatrix} 1 & -s/c \\ 0 & 1 \end{bmatrix},$$

Defining $D_0 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$, tem-se

$$L = D_0 (L_p D_p)^{-1} = \begin{bmatrix} \alpha/c & 0 \\ -\beta s & \beta c \end{bmatrix}.$$

Parameters of the visual tracking system

System's parameters

Initial conditions $x_c(0) = \begin{bmatrix} 5 & 5 \end{bmatrix}^T$.

Reference signal $r(t) = [10\sin(3t) \;\; 10\sin(0.5t)]^T$.

Model's constant $\lambda = 1$.

Orientation angle $\phi = 30°$.

Passifying matrix L

Nominal angle $\phi_n = 45°$.

D_0's constants $\alpha = 5$ e $\beta = 1$.

Controller's parameter M_θ

- With $\|\theta^*\| = \sqrt{2}$.
- It can be chose $M_\theta = 3$.

MRAC control with passivation and $\gamma = 5$

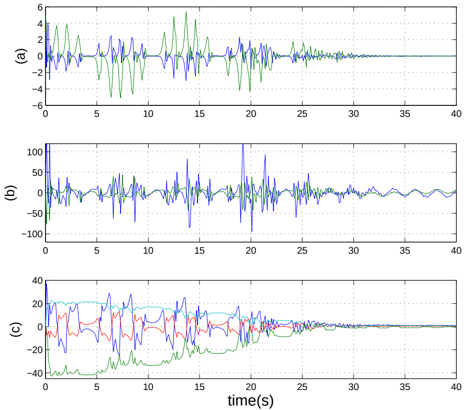


Figure: Behavior of the MRAC control with passivation and $\gamma = 5$: (a) Tracking errors e; (b) Plant control signals u; (c) Adaptive parameters

B-MRAC control without passivation and $\gamma = 5$

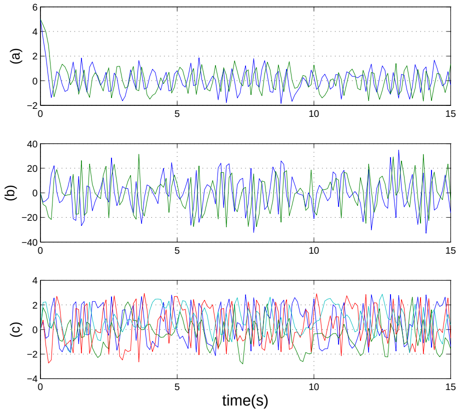


Figure: Behavior of the B-MRAC control without passivation and $\gamma = 5$: (a) Tracking errors e; (b) Plant control signals u; (c) Adaptive parameters

B-MRAC control with passivation and $\gamma = 5$

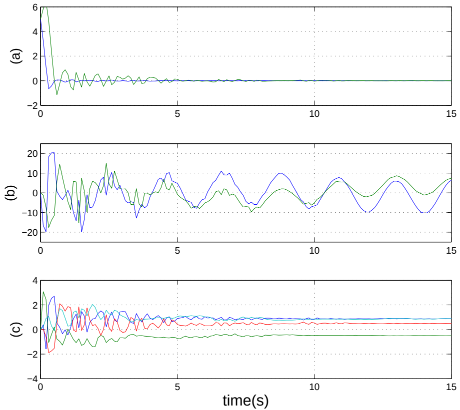


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 5$: (a) Tracking errors e; (b) Plant control signals u; (c) Adaptive parameters

B-MRAC control with passivation and $\gamma = 20$

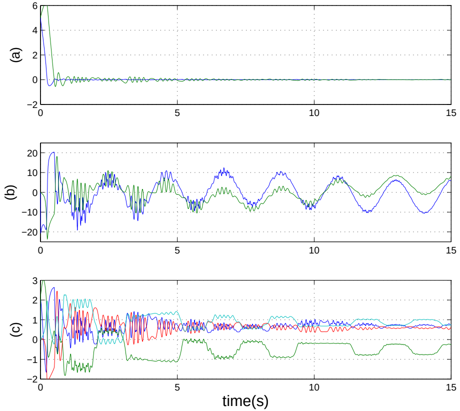


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 20$: (a) Tracking errors e; (b) Plant control signals u; (c) Adaptive parameters

UVC without passivation

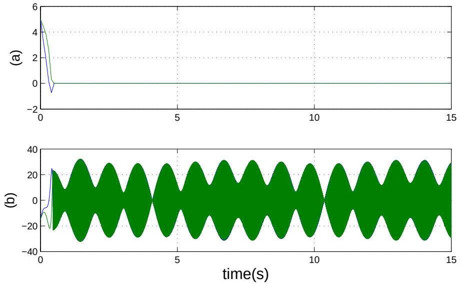


Figure: UVC without passivation: (a) Tracking errors e; (b) Plant control signals u

B-MRAC control without passivation and $\gamma = 100$

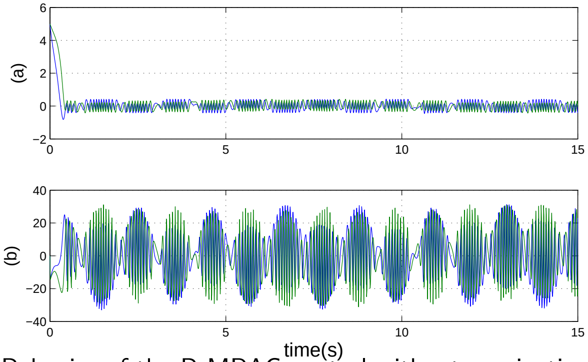


Figure: Behavior of the B-MRAC control without passivation and $\gamma = 100$: (a) Tracking errors e; (b) Plant control signals u

B-MRAC control with passivation and $\gamma = 100$

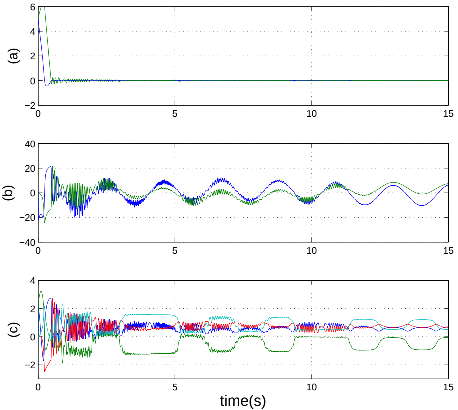


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 100$: (a) Tracking errors e; (b) Plant control signals u; (c) Adaptive parameters

7.3.1 Conclusions

- The B-MRAC was extended for MIMO systems.
- The generalized passivity concepts of WSPR and WSPR were used.
- With high adaptive gains B-MRAC's behavior gets closer to the UVC's behavior.
- The B-MRAC scheme improves the MRAC's transient.
- The passivation achieves chattering reduction.

The players

(Barkana, Teixeira, Costa, Assunção, Battistel, Nunes and Yanque (circa 2012))

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Questions?