Integral Sliding Modes

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Introduction

Conventional Sliding Mode Control (SMC) Utkin(1981)

- Main Steps
  - Reaching phase
  - Sliding phase
  - Finite-time convergence to sliding surface.
  - Reduced Order Sliding Dynamics.

Drawbacks

- Ignoring Nominal Controller
- Reaching phase
- Chattering
- Only Invariant with respect to matched perturbations.

Problem formulation

- Consider

$$\dot{x} = f(x) + B(x)\,u + \phi(x,t)\,, \tag{Unc}$$

- $x \in \mathbb{R}^n$ state
- $u \in \mathbb{R}^m$ control input
- $x(0) = x_0$
- $\phi(x,t)$ uncertainties
- Nominal System

$$\dot{x}_0 = f(x_0) + B(x_0)\,u_0. \tag{Nom}$$

- u_0 nominal control

If $\phi \neq 0$ the trajectories of (Unc) and (Nom) are different.

Problem formulation

Assumptions

A1) $\text{rank} B(x) = m$ for all $x \in \mathbb{R}^n$;

A2) $\phi(x,t) = B(x)\,\xi(x,t)$;

A3)
$$\|\xi(x,t)\| \leq \xi^{+}(x,t).$$

Control design objective

Objective

$x(0) = x_0(0)$, and $x(t) = x_0(t)$ for all $t \geq 0$.

Solution

- Design $u = u(t)$ as
$$u(t) = u_0(t) + u_1(t),$$
- $u_1(t)$ compensates $\phi(x, t)$, for all $t \geq 0$.

Linear case

LTI system

$$\dot{x}(t) = Ax(t) + B\left(u_0(t) + u_1(t)\right) + \phi(t,x), \tag{LTI}$$

$$\phi(t,x) = B\xi(t,x)$$

Sliding dynamic

$$\sigma\left(x\right) = G\left(x(t) - x\left(0\right)\right) - G\int_0^t \left(Ax\left(\tau\right) + Bu_0\left(\tau\right)\right)d\tau,$$

- $G \in \mathbb{R}^{m\times n}$ satisfies

$$\det GB \neq 0.$$

- $$\dot{\sigma}(x) = GB(u_1 + \xi)$$

Linear case

Control Design

- $$u_1 = -\rho(t,x)\frac{(GB)^T\sigma}{\|(GB)^T\sigma\|}, \quad \rho(t,x) > \|\xi^+(t,x)\|$$
- $V = \frac{1}{2}\sigma^T\sigma,$
- $$\begin{aligned} \dot{V} &= \sigma^T GB(u_1 + \gamma) \\ &\leq -\|(GB)^T\sigma\|(\rho(t,x) - \xi^+) < 0. \end{aligned}$$
- Integral sliding mode is guaranteed.