IEEE IES Distinguished Lecture

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IES History

- IES Timeline

- **February 21, 1951** First meeting of the IRE Industrial Electronics Group.
- **May 8, 1951** Name change to IRE Professional Group on Industrial Electronics (PG-IE).
- **August 1953** Published the first volume of the Transactions on Professional Group on Industrial Electronics.
- **August 21, 1963** IRE merged with AIEE and become IEEE. The PG-IE approved the merger of IRE's PG-IE and AIEE's group on control instrumentation (GCI).
- **August 22, 1963** The merger of the PG-IE and the GCI into Professional Group on Industrial Electronics and Control Instrumentation (PG-IECI)
- was approved by the IEEE Executive Committee.
- **October 14, 1964** Name change to Group on Industrial Electronics and Control Instrumentation (PG-IECI).
- **Year 1975** First IECON was held in Philadelphia. Until then there had been only a sponsored session at the Industry Applications Society Conference.
- **March 20, 1978** Name change to Industrial Electronics and Control Instrumentation Society (IECIS).
- **June, 1982** Name change to Industrial Electronics Society.

Fields of interest

- The Industrial Electronics Society through its members encompasses a diverse range of technical activities devoted to the application of electronics and electrical sciences for the enhancement of industrial and manufacturing processes.
- These technical activities address the latest developments in:
  - intelligent and computer control systems
  - robotics
  - factory communications and automation
  - flexible manufacturing
  - data acquisition and signal processing
  - vision systems
  - power electronics

Activities

- Over 5,000 members world-wide.
- 3 international, general interest conferences (spring, summer, autumn).
- Several other focused conferences (motion control, factory automation, informatics, electric machines and drives, etc).
- About two dozen technically co-sponsored conferences and workshops.
- Over 20 technical communities covering the scope of IE.

Technical Activities

1. Automotive Technology
2. Building Automation, Control and Management
3. Control, Robotics and Mechatronics
4. Data Driven Control and Monitoring
5. Education in Engineering and Industrial Technologies
6. Electrical Machines
7. Electronic Systems on Chip (ESOC)
8. Energy Storage
9. Factory Automation
10. Human Factors
11. Industrial Agents
12. Industrial Informatics
13. MEMS and Nanotechnologies
14. Motion Control
15. Network-based Control Systems and Applications
16. Power Electronics Technical Committee (PETC)
17. Renewable Energy Systems
18. Resilience and Security for Industrial Applications (ReSia)
19. Sensors and Actuators
20. Smart Grids
21. Standards

Events

- Large Conferences
  - IECON (1500)
  - ISIE (500-800)
- Small conferences and workshops
  - AMC (200)
  - ICM (200)
  - ETFA (250)
  - INDIN (150)
  - ...

Upcoming Conferences: Status

- ISIE2019 in Vancouver, Canada: June 12-14, 2019
  Acceptance/rejection decisions sent to authors.
- IECON2019 in Lisbon,Portugal: October 14-17, 2019
  Paper submission is now open.
- ICIT2020 in Buenos Aires, Argentina: Feb. 26-28, 2020
- ISIE2020 in Delft, Netherlands: June 17-19, 2020
- IECON2020 in Singapore: October 18-22, 2020
- IECON, ISIE and ICIT are the major conferences for IES. IES conferences move round the globe

IES Transactions/Journals

- IEEE Trans. on Industrial Electronics
  Impact factor: 7.05
- IEEE Trans. on Industrial Informatics
  Impact factor: 5.43
- IEEE Industrial Electronics Magazine
  Impact Factor: 10.429
- IES also supports other trans./journals e.g. IEEE Trans. on Sustainable Energy/ Smart Grid etc.

Discrete-Time Sliding Mode Control

Outline

Need for DSMC

- **With the increase in use of digital computers and microcontrollers for the implementation of control algorithms, a discrete time model of the system is justified.**
- **In many practical situations one cannot achieve performance one would expect on the basis of the continuous time theory.**
- **When continuous-time algorithm is implemented in practical systems by digital controllers, not only may the chattering be generated around the sliding mode, but the stability of the sliding mode may also be compromised.**
- **Relatively low switching frequencies are required than continuous time sliding mode control, so discrete control algorithm is more practical to implement.**
- **A large class of continuous time systems are computer controlled and information about the system measurements are available only at specific time instances and control inputs can only be changed at these time instances. Eg. - biological systems, thyristor, radar system, economic systems, etc.**

Early Works

- The first work in the area of discrete variable structure control system was published in Russia.
- But more famous works in discrete-time sliding mode control literature are done by Milososavljevic, Utkin, Darkunov, Sarpturk et.al., Furuta, Sira-Ramirez, Spurgeon, Hung et.al., Bartolini et. al., Gao et. al., Bartoszewicz and others.
- The term 'discrete-time sliding mode' was first introduced by Utkin and Darkunov.
- In case of DSM design, the control input is computed only at certain sampling instants and the control effort is held constant over the entire sampling period.

Two Schools of Thought

Switching Based Control

- Upon direct discretization of switching based control laws for continuous times cases, we get discrete control laws based on a switching function.
- However, Gao et al. shows that due to the use of the switching function, the system states would reach the vicinity of the origin , but cannot get arbitrarily close to the origin.
- This happens necessarily because of finite switching frequency in case of discrete time systems.

Control Without Switching

- Sliding mode may be achieved in discrete-time systems without the use of a switching function.
- This is due to the fact that discrete-time control is inherently discontinuous in nature and thus may not require an explicit discontinuity in the control law to bring out sliding mode control.
- Such control laws are proposed by Bartoszewicz and Bartolini et al.

Design Steps

Stable Sliding Surface

- The first step of the design procedure is exactly the same as for the continuous sliding mode surface design.
- It is assumed that the closed-loop system is kept close enough to the sliding surface to approximate the switching function by zero and design the surface for stable sliding mode dynamics.

A Reaching Control

- The second step of the design procedure is different for continuous and discrete sliding mode.
- Due to the limited switching frequency, the closed-loop system cannot be driven into a sliding mode.
- The reaching conditions are not as straightforward as for the continuous time case.

Milosavljevic's Reaching Law

Reaching Law by Milosavljevic

Milosavljevic proposed a necessary reaching condition

$$s(k)(s(k+1) - s(k)) < 0$$

where $s(k)$ is the sliding variable.

- This is direct discretization of the continuous counterpart

$$s\dot{s} < 0.$$

- However, this reaching condition only gives the direction of movement of $s(k)$, and does not guarantee convergence of $s(k)$. Hence it is only a necessary condition, and not sufficient one.

Sarpturk's Reaching Law

Reaching Law by Sarpturk

Sarpturk gave the reaching condition

$$|s(k+1)| < |s(k)|.$$

In this case not only the direction is given, but also the norm of the switching function is defined to be strictly decreasing. It can be decomposed as

- necessary Condition for existence of sliding mode

$$(s(k+1) - s(k))sgn(s(k)) < 0.$$

- The Condition for convergence, or stability, of the quasi sliding mode -

$$(s(k+1) + s(k))sgn(s(k)) > 0.$$

Lyapunov Based Reaching Law

An equivalent form of a Lyapunov-type reaching condition has been proposed for discrete case by Furuta

$$\Delta V = s^2(k+1) - s^2(k) = 2s(k)\Delta s(k) + \Delta s^2(k) < 0$$

which can be rewritten further as

$$s(k)(s(k+1) - s(k)) < -\frac{1}{2}(s(k+1) - s(k))^2.$$

Gao's Reaching Law

Reaching Law by Gao

This reaching law is given as

$$s(k+1) = (1 - Q\tau)s(k) - K\tau sgn(s(k)).$$

This is obtained by the Euler discretization of the continuous reaching law

$$\dot{s} = -Qs(t) - Ksgn(s(t))$$

where τ is the sampling interval and $Q, K > 0$ such that $0 < (1 - Q\tau) < 1$.

Such a reaching law possesses the following attributes:

1. Starting from any initial state, the trajectory will move monotonically towards the switching plane and cross it in finite time.
2. Once the trajectory has crossed the switching plane the first time, it will cross the plane again in every successive sampling period, resulting in a zig-zag motion about the switching plane.
3. The size of each successive zigzagging is non-increasing and the trajectory stays within a specified band.

The motion of a discrete SMC system satisfying the first two attributes is called a Quasi-Sliding Mode (QSM) motion.

Gao's Reaching Law

Sliding Mode Control

Let $x(k+1) = \Phi x(k) + \Gamma u(k)$ be the discrete model of the continuous system and $s(k) = c^T x(k)$ be a stable sliding surface.

$$s(k+1) = c^T x(k+1) = c^T \{\Phi x(k) + \Gamma u(k)\}$$
$$\Rightarrow u(k) = -(c^T \Gamma)^{-1}[(c^T \Phi - c^T + Q\tau c^T)x(k) + K\tau sgn(s(k))]$$

QSM Band

In QSMB, the sign of $s(k+1)$ will be opposite to $s(k)$. So, the quasi sliding mode band can be obtained by substituting $s(k+1) = -s(k)$ with $s(k) > 0$ as

$$\begin{aligned} s(k+1) &= (1 - Q\tau)s(k) - K\tau \text{sign}(s(k)) \\ -s(k) &= (1 - Q\tau)s(k) - K\tau \\ 0 &= (2 - Q\tau)s(k) - K\tau. \end{aligned}$$

So, QSMB is given as

$$s(k) = \frac{K\tau}{2 - Q\tau}.$$

Utkin's Reaching Law

Utkin's Reaching Law

$$s(k+1) = 0$$

Here the control input brings the sliding trajectory to sliding surface in a single time step.

Control Law Using Gao's Reaching Law

Linear Time Invariant Model

Let the discrete-time system be

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) + \widetilde{d}(k) \\ y(k) &= Cx(k) \end{aligned}$$

The disturbance term $\widetilde{d}(k)$ is matched and assumed to be bounded. The sliding variable is designed as

$$s(k) = c^T x(k)$$

such that it results a stable sliding mode dynamics.

Control Law Using Gao's Reaching Law

Design of Discrete-Time Sliding Mode Control

The control law can be designed as follows. First we consider the case disturbance free case, i.e., $\widetilde{d}(k) = 0$

$$\begin{aligned} s(k+1) &= c^T x(k+1) \\ &= c^T \left\{ \Phi x(k) + \Gamma u(k) \right\}. \end{aligned}$$

Thus if the control law is designed as

$$u(k) = -(c^T \Gamma)^{-1} [(c^T \Phi - c^T + Q\tau c^T) x(k) + K\tau sgn(s(k))]$$

then, the closed loop system becomes

$$s(k+1) = (1 - Q\tau)s(k) - K\tau sgn(s(k)).$$

Control Law Using Gao's Reaching Law

Design of Discrete-Time Sliding Mode Control

If the matched Uncertainty $\widetilde{d}(k)$ is considered, then

$$s(k+1) = c^T x(k+1) = c^T \left\{ \Phi x(k) + \Gamma u(k) + \widetilde{d}(k) \right\}.$$

Then the following control

$$u(k) = -(c^T \Gamma)^{-1} [(c^T \Phi - c^T + Q\tau c^T) x(k) + K\tau sgn(s(k))] - (c^T \Gamma)^{-1} c^T \widetilde{d}(k)$$

is not implementable because it contains unknown term $(c^T \Gamma)^{-1} c^T \widetilde{d}(k)$.

Control Law Using Gao's Reaching Law

Assumptions

In order to design discrete-time sliding mode control with matched disturbance, the following assumptions are made on the disturbances

$$d_l \leq c^T \widetilde{d}(k) \leq d_u.$$

The mean and spread of the disturbance are defined as $d_0 = \frac{d_u + d_l}{2}$ and $d_1 = \frac{d_u - d_l}{2}$, respectively.

Modified Reaching Law

The reaching law in case of disturbance is given as

$$s(k+1) = (1 - Q\tau)s(k) - (d_1 + K\tau)sgn(s(k)) + d(k) - d_0$$

where $d(k) = c^T \widetilde{d}(k)$. This law ensures for all $d(k)$

$$s(k)(s(k+1) - s(k)) < 0$$

Control Law Using Gao's Reaching Law

Design of Discrete-Time Sliding Mode Control

The control law can now be designed from the dynamics

$$s(k+1) = c^T \Phi x(k) + c^T \Gamma u(k) + d(k)$$

and is given as

$$u(k) = -(c^T \Gamma)^{-1}[(c^T \Phi - c^T + Q\tau c^T)x(k) + d_0 + (d_1 + K\tau)sgn(s(k))]$$

where $Q > 0, K > 0$ and $(1 - Q\tau) > 0$.

Condition for Quasi Sliding Mode

In order that trajectory cross and re-cross in each time step after reaching the sliding surface, it must be satisfied

$$sgn(s(k+2)) = -sgn(s(k+1)) = sgn(s(k)).$$

Control Law Using Gao's Reaching Law

Condition for Quasi Sliding Mode

Thus, we find first $s(k+2)$ as

$$\begin{aligned} s(k+2) &= (1-Q\tau)s(k+1) - (d_1 + K\tau)\text{sign}(s(k+1)) + d(k+1) - d_0 \\ &= sgn(s(k))\left((1-Q\tau)^2|s(k)| + Q\tau K\tau + d_1 Q\tau\right) + (1-Q\tau)(d(k)-d_0) \\ &\quad + d(k+1) - d_0. \end{aligned}$$

To ensure that $sgn(s(k+2)) = sgn(s(k))$ irrespective of $d(k)$ and $s(k)$

$$(Q\tau K\tau + d_1 Q\tau) + (1-Q\tau)(d(k)-d_0) + (d(k+1)-d_0) > 0.$$

The above is true if the lower bound of right hand side quantity greater than zero

$$\begin{aligned} & (Q\tau K\tau + d_1 Q\tau) + (1-Q\tau)(-d_1) + (-d_1) > 0 \\ \iff \quad & Q\tau\,(K\tau + d_1) > (2-Q\tau)d_1 \\ \iff \quad & d_1 < \frac{Q\tau K\tau}{2(1-Q\tau)}. \end{aligned}$$

Thus if the disturbance satisfies this there will be crossing and recrossing successively.

Quasi Sliding Mode Band

QSMB can be calculated for Modified Gao's Reaching Law from the condition that if $s(k) > 0$ then $s(k+1) < 0$. So, it gives with $s(k) = \delta$

$$(1 - Q\tau)\delta - (K\tau + d_1) + (d(k) - d_0) < 0$$
$$(1 - Q\tau)\delta < (K\tau + d_1) - (d(k) - d_0)$$
$$\delta < \frac{(K\tau + d_1) - (d(k) - d_0)}{1 - Q\tau} < \frac{K\tau + d_1 + d_1}{1 - Q\tau} < \frac{K\tau + 2d_1}{1 - Q\tau}.$$

Since $d_1 < \frac{Q\tau K\tau}{2(1-Q\tau)}$, we write further $\delta < \frac{K\tau + \frac{Q\tau K\tau}{1-Q\tau}}{1-Q\tau} = \frac{K\tau}{(1-Q\tau)^2}$.

Quasi Sliding Mode

- Discrete sliding mode is more practical for real time implementation.
- Not exact sliding mode.
- There is always chattering even in theory.

Analysis of Chattering in Quasi Sliding Mode

Why the chattering ?

QSMC is derived from discretizing continuous SMC logic

$$\dot{s} = -Qs(t) - K\text{sign}(s(t))$$
$$\Downarrow$$
$$s(k+1) = (1 - Q\tau)s(k) - K\tau\text{sign}(s(k)) \quad (1)$$

In discrete-time system the sign$(s(k))$ changes abruptly near $s(k) = 0$, but Control cannot be changed at any time. This results always chattering.

Fresh Approach:-Control Without Switching Function

Aim of Discrete Sliding Mode: To get the system to the sliding surface and maintain the state on the surface. Mathematically, one can write $s(k+1) = 0$ with $s(k) \neq 0$ for the fulfilling the both requirement. Actually this is motivated by the concepts of **Dead beat Control**.

Note:-In discrete-time control theory, the dead beat control problem consists of finding what input signal must be applied to a system in order to bring the output to the steady state in the smallest number of time steps.

Control Without Switching Function :- Utkin's Reaching Law

So, apparently the reaching law is simply $s(k+1) = 0$.

Consider the discrete time representation of continuous time linear time invariant system(where $r(t)$ is reference input)

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dr(t) \\ x(k+1) &= A^*x(k) + B^*u(k) + D^*r(k) \end{aligned} \tag{2}$$

where $A^* = e^{A\Delta t}$, $B^* = \int_0^{\Delta t} e^{A(\Delta t - t)} B d\tau$, $D^* = \int_0^{\Delta t} e^{A(\Delta t - t)} D d\tau$

For the existence of DTSM, control law is

$$\begin{aligned} s(k+1) &= CA^*x(k) + CB^*u(k) + CD^*r(k) = 0 \\ \Rightarrow u(k) &= -(CB^*)^{-1} \left(CA^*x(k) + CD^*r(k) \right) \end{aligned} \tag{3}$$

Control Without Switching Function :- Utkin's Reaching Law

Control Law can be represented as sum of two linear function as

$$u_{eq}(k) = -(CB^*)^{-1}s(k) - (CB^*)^{-1}\left((CA^* - C)\,x(k) + CD^*r(k)\right) \tag{4}$$

And

$$s(k+1) = s(k) + (CA^* - C)x(k) + CB^*u(k) + CD^*r(k) \tag{5}$$

Note above two extension are used to prove the forward move of trajectory.

Problem with above approach

Probably of too much control in the one step.

Control Without Switching Function :- Utkin's Reaching Law

Solution:-

Go slow but, be sure you are going forward direction.

Let the maximum applicable control be $\|u(k)\| \leq u_0$ and use the control as

$$u(k) = \begin{cases} u_{eq}(k) & \text{if } \|u_{eq}(k)\| \leq u_0 \\ u_0 \frac{u_{eq}(k)}{\|u_{eq}(k)\|} & \text{if } \|u_{eq}(k)\| > u_0 \end{cases} \tag{6}$$

Now, to be sure this is enough i.e., we are moving forward, we get the condition

$$|s(k+1)| < |s(k)| \tag{7}$$

Control Without Switching Function :- Utkin's Reaching Law

Proof for forward move

Consider the case $\|u_{eq}(k)\| \geq u_0$. Putting the value of $u(k)$ in the modified equation of $s(k+1)$

$$s(k+1) = s(k) + (CA^* - C)x(k) + CD^* r(k) - \left[s(k) + (CA^* - C)x(k) + CD^* r(k)\right] \frac{u_0(k)}{\|u_{eq}(k)\|} \quad (8)$$

Taking norm on both sides

$$\|s(k+1)\| = \|s(k) + (CA^* - C)x(k) + CD^* r(k) - \left[s(k) + (CA^* - C)x(k) + CD^* r(k)\right] \frac{u_0(k)}{\|u_{eq}(k)\|}\| \quad (9)$$

where

$$\|u_{eq}(k)\| = \| - (CB^*)^{-1} s(k) - (CB^*)^{-1} \left(\left(CA^* - C\right) x(k) + CD^* r(k)\right) \| \quad (10)$$

Proof for forward move

Using norm inequality

$$\|s(k+1)\| \leq \|s(k)\| + \|(CA^* - C)x(k) + CD^*r(k)\| - \frac{u_0(k)}{\|(CB^*)^{-1}\|} \tag{11}$$

For convergence

$$\begin{aligned} \|(CB^*)^{-1}\| \| \left(\left(CA^* - C \right) x(k) + CD^*r(k) \right) \| &< u_0(k) \\ \Rightarrow \|s(k+1)\| &< \|s(k)\| \end{aligned} \tag{12}$$

Robust Non-switching type DSMC

Consider the system

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) + D_\tau f(k) \tag{13}$$

Uncertainty satisfied matching condition

$$D_\tau = \Gamma_\tau \widetilde{D}_\tau \tag{14}$$

State equation becomes using above matching condition

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau (u(k) + \widetilde{D}_\tau f(k)) \tag{15}$$

Stable sliding function

$$s(k) = c^T x(k) \tag{16}$$

Control law

Using the reaching law $s(k+1)=0$, we get

$$c^T(\Phi_\tau x(k) + \Gamma_\tau(u(k) + \widetilde{D}_\tau f(k))) = 0$$

$$u(k) = -(c^T\Gamma\tau)^{-1}\left[C^T\Phi_\tau x(k)\right] - \underbrace{\widetilde{D}_\tau f(k)} \quad (17)$$

Above control is not feasible because control part contains uncertain term $\widetilde{D}_\tau f(k)$.

Robust Non-switching type DSMC

Modified Control law can be obtained by ensuring at all instants of the time that the maximum deviation of the trajectory form the sliding surface is the spread of the disturbance d_1. We define

$$\widetilde{d}(k) = c^T \Gamma \tau \widetilde{D}_\tau f(k)$$

Assumptions:-

$d_l \leq \widetilde{d}(k) \leq d_u$, and the mean and spread of the disturbance are defined as $d_0 = \frac{d_u + d_l}{2}, \quad d_1 = \frac{d_u - d_l}{2}$.

The modified reaching law is given by

$$s(k+1) = \widetilde{d}(k) - d_0 \tag{18}$$

And the modified control law is given as

$$u(k) = -(c^T \Gamma \tau)^{-1} \left[C^T \Phi_\tau x(k) + d_0 \right] \tag{19}$$

Bartoszewicz's Reaching Law

- **Another approach to the DSMC problem is to design a control algorithm that would inherently go slow.**
- **Instead of trying to reach the surface in one step. Try to reach it in say k* steps.**

Bartoszewicz proposed the reaching law

$$s(k+1) = d(k) - d_0 + s_d(k+1) \tag{20}$$

where $s_d(k)$ is an *a priori* known function such that the following applies

Conditions

If $|s(0)| > 2d_1$, then

$$s_d(0) = s(0) \tag{21}$$
$$s_d(k).s_d(0) \geq 0 \quad \text{for} \quad \text{any} \quad k \geq 0 \tag{22}$$
$$s_d(k) = 0 \quad \text{for} \quad \text{any} \quad k \geq k^* \tag{23}$$
$$|s_d(k+1)| < |s_d(k)| - 2d_1 \quad \text{for} \quad \text{any} \quad k < k^* \tag{24}$$

Bartoszewicz's Reaching Law...Cntd.

- These relations state that $s_d(k)$ converges monotonically and in finite time from its initial position $s_d(0) = s(0)$ to the sliding surface.
- Furthermore, in each control step, $s_d(k)$ moves by the distance greater than $2d_1$.
- This, together with the reaching law, imply that the reaching condition is satisfied, even in the case of the worst combination of disturbance.
- If $|s(0)| \leq 2d_1$, then $s_d(k) = 0$ for any $k \geq 0$ and the reaching law becomes same as Utkin's reaching law.

Parameter k^*

The constant k^* is a positive integer chosen by the designer in order to achieve good trade-off between the fast convergence rate of the system and the magnitude of the control $u(k)$. For any $k \geq k^*$, the quasi-sliding mode in the d_1 vicinity of the sliding plane $s(k) = c^T x(k) = 0$ is reached.

A Priori Function $s_d(k)$

When $|s(0)| > 2d_1$,

$$s_d(k) = \frac{k^* - k}{k^*} s(0) \;\; with \;\; k^* < \frac{|s(0)|}{2d_1} \tag{25}$$

Bartoszewicz's Reaching Law...Cntd.

A plot of $s_d(k)$

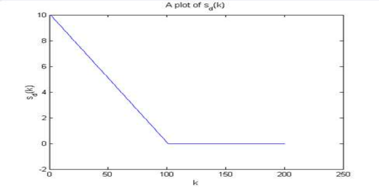

Figure: A plot of $s_d(k)$

Computed Control

$$u(k) = -(c^T \Gamma_\tau)^{-1} [c^T \Phi_\tau x(k) + d_0 - s_d(k+1)] \quad (26)$$

QSM Band

For any $k \geq k^*$,

$$|s(k)| = |d(k-1) - d_0| \leq d_1 \quad (27)$$

Thank You