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Design of Digital Notch Filters

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***Abstract*—It is shown that a second-order digital notch filter is uniquely characterized by two distinct parameters, the notch frequency and the 3-dB rejection bandwidth. As a result, such a filter can be realized using only two multipliers. Methods are outlined to design a notch filter for a prescribed notch frequency and a prescribed 3-dB rejection bandwidth, along with procedures for post-design adjustment of these parameters. All two-multiplier, canonic and noncanonic, notch filter configurations are developed using the multiplier extraction approach. These networks are then compared with regard to the effect of internal multiplication roundoff errors. Results of computer simulation of the notch filter configurations are also included.**

I. INTRODUCTION

DIGITAL filters are being increasingly used in many diverse areas, such as speech processing, picture processing, sonar and radar systems, digital control systems, etc., as they offer a number of attractive advantages with regard to stability, accuracy, reliability, and flexibility in comparison to the more conventional analog filters. In many applications, a digital filter is designed to have a prescribed magnitude characteristic. A very useful type of such filters is the notch filter which attenuates highly a particular frequency component in the input signal while leaving nearby frequencies relatively unattenuated. As an example of the application of a digital notch filter, consider the feedback control vehicle system in which a portion of the feedback signal detected in a sensing device is due to a bending mode, which can be eliminated with the aid of a notch filter to keep the system from destroying itself [1].

The general problem of designing a digital notch filter is considered here. The development of a suitable digital transfer function having a notch characteristic is first outlined in Section II. Here the aim is to minimize the number of parameters characterizing the transfer function. To this end, it is shown that two parameters are sufficient to characterize a second-order digital notch filter, and as a result, such a filter can be realized using only two multipliers. A method to design a digital notch filter for a given notch frequency and 3-dB rejection bandwidth is included here. Next, all canonic and noncanonic two-multiplier second-order digital notch filter configurations are developed in Section III using the multiplier extraction approach [2], in which the notch filter is considered as a digital three-pair [3] of which two pairs of input and output terminal variables are constrained by the two multipliers. The transfer matrix parameters of the three-pair, containing only delays and adders, are then identified, from which the realization is obtained by inspection. All of the realizations are compared in Section IV with regard to the effect of multiplication roundoff errors. Hardware requirements are discussed in Section V, along with a method of designing notch filters with voltage variable characteristics. Results of simulating these notch filters on a general-purpose digital computer are described in Section VI.

II. TRANSFER FUNCTION OF A DIGITAL NOTCH FILTER

The transfer function of an analog notch filter is given as

$$H(s) = \frac{s^2 + \lambda^2}{s^2 + bs + \lambda^2}. \tag{1}$$

The plot of the magnitude function $|H(j\omega)|$ is sketched in Fig. 1 and shows that $\omega = \lambda$ is the notch frequency at which there is no transmission through the filter. Within the frequency band centered at $\omega = \lambda$ and of width b,

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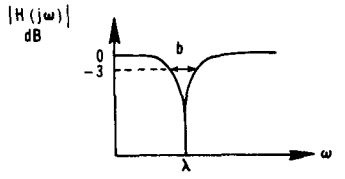


Fig. 1. Frequency characteristics of the notch filter for the continuous case.

all signal components are attenuated by more than 3 dB. Hence the 3-dB rejection bandwidth is b. Note also that the dc gain is 0 dB.

To develop a suitable transfer function for the digital notch filter, we apply the bilinear transformation [4]–[6]

$$s = \frac{z-1}{z+1} \tag{2}$$

to $H(s)$. This yields

$$G(z) = H(s)\,|_{s=(z-1)/(z+1)} = \frac{(1+\lambda^2) - 2(1-\lambda^2)z^{-1} + (1+\lambda^2)z^{-2}}{(1+\lambda^2+b) - 2(1-\lambda^2)z^{-1} + (1+\lambda^2-b)z^{-2}}. \tag{3}$$

To minimize the number of coefficients characterizing the digital notch filter transfer function $G(z)$, the right-hand side of (3) can be expressed in a different form as

$$G(z) = \frac{1}{2} \cdot \frac{(1+a_2) - 2a_1z^{-1} + (1+a_2)z^{-2}}{1 - a_1z^{-1} + a_2z^{-2}} \tag{4}$$

where

$$a_1 = \frac{2(1-\lambda^2)}{1+\lambda^2+b} \tag{5a}$$

$$a_2 = \frac{1+\lambda^2-b}{1+\lambda^2+b}. \tag{5b}$$

Neglecting the constant 1/2, which is equivalent to that of adding a flat 6-dB gain to the frequency response, we rewrite (4) in the desired form as

$$G(z) = \frac{(z^{-2}+1) - 2a_1z^{-1} + a_2(z^{-2}+1)}{1 - a_1z^{-1} + a_2z^{-2}}. \tag{6}$$

The dc gain obtained by letting $z = 1$ in (6) is 2, which is 6 dB, as expected. Note that the notch filter transfer function given by (6) is characterized by two distinct parameters a_1 and a_2. It will be shown in the next section that $G(z)$ as given above can be realized by a digital filter containing two multipliers of coefficients a_1 and a_2.

We can relate the constants a_1 and a_2 to the notch frequency ω_0 and the 3-dB rejection band Ω. From (3), the magnitude is given as

$$|\,G[\exp(j\omega T)]\,|^2 = \frac{\{(1+\lambda^2)\cos\omega T - (1-\lambda^2)\}^2}{\{(1+\lambda^2)\cos\omega T - (1-\lambda^2)\}^2 + b^2\sin^2\omega T}. \tag{7}$$

The notch frequency ω_0 is obtained by setting the numerator of (7) equal to zero, which yields

$$\cos\omega_0 T = \frac{1-\lambda^2}{1+\lambda^2}, \tag{8}$$

i.e.,

$$\omega_0 = \frac{1}{T}\cos^{-1}\frac{1-\lambda^2}{1+\lambda^2}. \tag{9}$$

From (7) we also note that the frequencies ω_1 and ω_2 where the magnitude $|\,G[\exp(j\omega T)]\,|$ is down 3 dB from its dc value are obtained by solving

$$\{(\lambda^2+1)\cos\omega T - (1-\lambda^2)\}^2 = b^2\sin^2\omega T,$$

which after some algebra yields

$$\cos(\Omega T) = \cos(\omega_2-\omega_1)T = \frac{(1+\lambda^2)^2 - b^2}{(1+\lambda^2)^2 + b^2} \tag{10}$$

or

$$\Omega = \frac{1}{T}\cos^{-1}\frac{(1+\lambda^2)^2 - b^2}{(1+\lambda^2)^2 + b^2}. \tag{11}$$

Equations (8) and (10) can be solved for λ^2 and b, which when substituted in (5a) and (5b) give the values of the constants a_1 and a_2 as a function of ω_0 and Ω. From (8)

$$\lambda^2 = \tan^2(\omega_0 T/2) \tag{12}$$

and from (10)

$$b = (1+\lambda^2)\tan(\Omega T/2). \tag{13}$$

Substituting (12) and (13) in (5), we arrive at the desired expressions:

$$a_1 = (1+a_2)\cos(\omega_0 T) = \frac{2\cos\omega_0 T}{1+\tan(\Omega T/2)} \tag{14}$$

$$a_2 = \frac{1-\tan(\Omega T/2)}{1+\tan(\Omega T/2)}. \tag{15}$$

Observe that the notch frequency ω_0 can be changed while keeping the 3-dB rejection band and dc gain constants just by varying a_1. Similarly, the rejection bandwidth can be changed by varying only a_2. However, to keep the notch frequency invariant, a_1 should be adjusted accordingly.

Example 1: Let us determine the transfer function of a digital notch filter having a notch frequency of 1250 Hz and a 3-dB rejection band of 100 Hz. The sampling rate is 10 kHz.

Thus

$$\omega_0 T = 2\pi \times 1250 \times 10^{-4} = 0.25\pi$$

$$\Omega T = 2\pi \times 100 \times 10^{-4} = 0.02\pi,$$

which yields

$$\cos(\omega_0 T) = \cos(0.25\pi) = 0.7071068$$

$$\tan(\Omega T/2) = \tan(0.01\pi) = 0.0314263.$$

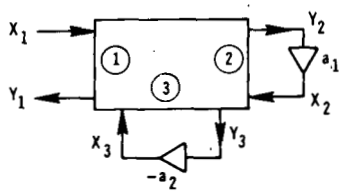


Fig. 2. Constrained three-pair.

Using the above in (14) and (15), we obtain

$$a_1 = 1.3711242 \qquad a_2 = 0.93906244.$$

Hence from (6) the desired notch transfer function is

$$G(z) = \frac{(z^{-2}+1) - 2.7422484z^{-1} + 0.93906244(z^{-2}+1)}{1 - 1.3711242z^{-1} + 0.93906244z^{-2}}. \tag{16}$$

III. NOTCH FILTER CONFIGURATIONS

In the case of realization of the transfer function of (6), we consider the final realization as a constrained three-pair, as shown in Fig. 2 where the three-pair is assumed to contain only delays and adders.

The equations characterizing the network of Fig. 2 are

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{17}$$

and

$$X_2 = a_1 Y_2 \qquad X_3 = -a_2 Y_3 \tag{18}$$

where the t_{ij}'s are the transfer parameters of the three-pair. From (17) and (18) we arrive at

$$\frac{Y_1}{X_1} = t_{11} + \begin{bmatrix} a_1 t_{12} & -a_2 t_{13} \end{bmatrix} \begin{bmatrix} 1 - a_1 t_{22} & a_2 t_{23} \\ -a_1 t_{32} & 1 + a_2 t_{33} \end{bmatrix}^{-1} \begin{bmatrix} t_{21} \\ t_{31} \end{bmatrix}. \tag{19}$$

The denominator of the right-hand side of (19) is the expression

$$1 - a_1 t_{22} + a_2 t_{33} + a_1 a_2 (t_{23} t_{32} - t_{22} t_{33}) \tag{20}$$

and the numerator is the expression

$$\begin{aligned} &t_{11} - a_1(t_{11}t_{22} - t_{12}t_{21}) + a_2(t_{11}t_{33} - t_{13}t_{31}) \\ &\quad + a_1 a_2 \{ t_{21}(t_{12}t_{33} - t_{13}t_{32}) + t_{31}(t_{22}t_{13} - t_{12}t_{23}) \\ &\quad + t_{11}(t_{23}t_{32} - t_{22}t_{33}) \}. \end{aligned} \tag{21}$$

Comparing now the denominator of (6) with (20), we identify

$$t_{22} = z^{-1} \qquad t_{33} = z^{-2} \tag{22}$$

$$t_{23}t_{32} - t_{22}t_{33} = 0. \tag{23}$$

Comparing next the numerator of (6) with (21), we identify

$$t_{11} = z^{-2} + 1 \tag{24}$$

$$t_{11}t_{22} - t_{12}t_{21} = 2z^{-1} \tag{25}$$

$$t_{11}t_{33} - t_{13}t_{31} = z^{-2} + 1 \tag{26}$$

$$t_{21}(t_{12}t_{33} - t_{13}t_{32}) + t_{31}(t_{22}t_{13} - t_{12}t_{23}) + t_{11}(t_{23}t_{32} - t_{22}t_{33}) = 0. \tag{27}$$

Making use of (22) and (24), we obtain from (23), (25), (26), and (27) the following relations:

$$t_{12}t_{21} = z^{-1}(z^{-2} - 1) \tag{28}$$

$$t_{23}t_{32} = z^{-3} \tag{29}$$

$$t_{13}t_{31} = z^{-4} - 1 \tag{30}$$

$$t_{13}t_{21}t_{32} + t_{31}t_{12}t_{23} = z^{-3}(z^{-2} - 1) + z^{-1}(z^{-4} - 1). \tag{31}$$

There are 16 possible realizable sets of values of $t_{12}, t_{21}, t_{13}, t_{31}, t_{23}$, and t_{32} satisfying the last four equations The corresponding transfer matrices of the three-pair N are given by[1]

Notch Filter A:

$$\begin{bmatrix} z^{-2}+1 & z^{-2}-1 & z^{-2}-1 \\ z^{-1} & z^{-1} & z^{-1} \\ z^{-2}+1 & z^{-2} & z^{-2} \end{bmatrix} \tag{32a}$$

Notch Filter B:

$$\begin{bmatrix} z^{-2}+1 & z^{-1}+1 & z^{-1}+1 \\ z^{-1}(z^{-1}-1) & z^{-1} & z^{-1} \\ (z^{-1}-1)(z^{-2}+1) & z^{-2} & z^{-2} \end{bmatrix} \tag{32b}$$

Notch Filter C:

$$\begin{bmatrix} z^{-2}+1 & z^{-1}-1 & z^{-1}-1 \\ z^{-1}(z^{-1}+1) & z^{-1} & z^{-1} \\ (z^{-1}+1)(z^{-2}+1) & z^{-2} & z^{-2} \end{bmatrix} \tag{32c}$$

Notch Filter D:

$$\begin{bmatrix} z^{-2}+1 & 1 & 1 \\ z^{-1}(z^{-2}-1) & z^{-1} & z^{-1} \\ z^{-4}-1 & z^{-2} & z^{-2} \end{bmatrix} \tag{32d}$$

[1] It should be noted that (22) and (28)–(31) are identical to those obtained in the realization of Type 3 second-order all-pass networks described elsewhere [2]. As a result, the values of the transfer parameters are identical to those obtained for the Type 3 all-pass networks, except that the value of t_{11} is that given by (24).

Notch Filter E:

$$\begin{bmatrix} z^{-2}+1 & z^{-1}(z^{-2}-1) & z^{-2}-1 \\ 1 & z^{-1} & 1 \\ z^{-2}+1 & z^{-3} & z^{-2} \end{bmatrix} \tag{32e}$$

Notch Filter F:

$$\begin{bmatrix} z^{-2}+1 & z^{-1}(z^{-1}+1) & z^{-1}+1 \\ z^{-1}-1 & z^{-1} & 1 \\ (z^{-1}-1)(z^{-2}+1) & z^{-3} & z^{-2} \end{bmatrix} \tag{32f}$$

Notch Filter G:

$$\begin{bmatrix} z^{-2}+1 & z^{-1}(z^{-1}-1) & z^{-1}-1 \\ z^{-1}+1 & z^{-1} & 1 \\ (z^{-1}+1)(z^{-2}+1) & z^{-3} & z^{-2} \end{bmatrix} \tag{32g}$$

Notch Filter H:

$$\begin{bmatrix} z^{-2}+1 & z^{-1} & 1 \\ z^{-2}-1 & z^{-1} & 1 \\ z^{-4}-1 & z^{-3} & z^{-2} \end{bmatrix} \tag{32h}$$

and their transposes. Realizations of notch filters A–H are obtained by implementing (32) along with (18), and are sketched in Fig. 3. The remaining eight notch filters can be obtained by taking the transpose [7] of each network of Fig. 3, and will be denoted by the subscript "t" for future reference. Thus, filter A_t will denote the transpose of filter A, and so on.

IV. MULTIPLICATION ROUNDOFF ERRORS

All of the two-multiplier notch filters developed in the previous section have identical coefficient truncation errors. Thus, to compare the performance of these networks in practice, we evaluate next the noise generated at the filter output due to the multiplication roundoff caused by the multipliers a_1 and $-a_2$. To this end, observe that $G(z)$ given by (6) can be rewritten as

$$G(z) = 1 + \frac{z^{-2} - a_1 z^{-1} + a_2}{1 - a_1 z^{-1} + a_2 z^{-2}}. \tag{33}$$

The second term on the right-hand side of (33) is recognized as the transfer function of a Type 3 second-order all-pass network which can be realized 16 different ways using two multipliers a_1 and $-a_2$ [2]. As a result, the Type A notch filter configuration can be considered as a Type 3A all-pass network configuration with a direct path in parallel from the input to the output with unity gain. Consequently, the multiplication roundoff noise properties of the Type A notch filter will be identical to those

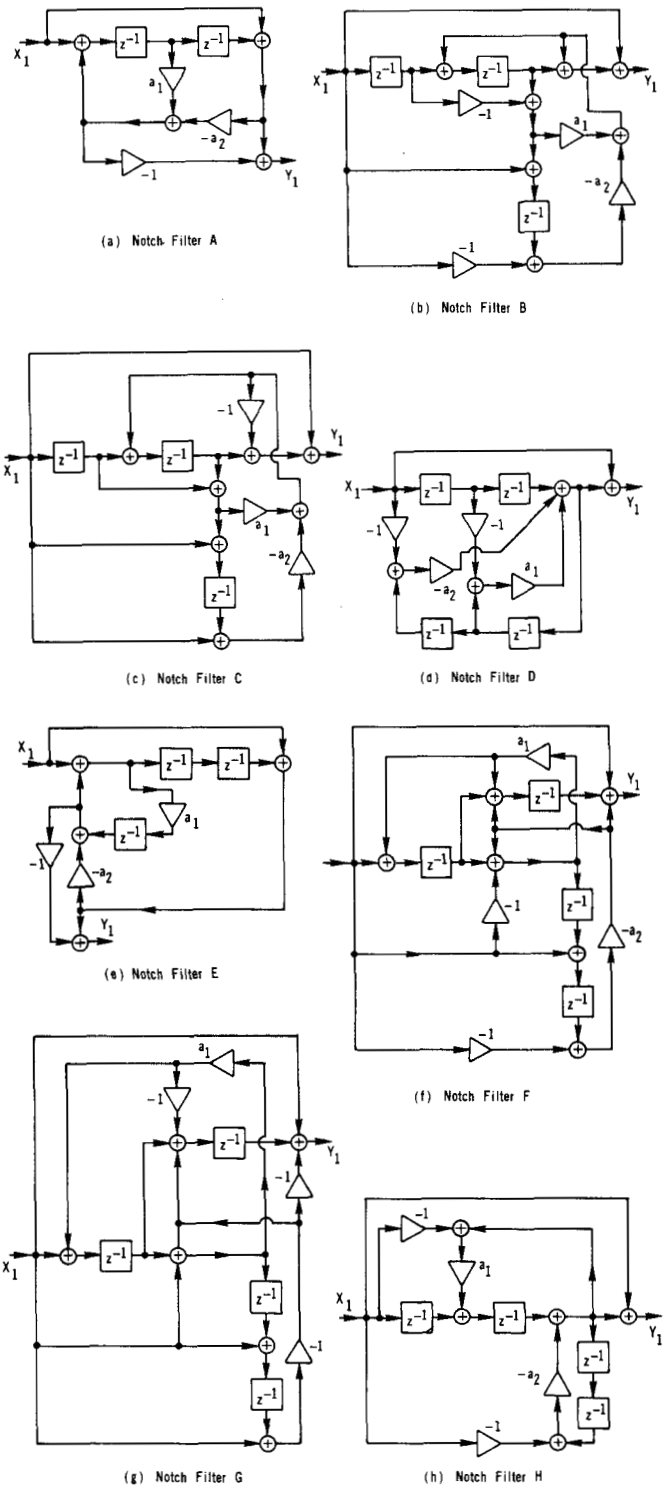


Fig. 3. Notch filter configurations. (a) Notch filter A. (b) Notch filter B. (c) Notch filter C. (d) Notch filter D. (e) Notch filter E. (f) Notch filter F. (g) Notch filter G. (h) Notch filter H.

of the Type 3A all-pass network. Similarly, the other notch filter and all-pass network configurations are related. For convenience, we list in Table I the steady-state variance σ_0^2 of the output noise caused by the multiplication roundoff of each notch filter configuration, as obtained from [2, table III] where the plots of these output noises as a function of the pole positions may be found.

TABLE I
OUTPUT NOISE OF THE NOTCH FILTER CONFIGURATIONS

Notch Filter	$A(E)$	$B(F)$	$C(G)$	D, H
σ_0^2	$\frac{q^2}{6} R_1$	$\frac{q^2}{6} R_2$	$\frac{q^2}{6} R_3$	$\frac{q^2}{6} R_4$
Notch Filter	D_t, H_t	$C_t(G_t)$	$B_t(F_t)$	$A_t(E_t)$
σ_0^2	$\frac{q^2}{12} (\rho R_1 + R_{11})$	$\frac{q^2}{12} (\rho R_2 + R_9)$	$\frac{q^2}{12} (\rho R_3 + R_7)$	$\frac{q^2}{12} (\rho R_4 + R_5)$

$$R_1 = \frac{2}{1 - r^2} \qquad R_5 = \frac{2(1 - r^2 \cos 2\psi)}{1 - 2r^2 \cos 2\psi + r^4}$$

$$R_2 = \frac{2}{(1 - r^2)(1 - 2r \cos \psi + r^2)} \qquad R_7 = \frac{4(1 + r \cos \psi)}{1 + 2r \cos \psi + r^2}$$

$$R_3 = \frac{2}{(1 - r^2)(1 + 2r \cos \psi + r^2)} \qquad R_9 = \frac{4(1 - r \cos \psi)}{1 - 2r \cos \psi + r^2}$$

$$R_4 = \frac{1 + r^2}{(1 - r^2)(1 - 2r^2 \cos 2\psi + r^4)} \qquad R_{11} = 2$$

$$\rho = (1 - r^2)^2 \qquad a_1 = 2r \cos \psi \qquad a_2 = r^2.$$

TABLE II
HARDWARE REQUIREMENTS TO IMPLEMENT A SECOND-ORDER NOTCH FILTER

Notch Filters	Delays	Two-Input Adders
A, A_t	2	4
B, B_t, C, C_t	3	7
D, D_t	4	5
E, E_t	3	4
F, F_t, G, G_t	4	9
H, H_t	4	5

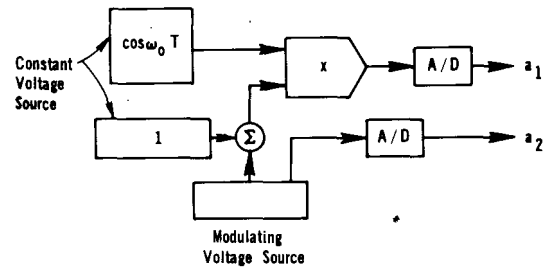


Fig. 4. Generation of multiplier coefficients to vary the 3-dB rejection bandwidth, keeping the notch frequency invariant.

V. HARDWARE CONSIDERATIONS

As can be seen from Table I, from the point of view of output noise there are only eight distinct groups of notch filters, with two different configurations in each group, and having identical output noise. Hardware requirements as given by Table II may then be used to select a configuration from a given group. For example, as Table II points out, notch filter A requires fewer components than notch filter E, even though both of them have identical output noise characteristics. In a similar manner, notch filters in other groups can be selected. In Table I, notch filters requiring more components have been placed inside parentheses.

It should be further observed that the two notch filter configurations in a given group are also topologically related, and any one can be derived from the other by simple topological manipulations.

Another interesting feature of the notch filters developed here is that they remain the notch characteristic irrespective of the actual values of the multiplier coefficients a_1 and $-a_2$. As a result, any one of the proposed circuits can be easily converted to a variable notch filter by making the multiplier coefficients variable. As pointed out earlier, the notch frequency can be varied, keeping the 3-dB rejection band and the dc gain constant, just by varying a_1. In a practical implementation, a_1 is usually stored in a shift register. To design a notch filter with voltage variable notch frequency (as may be required for tracking), the shift register can be replaced by a modulating voltage source followed by an A/D converter whose output is then used as a multiplier coefficient. By varying the modulating voltage, the notch frequency can be varied. On the other hand, if it is desired to vary the 3-dB bandwidth while keeping the notch frequency invariant it is necessary to vary a_1 and a_2 such that $a_1/(1 + a_2)$ remains invariant. This can be accomplished with the aid of an analog multiplier, a summing amplifier, and two constant voltage sources in addition to the modulating voltage source, as sketched in Fig. 4. Here the output of the analog multiplier is fed into an A/D converter whose output is used as the multiplier coefficient a_1. The output of the modulating voltage source is also fed directly into another A/D converter whose output is used as the multiplier coefficient a_2.

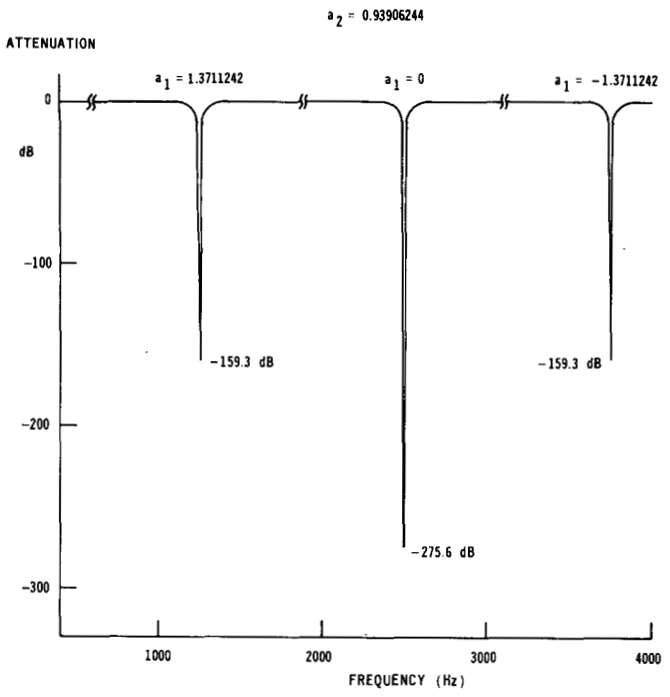


Fig. 5. Change of notch frequency due to the variation of a_1, keeping the notch bandwidth and passband gain constant.

VI. COMPUTER EVALUATION RESULTS

In order to evaluate the proposed design method, the frequency response of the notch filter of Example 1 was calculated on the general-purpose digital computer FACOM 230 using double-precision arithmetic. The plot of the magnitude of the notch filter transfer function of (16) is shown in Fig. 5 along with that for two other notch frequencies obtained by varying a_1 only. It should be noted that the attenuations at the notch frequencies for the three different designs are different, with maximum attenuation of over 275.6 dB being provided by the filter having the notch frequency at 2500 Hz. This is due to the inherent inaccuracy in the calculation, where the significant number of bits of a_1 used is restricted to seven digits except for $a_1 = 0$.

In the calculation of the frequency response using single-precision arithmetic, the attenuations at the three notch frequencies, i.e., 1250, 2500, and 3750 Hz, were reduced to 83.4, 86.0, and 82.8 dB, respectively. Because of these internal computational errors, in the first case, the apparent notch frequency shifted to 1250.002 Hz with a resultant attenuation of more than 93 dB; likewise, in the second case, the apparent notch frequency shifted to 2500.003 Hz with an increase of attenuation to 99.7 dB; and in the last case, the apparent notch frequency was at 3750.001 Hz with an attenuation of 94.3 dB.

In addition to the above, two different notch filter configurations, Types A and D_t, realizing the transfer function of (16), were also simulated on the computer. Fig. 6 shows the output time response of a Type A notch

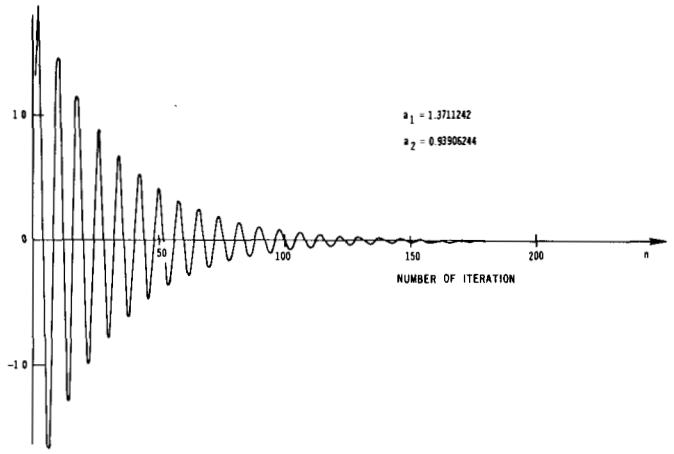


Fig. 6. Output time response of notch filter A designed for a notch frequency of 1250 Hz when an input sine wave of frequency 1250 Hz was applied.

filter when a sine wave of frequency 1250 Hz was applied at the input. Observe that, as expected, the transient response decays to −159.3 dB after 1000 iterations. Similar responses were observed for the other configurations.

VII. CONCLUSION

It has been shown that a digital second-order notch filter transfer function as given by (6) can be uniquely characterized by two parameters a_1 and a_2 which are related to the notch frequency ω_0 and the 3-dB rejection band Ω in accordance with (14) and (15). Such transfer functions can be realized using only two multipliers of coefficients a_1 and a_2 in 16 different forms, of which 8 are shown in Fig. 3. The other 8 are the transposes of these configurations. As indicated by (33), each of these configurations can also be derived from the Type 3 all-pass networks developed elsewhere [2] by providing a direct path from the input to the output with a unity gain. Consequently, the output noise of a given notch filter configuration is identical to that of its associated all-pass network configuration. For convenience, the expressions for the output noise as obtained from [2] have been tabulated again in Table I of this paper. From the plots of these output noises as given in [2, fig. 11], we observe that for a notch filter transfer function having very low values of the pole radius, i.e., for a notch filter with wide rejection band, the notch filter configurations D and H have the lowest value of the output noise, whereas for very large values of the pole radius, i.e., for a narrow rejection band, configurations D_t and H_t are preferable. For intermediate values of the pole radius, selection of the optimum configuration depends on the actual values of the pole radius and angle.

The proposed configurations are suitable for the design of notch filters with variable characteristics. For example, by varying the multiplier coefficient, the notch frequency can be varied without changing the 3-dB rejection band

and the dc gain. On the other hand, the 3-dB rejection band can be varied by varying the multiplier coefficient a_2. However, to keep the notch frequency invariant, it is necessary to adjust a_1 such that $a_1/(1 + a_2)$ remains invariant. One solution to this second problem is shown in Fig. 4.

It should be pointed out here that the notch filter function of (6) can also be realized by other methods, but the method suggested in this paper leads to realizations using the minimum number of multipliers.

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Digital DPCM Codec for TV Signals Based on ΔM/DPCM Digital Conversions

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***Abstract*—A digital differential pulse-code modulation (DPCM) codec based on ΔM/DPCM and DPCM/ΔM digital conversions is presented as an equivalent replacement of a conventional DPCM codec for TV signals by using digital processing technology. A generalized model and equivalent circuit are shown for the digital DPCM codec, and transfer characteristics and quantizing noises are analyzed. The digital DPCM codec is designed for 1-MHz TV signals and the performance is verified by experiments. By the use of double integration ΔM and digital filter technology, it is shown that at the ΔM sampling rate of 16 MHz the overall SNR performance is sufficiently good for use as a conventional DPCM codec, and the digital code converters can be realized with commercially available transistor–transistor-logic (TTL) medium-scale-integrated (MSI) circuits and low-power emitter-coupled-logic (ECL) integrated circuits (IC's).**

INTRODUCTION

DUE to large-scale-integration (LSI) technologies development, existing analog circuits in communication equipments will be replaced by digital processors in the future. Digital processing in communication equipments may provide more uniformity of performance, better tracking of transfer characteristics, smaller size, and better economy. A few attempts have been reported on such a digitization of communication circuits as a digital frequency-division-multiplexing (FDM) terminal [1], a digital electronic switching system for video signals [2] and so on. In these cases, delta modulation (ΔM) has been extensively used as a simple and economical A/D and D/A conversion method [3], [4].

In TV signal coding, the nonlinear differential pulse-code modulation (DPCM) provides an efficient coding technique [5]. Here, a digital DPCM codec is presented, which is composed of a double integration ΔM codec and digital processing circuits converting between ΔM and DPCM signals. This technique avoids tracking problems and stringent requirements existing in conventional DPCM codecs. By the use of double integration ΔM, the ΔM sampling rate can be greatly reduced so that the circuit can be built with presently available circuit elements.

This paper discusses a generalized model and an equivalent circuit of the digital DPCM codec, the quantizing noise, and digital filter performances. Based on these results, a full layout of the digital DPCM codec is designed, and overall performance is verified through experimentation. The digital DPCM codec thus designed is compatible with the conventional DPCM codec and is shown to be realized with reasonable size hardware.

DIGITAL DPCM CODEC MODEL

In a conventional DPCM codec [5], the main portion of the DPCM coder is equivalently described by a noise feedback model [6] having a sampler with sampling rate

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