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Signal Processing

Digital pulse-shaping FIR filter design with reduced intersymbol and interchannel interference

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SUMMARY

An important requirement in the design of data transmission filters is the minimization of intersymbol interference, which is zero if the overall impulse response (transmit filter, channel and receive filter) satisfies the first Nyquist criterion. In this context, an important class of transfer functions, satisfying the Nyquist criterion, is the raised-cosine filter family. In order to guarantee low interference between adjacent channels, the transmit and receive filters must have a high value of stopband attenuation, so as to reduce the interchannel interference as much as possible. In this paper, a design method for finding the coefficients of a pair of linear-phase transmit/receive FIR filters, that when cascaded have raised-cosine frequency response, is presented. The design is based on Frequency Sampling techniques, and the filter parameters are chosen in order to obtain maximum stopband attenuation and low intersymbol interference. The filter coefficients can be easily evaluated and the optimal filter parameters can be obtained with tables or equations. The design method is very simple, completely automatic and suited for non-filter-oriented users. Look-up table techniques can be used for automatic re-design of the transmit and receive filters, making the proposed solution well suited to programmable computing platforms (FPGA—Field Programmable Gate Arrays- and PLD—Programmable Logic Devices-based platforms), or for applications where the design must be performed without any user intervention. The proposed filter design technique is quite simple and the results obtained often match the performance of filter designed using computationally more complex and conceptually more difficult methods. Copyright © 2003 AEI.

1. INTRODUCTION

Different requirements are imposed on the transmit and receive filters in the design of a data transmission system: the transmit filter is used to band-limit the signal spectrum to the Nyquist bandwidth, whereas the receive filter must reject both the out-of-band noise and the side-channels, and it needs therefore to have high stopband attenuation.

The stopband attenuation is in fact a very important constraint and it must be minimized in order to reduce the Interchannel Interference (ICI). Besides, the cascade of the transmit and receive filters must satisfy the first Nyquist criterion, in order to avoid Intersymbol Interference (ISI).

Various techniques have been proposed for the design of digital filters satisfying the first Nyquist criterion. A very common solution is the use of Finite Impulse Response (FIR) filters, which have been successfully designed using linear programming techniques [1–5]. In References [3–5], in particular, an equiripple stopband behavior can be obtained, while in Reference [3, 5] the transmit/receive

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filter pair can be jointly designed. Although linear programming is a very flexible and powerful technique for designing digital filters, it does suffer from some numerical ill-conditioning problems for high-order filter design.

In Reference [5], a simple modification of the standard linear programming approach for FIR filters design has been proposed, which avoids the necessity of a dense grid of frequency points, achieving maximum possible stopband attenuation for a given filter order and stopband edge frequency. However, the design method cannot guarantee the convergence of the proposed algorithm.

A valid alternative to the use of linear programming methods has been proposed in Reference [7], in which an iterative technique for designing equiripple FIR Nyquist filters using a multistage structure is described. The multistage implementation shown in Reference [7] is very efficient, but the resulting filter cannot be easily split into a transmit/receive filter pair, because the design method does not guarantee the ICI specifications on the two separate filters. This problem has been dealt with in Reference [8], where the authors proposed a method to extend the design procedure presented in Reference [7] to the transmit-receive filter pair.

Design methods for FIR structures are very common because FIR filters can be easily constrained to have linear phase, which turns out to be a good property from the system performance point of view. On the other side, FIR structures generally require a large number of multipliers to meet the design specifications. Some alternative solutions, able to avoid this problem, can be found in the literature. In this paper however, only linear phase FIR structures have been considered, taking into account both the sidelobe amplitude in the stopband (which characterizes the amount of ICI) and the ISI.

A design procedure based on Frequency Sampling (FS) techniques is described in the following. The potential of this design procedure derives from the behavior of the out-of-band power of FS-designed raised-cosine filters. In fact, as it will be demonstrated later, both numerically and analytically, the out-of-band power does not depend on the oversampling factor and it presents local minima for certain particular values of the filter parameters. The proposed design method controls the amount of ISI and ICI of the global transfer function, and it constraints the stopband attenuation of both the transmit and the receive filters as well. This is a very important requirement, especially when the transmission channel is characterized by a low signal-to-noise ratio. In some sense, the proposed method can be considered as an FS technique especially tailored for Transmission/Reception (TX/RX) Nyquist filters. For this reason, it will be referred in the following as FS for Nyquist filters (FS-N).

The FS-N method has been introduced with the aim of providing a simple, automatic and efficient way of designing reconfigurable TX/RX Nyquist filters. The method is not optimal, but leads to a very interesting ISI/ICI tradeoff, and exhibits a number of advantages and features with respect to other techniques. First, it always generates stable and causal filters and, it does not have convergence problems, and it does not require any intervention from the user. Furthermore, both transmit and receive filters have symmetric impulse response; this latter characteristic allows a reduction of 50% in the number of multipliers and guarantees linear phase, thus avoiding the need for group delay equalizers. The proposed method also allows to specify the required splitting of the Nyquist characteristic between the transmit and receive filters. This feature may be very useful in non-linear channels where asymmetrical splitting of the Nyquist characteristic is often required [9]. Finally, the design parameters of the proposed method can be obtained from design charts and simple formulas whose coefficients may be stored in a Read Only Memory (ROM). In this way, it is possible to efficiently retrieve the optimum combination of design parameters and modify the filter characteristics at run-time, just looking for the optimum combinations located into look-up tables, without executing complex design procedures. This makes the FS-N method a user-friendly design tool, especially for users more expert (or interested) in the telecommunication system aspects, than in the digital filter design. Being completely automatic, the FS-N method is also particularly suited for all those applications where the filter design must be performed without user intervention, like for instance simulation or software radio (SR). The proposed FS-N filter design method could be used both for '*a priori* design' to design a limited set of pulse-shaping filters to be stored into a transceiver system, or for 'run-time design', where no alternative techniques with equivalent simplicity are available.

Since the main scope of this work is to design a pair of digital Nyquist filters, throughout this paper it has been assumed that imaging and aliasing are perfectly avoided by ideal analog filters, inserted before the analog to digital converter at the receiver side, which do not introduce phase and amplitude distortions on the received signal. It has also been assumed that digital to analog conversion is ideally accomplished at the transmitter side. These hypotheses are practically adopted throughout the literature like, for instance, in the recent papers [5, 10, 13–15].

The structure of the paper is as follows. Sections 2 summarizes some basic concepts of the FS design applied to the raised-cosine function. Section 3 discusses the design methodology with respect to the ICI performances. At the end of this section, some design charts and formulas suitable for an automatic implementation of the proposed method are given. Section 4 deals with the ISI performances. Section 5 shows some design examples, comparing them with other filter design methods, while Section 6 contains the final conclusions.

2. BACKGROUND THEORY AND PROBLEM SETUP

Denoting by $h[n]$ the overall system impulse response, the first Nyquist criterion in the digital domain states that

$$h[n] = \begin{cases} A & \text{if} \quad n = n_0 \\ 0 & \text{if} \quad n = n_0 \pm kN_s, \quad k = 1, 2, \ldots \end{cases} \tag{1}$$

where A is a non-zero constant, n_0 is the discrete sampling instant and N_s is the number of samples per symbol. N_s represents the oversampling factor, where T is the sampling interval, F the sampling frequency and R_s the baud rate (in symbols per second), we have $F = 1/T$, $R_s = 1/N_sT$ and $N_s = F/R_s$. The first Nyquist criterion can be expressed in the frequency domain as well: being $H(e^{j2\pi fT})$ the z-transform of $h[n]$ evaluated for $z = e^{j2\pi fT}$, and $H_T(e^{j2\pi fT})$ a function equal to $H(e^{j2\pi fT})$ for $-1/2T < f < 1/2T$ and zero elsewhere, the following condition must hold

$$\sum_{k=-\infty}^{+\infty} H_T\left(e^{j2\pi(f-k/N_sT)T}\right) = AN_s \tag{2}$$

(the frequency range $(-1/2T, 1/2T)$ will be called in the following the 'Nyquist bandwidth'). Note that the transfer function has an odd symmetry around the point $(1/2N_sT, AN_s/2)$.

From Equations (1) and (2), it is possible to see that the Nyquist criterion in the digital domain is practically the same as in the analog domain, therefore the digital filter design can be performed starting from analog results, i.e. digitizing some well-known analog Nyquist filters.

It is well known from the analog filter theory that the so called raised-cosine transfer function [11], denoted as $R(f_a, \rho, T_s)$, where f_a is the analog frequency, ρ the roll-off and T_s the symbol period of transmission (i.e. $T_s = N_sT$), satisfies the first Nyquist criterion. The expression of a raised-cosine function is

$$R(f_a, \rho, T_S) = \begin{cases} 1 & \text{for } |f_a| \le f_1 \\ \cos^2\left[\frac{\pi}{4\rho}(2|f_a|T_s - 1 + \rho)\right] & \text{for } f_1 < |f_a| \le f_2 \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

where $f_1 = (1-\rho)/2T_s$ and $f_2 = (1+\rho)/2T_s$. The function is commonly split between transmit and receive filters, as $R^\alpha(f_a, \rho, T_s)$ and $R^{1-\alpha}(f_a, \rho, T_s)$, where α is chosen in order to optimize the overall performances ($0 \le \alpha \le 1$).

Since the expressions of the first Nyquist criterion in the analog and digital domain are equivalent, we will try to approximate a transmit and a receive filter with transfer functions $R^\alpha(f/T, \rho, T_s)$ and $R^{1-\alpha}(f/T, \rho, T_s)$ in the Nyquist bandwidth (where f is the digital frequency), that will satisfy Equation (2) when cascaded.

The problem considered in this paper is, therefore, the design of a digital FIR filter with linear phase and transfer function magnitude equal to $R^\alpha(f/T, \rho, T_s)$ with $0 \le \alpha \le 1$. Since we are looking for a causal FIR filter, we introduce a linear phase $\Phi(f) = -\pi fT(N-1)$, where N is the filter length. Therefore we will start our design from the ideal digital transfer function

$$H_I(f) = R^\alpha(f/T, \rho, T_s)e^{j\Phi(f)} \tag{4}$$

2.1. Filter design

Various FIR design methods have been proposed in the literature [12]. We chose the FS method which allows the evaluation of the filter coefficients $h[n]$ with a simple direct formula [12]

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H_k e^{j\frac{2\pi}{N}nk} \tag{5}$$

where H_k are the samples of the ideal transfer function $H_I(f)$ at the equally spaced points $f_k = \frac{k}{N}, k = 0, \ldots, N-1$.

The FS method produces a digital transfer function $H_D(f)$, which is forced to assume the ideal values H_k at the frequencies f_k, but it is not directly controlled in all the other frequencies.

In our design method, the filter length N is set equal to $2N_sE_p$, where E_p is the number of periods on the right (left) side of the impulse response. The parameter E_p has a real value compatible with the constraint that $2N_sE_p$ must be an integer number. We will examine both integer and non-integer values of E_p. The samples H_k to be plugged in Equation (5) can be evaluated from Equations (3) and (4):

$$H_k = \begin{cases} e^{-jk\pi\frac{N-1}{N}} & \text{for } 0 \le k < N_A \\ \cos^{2\alpha}\left[\frac{\pi}{4\rho}\left(\frac{k}{E_p} - 1 + \rho\right)\right] e^{-jk\pi\frac{N-1}{N}} & \text{for } N_A \le k < N_B \\ 0 & \text{for } N_B \le k \le N - N_B \\ \cos^{2\alpha}\left[\frac{\pi}{4\rho}\left(\frac{N-k}{E_p} - 1 + \rho\right)\right] e^{\frac{-j\pi(k-N)(N-1)}{N}} & \text{for } N - N_B < k \le N - N_A \\ e^{-j(k-N)\pi\frac{N-1}{N}} & \text{for } N - N_A < k \le N - 1 \end{cases}$$

where we have set $N_A = \lceil E_p(1-\rho) \rceil$ and $N_B = \lceil E_p(1+\rho) \rceil$ ($\lceil . \rceil$ is the ceiling operation).

3. ICI PERFORMANCES

In order to evaluate the effects of ICI, we first define the filter bandwidth B_s as twice the 'gross bandwidth' of the useful channel (see Figure 1), i.e. the frequency range from the origin to the first null of the designed filter transfer function, that will obviously be located in $B_s/2$. In usual applications, the ICI is due to an adjacent channel with the same frequency characteristics of the useful channel, starting at the frequency $B_s/2$ and of bandwidth B_s (see Figure 1).

In order to evaluate the amount of ICI introduced by the filter stopband characteristics, we introduce the parameter P_s, defined as the amplitude of the 'equivalent' (or average) lobe in the adjacent channel with bandwidth B_s

$$P_s = \frac{1}{B_s} \int_{\frac{B_s}{2}}^{\frac{3}{2}B_s} |H_D(f)| \, df \tag{7}$$

It has been numerically verified that the value of P_s does not depend on the number of samples per symbol N_s (the so called oversampling factor), but only on the roll-off parameter ρ and the number of periods E_p. For example, Figures 2 and 3 show the behavior under examination for three possible values of N_s, in the case of a square-root raised-cosine filter ($\alpha = 0.5$). This behavior can be explained as follows: considering the digital transfer function $H_D(f)$, the first null is generally placed in $k = N_B$ (see

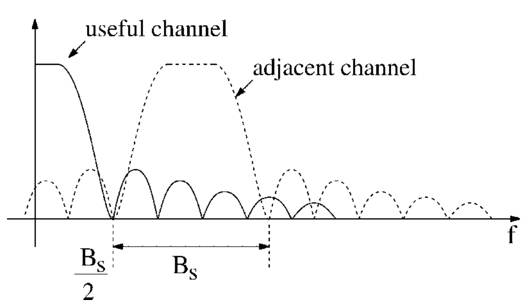

Figure 1. Spectral position of useful and adjacent channels.

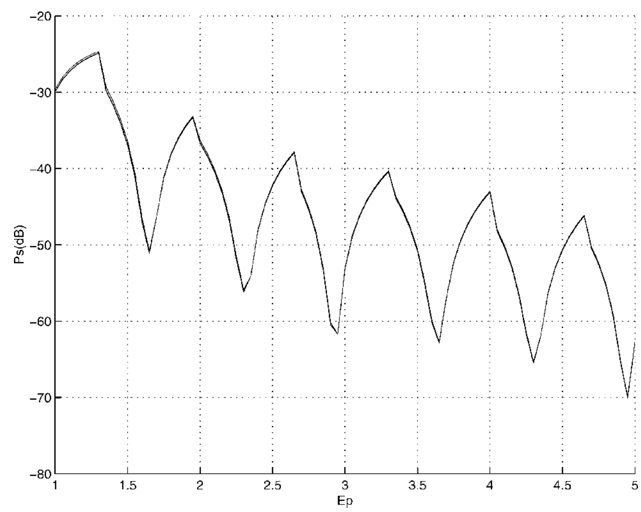

Figure 2. Value of P_s as a function of E_p for $\rho = 0.5$, $\alpha = 0.5$ and N_s =10, 20 and 30 (the three curves are superimposed).

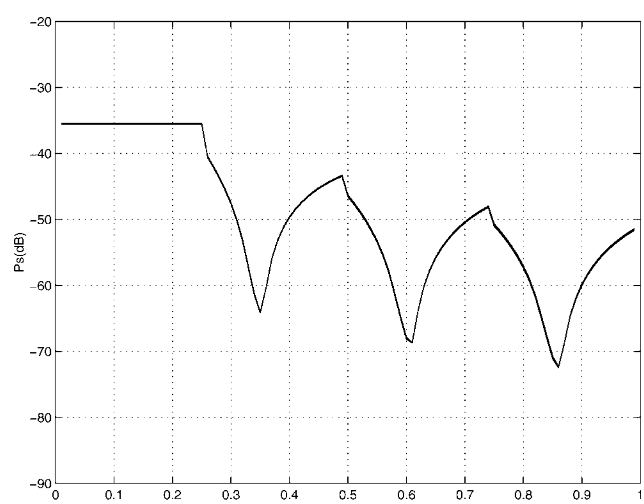

Figure 3. Value of P_s as a function of ρ for $\alpha = 0.5$, $E_p = 4$ and $N_s = 10$, 20 and 30 (the three curves are superimposed).

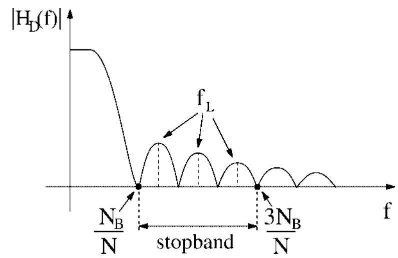

Figure 4. Magnitude of the digital filter $H_{\mathrm{D}}(f)$.

Figure 4), that is at the frequency $f = N_B/N$. Within the frequency interval $[B_{\mathrm{s}}/2, 3B_{\mathrm{s}}/2]$, the maxima of the magnitude sidelobes are placed very near to the middle point between two null samples, that is at the frequencies $f_L = (L+0.5)/N$, $N_{\mathrm{B}} \leq L < N-(N_{\mathrm{B}}-1)$. The parameter P_{s} defined in Equation (7) can be approximated as:

$$P_{\mathrm{s}} \approx \frac{1}{2N_{\mathrm{B}}} \sum_{L=N_{\mathrm{B}}}^{3N_{\mathrm{B}}-1} \left| H_{\mathrm{D}}\left(\frac{L+0.5}{N}\right)\right| = \frac{1}{2N_{\mathrm{B}}} \sum_{L=N_{\mathrm{B}}}^{3N_{\mathrm{B}}-1} |H_{\mathrm{D}}(f_L)| \tag{8}$$

The transfer function of the digital filter $H_{\mathrm{D}}(f)$ designed according to the FS method can be expressed as:

$$H_{\mathrm{D}}(f) = \frac{e^{-j\pi f(N-1)}}{N} \sum_{k=0}^{N-1} H_k \mathrm{e}^{-j\pi k/N} \frac{\sin(\pi f N)}{\sin(\pi(f-k/N))} \tag{9}$$

which is a linear combination of the samples H_k with frequency interpolating functions $A(f,k) = \left(\mathrm{e}^{-j\pi k/N}\right) \sin(\pi f N)/\sin(\pi(f-k/N))$ [12]. The quantity $|H_{\mathrm{D}}(f_L)|$ can therefore be expressed as:

$$\begin{aligned} |H_{\mathrm{D}}(f_L)| = \frac{1}{N} \Bigg| \sin\left(\frac{\pi L}{N}\right)^{-1} &+ \sum_{k=1}^{N_{\mathrm{A}}-1} (-1)^{-k} \\ &\times \left[\sin\left(\frac{\pi(L-k)}{N}\right)^{-1} + \sin\left(\frac{\pi(L+k)}{N}\right)^{-1}\right] \\ &+ \sum_{k=N_{\mathrm{A}}}^{N_{\mathrm{B}}-1} (-1)^{-k} R_k \left[\sin\left(\frac{\pi(L-k)}{N}\right)^{-1}\right. \\ &\left.+ \sin\left(\frac{\pi(L+k)}{N}\right)^{-1}\right]\Bigg| \end{aligned}$$

where

$$R_k = \cos^{2\alpha}\left[\frac{\pi}{4\rho}\left(\frac{k}{E_{\mathrm{p}}} - 1 + \rho\right)\right] \mathrm{e}^{-jk\pi\frac{N-1}{N}} \tag{11}$$

When $N \gg 1$, as it is generally verified in practical filters, the quantities $\sin(\cdot)$ in Equation (10) can be approximated with their arguments, obtaining the expression:

$$\begin{aligned} |H_{\mathrm{D}}(f_L)| \approx \frac{1}{\pi} \Bigg| \frac{1}{L} &+ \sum_{k=1}^{N_A-1} (-1)^{-k} \left[\frac{1}{L-k} + \frac{1}{L+k}\right] \\ &+ \sum_{k=N_{\mathrm{A}}}^{N_{\mathrm{B}}-1} (-1)^{-k} R_k \left[\frac{1}{L-k} + \frac{1}{L+k}\right]\Bigg| \end{aligned} \tag{12}$$

Substituting Equation (12) in Equation (8), we obtain an approximated expression of P_{s} that does not depend on the number of samples per symbol N_{s} and that justifies the behavior of Figures 2 and 3:

$$\begin{aligned} P_{\mathrm{s}} \approx \frac{1}{2N_{\mathrm{B}}} \sum_{L=N_{\mathrm{B}}}^{3N_{\mathrm{B}}-1} \frac{1}{\pi} \Bigg| \frac{1}{L} &+ \sum_{k=1}^{N_{\mathrm{A}}-1} (-1)^{-k} \left[\frac{1}{L-k} + \frac{1}{L+k}\right] \\ &+ \sum_{k=N_{\mathrm{A}}}^{N_{\mathrm{B}}-1} (-1)^{-k} R_k \left[\frac{1}{L-k} + \frac{1}{L+k}\right]\Bigg| \end{aligned} \tag{13}$$

As it can be seen in Figure 2, P_{s} shows some minimum values for certain optimal values of E_{p}. This situation repeats itself for every value of ρ, i.e given a value of ρ, P_{s} shows a certain number of minima for some 'optimal' values of E_{p}. This behavior can be verified in Figure 5, where the values of P_{s} as a function of E_{p} are shown for different values of ρ. In this latter figure, it can be observed that the optimal (minimum) values of P_{s} decrease when ρ increases. We would like to point out that the maxima of $|H_{\mathrm{D}}(f)|$ in the filter stopband typically show a decreasing amplitude as f increases, and therefore optimizing the filter

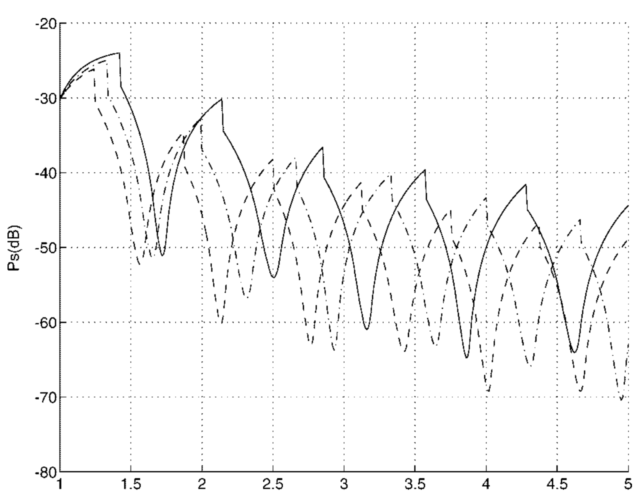

Figure 5. Value of P_{s} as a function of E_{p} for $\rho = 0.4$ (–), $\rho = 0.5$ (–·), $\rho = 0.6$ (– –) and $\alpha = 0.5$.

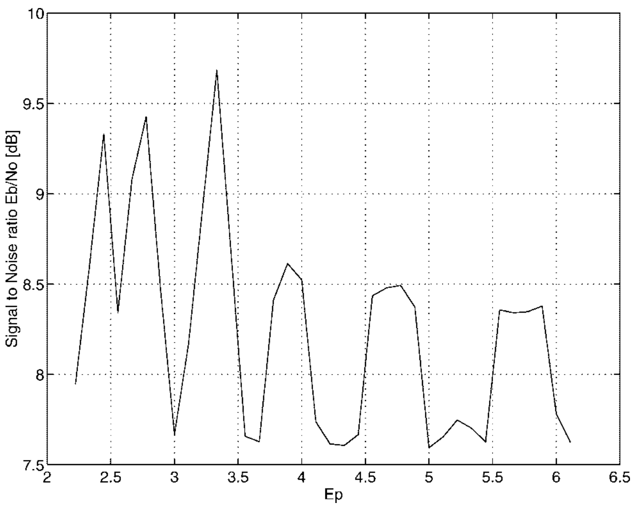

Figure 6. Signal to noise ratio required to achieve a bit error rate equal to 10^{-4} as a function of E_p for $N_s = 18$, $\rho = 0.5$ and $\alpha = 0.5$.

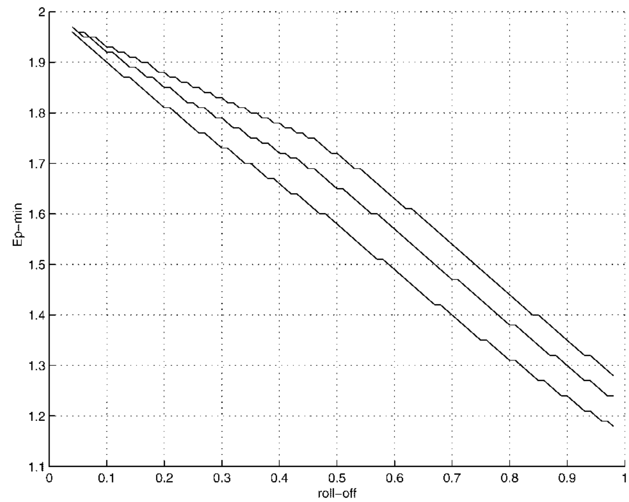

Figure 7. $(\rho, E_{p-\min})$ design table for 'optimal' FS-N raised-cosine filters. The upper curve refers to $\alpha = 0.6$, the middle curve to $\alpha = 0.5$ and the lower to $\alpha = 0.4$.

attenuation in the frequency range $[B_s/2, 3B_s/2]$ leads to an excellent filter behavior in all the stopband.

For a given value of α, a proper choice of the design parameters E_p and ρ can therefore result in a filter with very high stopband attenuation. This behavior has also been verified by simulation. In Figure 6, the signal-to-noise ratio $(E_b/N_o)^\dagger$ required to obtain a target bit-error-rate (BER) of 10^{-4} for a satellite Quadrature Phase Shift Keying (QPSK) transmission scheme employing Frequency Division Multiplexing (FDM) techniques is shown. The graph refers to a useful channel transmitting with bit-rate $R_b = 2.498$ Mbit/s, with center frequency $f_o = 29.51847$ GHz and two FDM adjacent channels located at $f_r = f_o + ((1+\rho)/2)R_b$ and $f_l = f_o - ((1+\rho)/2)R_b$, where $\rho = 0.5$. Useful and interfering adjacent channels use a square-root ($\alpha = 0.5$) of raised-cosine FIR shaping filter with roll-off ρ and $N = 2N_sE_p$ taps, where $N_s = 18$ and E_p is variable. The useful channel is attenuated of 30 dB with respect to the interfering adjacent channels, to account for worst-case propagation conditions. As it can be observed from Figure 6, the value of E_b/N_o required to achieve the target BER varies with E_p, i.e. with the stopband attenuation of the transmit and receive filters, and some optimal values of E_p can be identified.

In most applications, it is important to minimize the number of filter taps, while maximizing (or locally maximizing) the stopband attenuation. It is often important to have simple filter design methods, possibly completely automatic, that allow to select good filters without the need for interactive optimization, with low computational complexity and low memory storage requirements. These objectives can be achieved by means of graphs, which can be stored as look-up tables, easily accessed by an automatic filter design method. In the following, a method for constructing these graphs is described.

Graphs showing the values (ρ, E_p) in which the maxima stopband attenuations (i.e. the minimum values of P_s) are located can be easily obtained for different values of α. An example is shown in Figure 7 for three different values of α (i.e. $\alpha = 0.4$, 0.5 and 0.6). In particular, the upper curve refers to $\alpha = 0.6$, the lower to $\alpha = 0.4$ and the middle curve to $\alpha = 0.5$. Since for a given value of ρ different optimal choices of E_p are possible, the minimum 'optimal' value of E_p has been reported in Figure 7, i.e. the value that minimizes the number of taps N (we will denote this value as $E_{p-\min}$). The values of the minima of P_s considered in Figure 7 are all smaller than -40 dB, and their value decreases as $E_{p-\min}$ increases. Figure 7 can therefore be used as a design chart: given the required values of α and N_s, the optimal choice of the parameters $(\rho, E_{p-\min})$ corresponding to a minimum of P_s (that is a minimum of the ICI) can be directly read from the chart (paying attention to the constraint that $2N_sE_{p-\min}$ must be an integer number). The designed filter will have the minimum possible value of N for the given N_s. Once the filter has been designed, the value of its actual normalized excess bandwidth ρ_e can be evaluated, as we shall explain in the

$^\dagger E_b$ is the energy-per-bit, and N_o is the gaussian noise one-sided power spectral density.

following (notice that the nominal roll-off parameter ρ of the ideal transfer function (4) and the actual normalized excess bandwidth ρ_e of the digital filter $H_D(f)$ measured with respect to the first-null bandwidth can be different).

Note that the couple of values $(\rho, E_{p-\min})$ shown in Figure 7 are 'optimal' in the sense that they locally minimize the parameter P_s for the selected value of α. Since N_s does not affect the value of P_s, it may be chosen as a small integer value.

In order to make the design method completely automatic, a linear approximation of the curves of Figure 7 can be obtained. In particular, the values of α, ρ and $E_{p-\min}$ in Figure 7 are related to each other by the following approximation:

$$E_{p-\min} = \begin{cases} 1.945\rho\alpha - 1.611\rho + 1.97 & \text{for } 0 < \rho \leq 0.5 \\ -0.9\rho + 0.5\alpha + 1.83 & \text{for } 0.5 < \rho \leq 1 \end{cases} \tag{14}$$

Equation (14) allows to automatically determine the optimal value of ρ when α and $E_{p-\min}$ are given (this choice will be optimal for every value of N_s), or to select a possible value for $E_{p-\min}$ if ρ and α are given. In this second case, the final value for $E_{p-\min}$, denoted $\hat{E}_{p-\min}$, can be chosen as

$$\hat{E}_{p-\min} = \frac{\text{round}(2E_{p-\min}N_s)}{2N_s} \tag{15}$$

where round$(\cdot)$ is the function rounding its argument to the closest integer. Equation (15) selects $\hat{E}_{p-\min}$ as the value closest to $E_{p-\min}$ among those satisfying the constraint of having $N = 2\hat{E}_{p-\min}N_s$ an integer number.

With the aim of expanding the obtained results to a more general scenario, the design chart shown in Figure 8 has been obtained. In this latter figure, the abscissa ρ_e is the actual filter normalized excess bandwidth measured with respect to the first-null bandwidth, i.e. satisfying the formula $H_D(1 + \rho_e/2N_s) = 0$. We denote the parameter ρ_e as 'equivalent roll-off'. From Equation (6), we have that:

$$\rho_e = \frac{\lceil E_p(1+\rho) \rceil}{E_p} - 1 \tag{16}$$

where ρ is the nominal roll-off parameter of Equation (3) and ρ_e is the equivalent roll-off.

In Figure 9 the pair of values (ρ, E_p) that locally minimize P_s are also shown. When the value of E_p is given, Figure 9 can be used to determine the value of ρ that minimizes ICI. When a certain excess bandwidth ρ_e is required, Figure 8 can be used to select E_p and, then, the required value of ρ

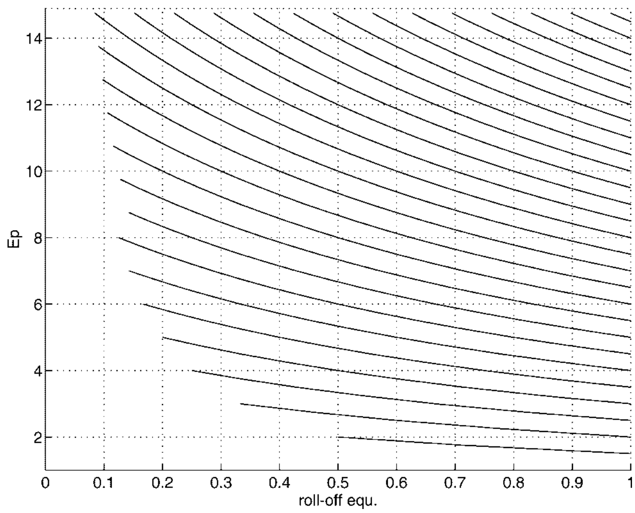

Figure 8. (ρ_e, E_p) design table for 'optimal' FS-N square-root $(\alpha = 0.5)$ raised-cosine filters.

can be determined from Figure 9. It is important to say that the whole procedure can be implemented by an automatic program, able to properly read the design charts.

4. ISI PERFORMANCES

As described in the previous section, the choice of a point (ρ, E_p) with maximum stopband attenuation will very often result in a non-integer value of E_p. The release of the integer constraint on E_p, while allowing very low ICI, negatively affects the ISI performances, especially

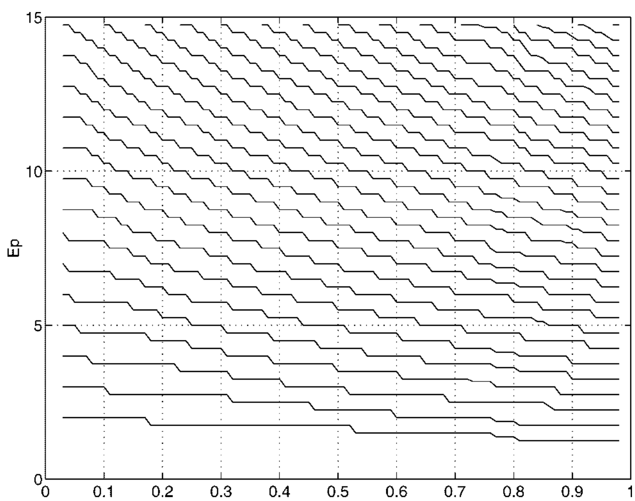

Figure 9. (ρ, E_p) design table for 'optimal' FS-N square-root $(\alpha = 0.5)$ raised-cosine filters.

for low values of ρ and E_p. This is due to a lack of symmetry in the digital filter transition bandwidth.

The width of the digital filter transition bandwidth is $2E_p\rho/N$; for $E_p\rho < 0.5$ in Equation (6) only one value of H_k in the transition bandwidth is fixed by the FS method. If E_p is an integer value, this point is the 'symmetry point' $(1/2N_s, AN_s/2)$, and the symmetry of the filter shape is somehow preserved. If E_p is not an integer and it assumes a small value, the symmetry in the transition bandwidth is lost, with a degradation in the ISI performances. In order to verify this behavior, the peak distortion, which is the key parameter characterizing the time domain response of the designed pulses and their sensitivity to timing errors (i.e., the extent of the eye opening under the worst case binary input combination), has been evaluated for two possible values of E_p ($E_p = 8$ and 8.5) (see Figures 10 and 11). The peak distortion has been evaluated as [11]

$$D_p = \frac{\sum_{k=-\infty, N_s k \neq n_0}^{+\infty} |h[n_o - k \cdot N_s]|}{|h[n_o]|}$$

where n_o is the discrete time instant in which the $h[n]$ takes on its maximum value.

Comparing Figures 10 and 11, it can be observed that, for a given value of ρ, a much smaller value of D_p is obtained if E_p is an integer number. This behavior is general and it has been observed for every other value of E_p.

5. DESIGN EXAMPLES

The design examples presented in the following have been obtained considering square-root of raised-cosine filters

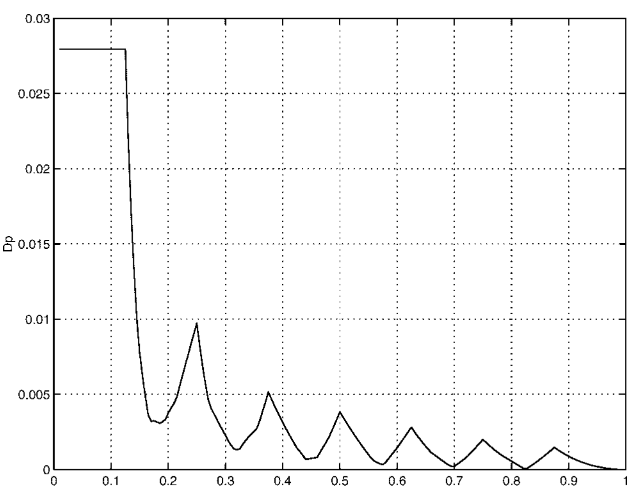

Figure 10. D_p as a function of the roll-off ρ, for $\alpha = 0.5$, $N_s = 2$, $E_p = 8$.

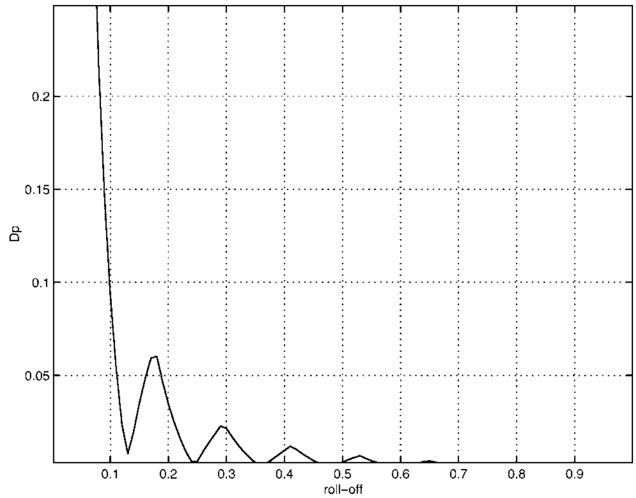

Figure 11. D_p as a function of the roll-off ρ, for $\alpha = 0.5$, $N_s = 2$, $E_p = 8.5$.

($\alpha = 0.5$), in order to obtain filters with the same characteristic of those that are commonly found in the literature. However, we want to point out that the FS-N filter design method is general, and it is valid also for values of α different from 0.5.

In the proposed examples, both symmetric filters with, say, $2M$ taps and asymmetric ones with M taps have been considered, using therefore M degrees of freedom and M multiplications for both structures. This implies that both filters have the same complexity and the same degrees of freedom, so that they can be properly compared, observing in particular their ISI and ICI performance. The comparison of filter design techniques based on the number of multiplications required to implement the designed Nyquist filter is, in fact, a performance indicator extensively used throughout the literature [6], and valid for many practical applications. As an example, symmetrical design methods [13, 14] are compared with asymmetrical ones [5] by observing the number of filter taps for the reasons just produced. Notice that since the FS-N method is based on ISI/ICI mitigation, other classical filter parameters, as for example the in-band ripple, are not considered. In fact, Nyquist filters are narrow-band pulse shaping waveform generators, whose performances do not directly depend on the in-band ripple, provided that the Nyquist criterion and the narrow-band requirements are satisfied. Also the peak-to-average power ratio of the output waveform has not been considered as a design parameter, since the FS-N method does not suggest a new shaping waveform, but only allows to efficiently implement in the discrete time domain a classic raised-cosine shaping pulse. It has however been verified that, given the same actual normalized excess bandwidth ρ_e, the shaping pulses obtained with the FS-N method for $\alpha = 0.5$ have a peak-to-average power ratio (evaluated in the discrete time) which is always smaller than the peak-to-average power ratio of a continuous time classic root-raised-cosine shaping pulse. Notice that for a classic continuous time root-raised-cosine filter, ρ and ρ_e coincide.

With the aim of correctly assessing all the possible problems that could arise in a finite length implementation, the effect on P_s of the filter coefficients quantization has been evaluated. We assume that quantization consists in the rounding of every coefficient and every computation result on N_b bits (including the sign bit). Extensive analysis have shown that the choice $N_b \geq 13$ practically avoids numerical errors, and the operations are completely equivalent to those obtained with infinite precision.

Example 1. In order to illustrate the design method proposed in this paper, we consider the specifications drawn from the first example of Reference [5] in which, in order to guarantee an actual normalized excess bandwidth $\rho_e = 0.23$, a FIR solution with asymmetric impulse response and 24 taps has been proposed, obtaining an equiripple non-linear phase filter with stopband attenuation of 23.4 dB and $D_p = 0$.

In order to satisfy the same specifications, we propose a transmit/receive filter with the following parameters:

$$\alpha = 0.5, N = 40, N_s = 4, E_p = 5, \rho_e = 0.2 \text{ and } \rho = 0.05$$

derived from the design charts shown in Figures 8 and 9. The specified parameter values correspond to a local maximum of the stopband attenuation. To perform the design, we added the constraint of having a low value of peak distortion: this has been obtained assuming an integer value for E_p.

The transfer function magnitude of the designed filter is shown in Figure 12. The peak distortion is equal to $D_p = 1.1 \times 10^{-2}$.

The FS-N filter has the first sidelobe at -22 dB, hence 1.4 dB higher than the first sidelobe of the filter designed in Reference [5]. However, the successive sidelobes are all lower than those in Reference [5] with a large margin.

Furthermore, being the filter impulse response symmetric, the FS-N filter only requires $N/2$ multiplications, and has, therefore, the same complexity of a 20 taps asymmetric filter, which is four taps smaller than the complexity required by the solution in Reference [5]. Finally, the FS-N filter has linear-phase, which is very helpful for synchronization and in presence of non-linear distortion [16].

Example 2. As second example, we consider the design presented in Reference [15], where a linear-phase symmetric filter with 48 coefficients, excess bandwidth factor 0.5, stopband attenuation of about 55 dB and $D_p = 0$ has been proposed. The FS-N solution yields to a filter with the followings parameters:

$$\alpha = 0.5, N_s = 4, \rho_e = 0.5, E_p = 6 \text{ and } \rho = 0.39$$

The designed FS-N filter has 48 coefficients, and its transfer function is shown in Figure 13.

As it can be observed from Figure 13, the FS-N filter has stopband attenuation larger than 55 dB, it therefore compares slightly favorably to the solution proposed in [15].

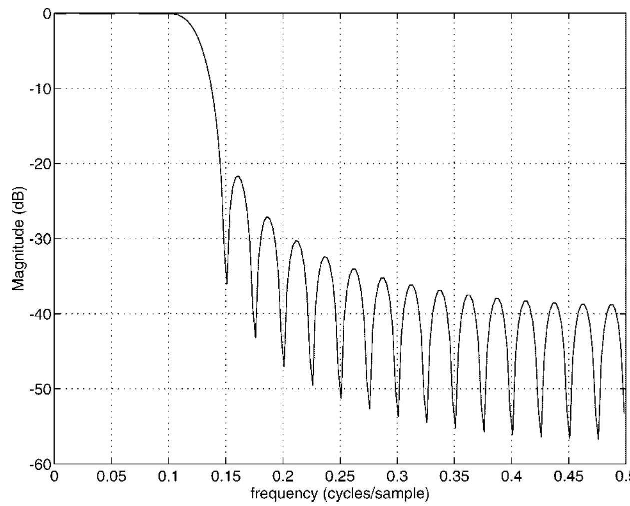

Figure 12. Transfer function magnitude of the square-root raised-cosine filter described in Example 1.

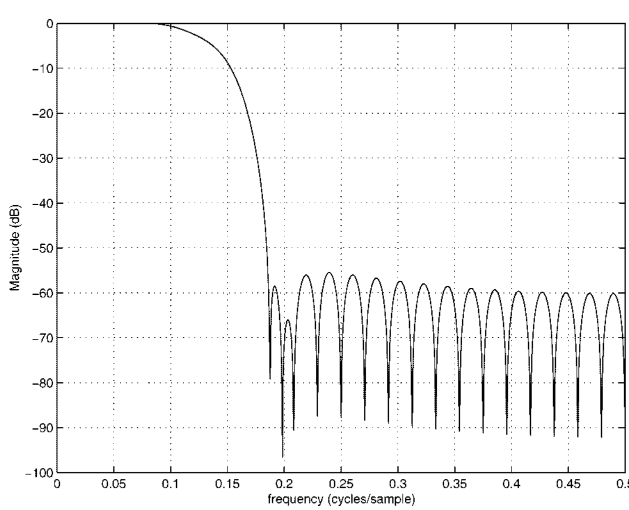

Figure 13. Transfer function magnitude of the square-root raised-cosine filter described in Example 2.

Furthermore, the peak distortion is equal to $D_{\mathrm{p}} = 1.3 \times 10^{-3}$.

6. CONCLUSIONS

A simple and completely automatic design method for low ICI raised-cosine FIR filter pairs based on FS techniques has been described. The proposed design method separately controls the stopband attenuation of both the transmit and the receive filters.

Simple design tables for raised-cosine digital filters have been presented. Filters with very high out-of-band attenuation can be obtained for every value of the raised-cosine splitting factor α, which can be an important design parameter in presence of non-linear transmission channels. Look-up table techniques can be used to obtain very short FIR filters able to meet fixed specifications, having N_{s}, ρ, E_{p} and α as parameters. Examples have illustrated that the proposed method can be used as an alternative to more complicated and time consuming FIR design techniques.

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