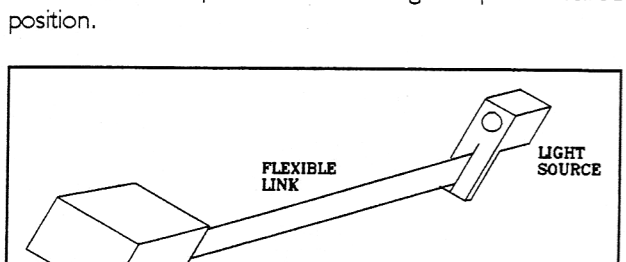
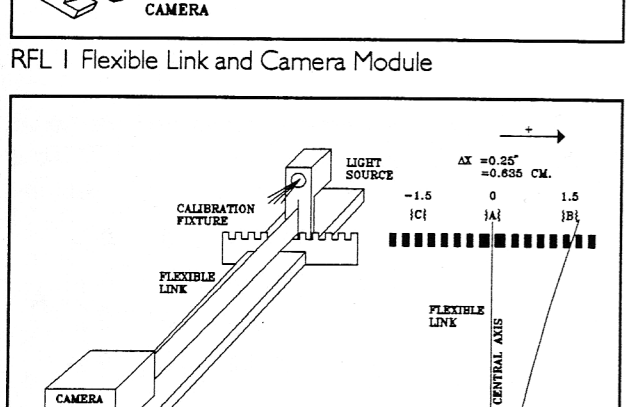
2.5.1 DESCRIPTION

The flexible link experiment is another module that can be attached to the servo plant when configured as a high gear ratio system. This experiment is similar to the flexible joint experiment except that the link is flexible rather than the joint. As shown in Figure RFL1, a light source is attached to the tip of the beam which is detected by a camera mounted to the rotating base. The purpose of the experiment is to design a feedback system which reduces (eliminates) the undamped oscillations of the tip while commanding the tip to a desired position.



RFL 1 Flexible Link and Camera Module

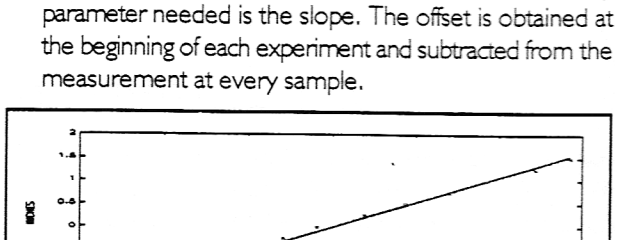


RFL 2 Calibration procedure for tip measurement

2.5.1.2 CALIBRATION

The camera output is an analog signal which is proportional to the relative deflection of the light source from the central axis. The gain and offset of the measurement are adjustable through two trimpots located on the camera. The output of the camera must be calibrated in order to obtain a measurement of tip deflection. A calibration fixture is supplied and is used to obtain measurements from the camera at 13 equidistant deflection points. The calibration procedure is the following:

a) attach the camera to the calibration fixture as shown in Figure RFL2.
b) measure the voltage at the camera output using a voltmeter or a computer program that prints out the sampled voltage using an A/D.
c) adjust the offset until the reading is zero when the light source is in position {A}.
d) move the light source to position {B} and adjust the gain until the voltage is 1.5 volts.
e) repeat (c) and (d) a few times until the measurements are within 100 mV from the required values. (0 and 1.5 V);
f) measure the voltages at all the other points (from {B} to {C}) and fit the data to a least squares straight line (eg matlab, see calcam.m, see example Figure RFL3). Note the distance between the points is 0.25" or (0.635 cm.) and ranges from -1.5 to 1.5 " (-3.81 to 3.81 cm.) using 13 points.
g) the straight line fit results in a slope and an offset. The only parameter needed is the slope. The offset is obtained at the beginning of each experiment and subtracted from the measurement at every sample.



RFL 3 Typical calibration curve of tip position versus measured voltage

2.5.2 MATHEMATICAL MODEL

A lumped parameter model for the system can be derived using the following (simple) approach:

1) obtain the natural frequency of the link with the base clamped. This is obtained experimentally to be fc = 1.8 Hz.

then, the **angular** deflection of the tip with respect to the clamped camera frame is given by:

$$\ddot{\alpha} = -\omega_c^2 \alpha$$

2) estimate the stiffness of the clamped joint since:

$$\omega_c = \sqrt{\frac{K_{STIFF}}{J_L + J_b}} \quad then \quad K_{STIFF} = (2\pi f_c)^2 (J_L + J_b)$$

where J_l is the inertia of the link assuming a rigid body and J_b is the inertia of the bulb at the end of the link.

$$J_L = \frac{1}{3} m_L L^2$$
$$J_b = m_b L^2$$

3) Assume the stiffness is *collocated* at the joint and write the state space model with α and θ (see Figure RFL4). Proceed with developing the differential equations as in the rotary flexible joint experiment (see Section 2.4.2.2). This approach is a gross approximation but it will work. More complete models can be derived using assumed mode shapes. These are however beyond the scope of this handbook.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{STIFF}}{J_{hub}} & -\frac{Km^2\,Kg^2}{RJ_{hub}} & 0 \\ 0 & -\frac{K_{STIFF}(J_{load} + J_{hub})}{J_{hub}J_{load}} & \frac{Km^2\,Kg^2}{RJ_{hub}} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{KmKg}{RJ_{hub}} \\ -\frac{KmKg}{RJ_hub} \end{bmatrix}$$

$$X = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$$

5) but the measurement is (for d « L)

$$d = \alpha L \Rightarrow \alpha = \frac{d}{L}$$

substituting , we obtain:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{STIFF}}{J_{hub} L} & -\frac{Km^2\,Kg^2}{RJ_{hub}} & 0 \\ 0 & -\frac{K_{STIFF}(J_{load} + J_{hub})}{J_{hub}J_{load}} & \frac{L\,Km^2\,Kg^2}{RJ_{hub}} & 0 \end{bmatrix}$$

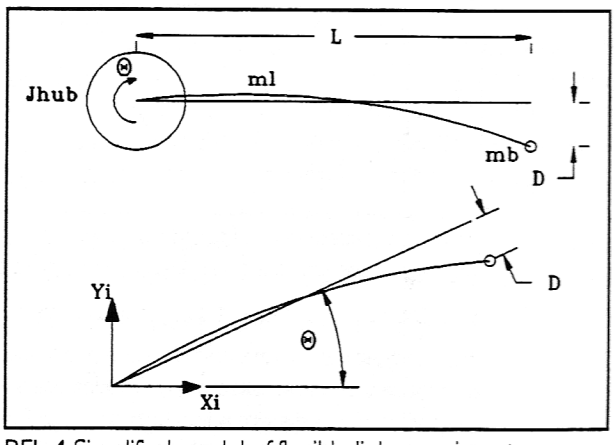
$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{KmKg}{RJ_{hub}} \\ -\frac{L\,KmKg}{RJ_{hub}} \end{bmatrix}$$

$$X = \begin{bmatrix} \theta & d & \dot{\theta} & \dot{d} \end{bmatrix}^T$$

$$Y = \begin{bmatrix} 1 & \frac{1}{L} & 0 & 0 \end{bmatrix}$$

where

$J_{load} = J_L + J_b$ *is the total load inertia*



RFL 4 Simplified model of flexible link experiment

using the given system parameters we have

$$\begin{bmatrix} \dot{\theta} \\ \dot{d} \\ \ddot{\theta} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1919 & -57 & 0 \\ 0 & -932 & 24 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ d \\ \dot{\theta} \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 106 \\ -44 \end{bmatrix} V_m$$

2.5.3 CONTROL SYSTEM DESIGN

The design is performed using LQR with

$$Q = diag\ [300 \quad 30000 \quad 1 \quad 0] \ and \ r = 2$$

resulting in

$$k = [12 \quad -95 \quad 2.6 \quad 4.4]$$

for units in V/rad and V/m

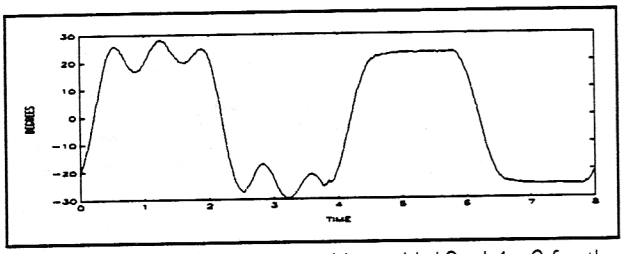
or

$$k = [0.21 \quad -0.95 \quad 0.046 \quad 0.045]$$

for units in V/deg and V/cm.

2.5.4 RESULTS

The systems is implemented and it is found to perform well. Figure RFL5 compares the response of the tip position to ramp inputs (100 deg/sec) using full state feedback versus only motor feedback (k2=k4=0). Clearly, camera feedback plays a major role in obtaining a well behaved response.



RFL 5 Step responses of tip position with k2=k4=0 for the first two steps and full state feedback for next two steps.

The system is very sensitive to variations in low pass filter cutoff frequency, sampling rate and derivative feedback gains. This is because the unmodelled higher modes will be excited if the bandwidth of the system is too high or if high frequency noise (due to derivative feedback) is present.

Note that the C matrix in the system is defined as:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \frac{1}{L} & 0 & 0 \end{bmatrix}$$

where the third output is total deflection of the tip. This is the variable which we are trying to control (and is shown in the plot).

The system can be tuned by changing the gains k2 and k4. Increasing k2 (in magnitude only, the sign should always be negative relative to the other gains!) will increase the overshoot in the total deflection. Too much k2 would lead to instability. Reducing the magnitude would reduce the overshoot but system response would slow down and eventually the system will be too underdamped. Increasing k4 will dampen the overshoot up to a certain point after which you start exciting the higher modes and again instability results. Too little k4 would also result in instability.

CAUTION

1)The camera has an active circuit which is biased with +12V and -12V. DO NOT CONFUSE THE TWO! this may damage the circuitry.

2) The camera detects the position of the light source. Make sure there are no other bright sources or large variations in light in the direction to which the camera is pointing. Do not point the camera to a window for example since the light from the window will deteriorate the fidelity of the measurement.