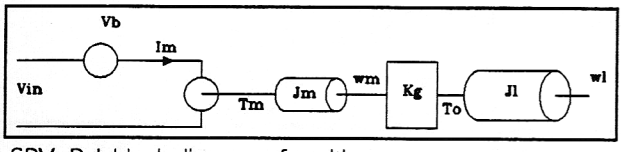
The rotary motion experiments are based on the Rotary Servo Plant SRV-02. The single input single output experiments that can be performed with the servo, using the several modules available are the position servo, ball and beam, rotary inverted pendulum, rotary flexible joint, flexible link, gyro / stable platform.

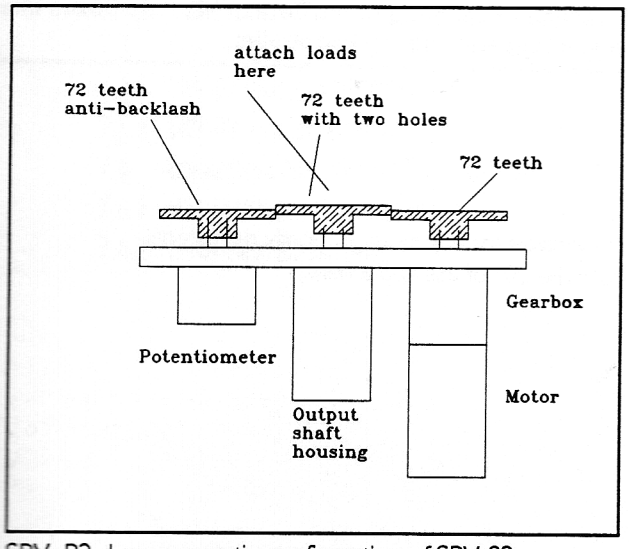
The two input, two output experiments that can be performed are 2D-ROBOT, 2D-GANTRY and 2D-inverted pendulum.

2.1.1 DESCRIPTION

The block diagram for the position Servo SRV-02 is shown in Figure SRV_R1. It consists of a DC servomotor with built in gearbox whose ratio is 14 to 1. The output of the gearbox drives a potentiometer and an independent output shaft to which a load can be attached. The purpose of the experiment is to implement a controller which controls the position of the output shaft. Note that for this experiment, the Servo is used in the low gear ratio configuration (3 equal external gears). (See Figure SRV_R2)



SRV_R 1 block diagram of position servo



SRV_R2 Low gear ratio configuration of SRV-02.

2.1.2 MATHEMATICAL MODEL

The model is derived from the basic equations of a DC motor:

θ = angle of output shaft
w_m = angular velocity of motor shaft
w_l = angular velocity of output shaft
K_g = gear ratio

Electrically:

$$V_{in} = I_m R_m + K_m \omega_m$$
$$= I_m R_m + K_m K_g \omega_l$$

Mechanically:

T_m = torque generated by motor
T_o = torque at the output after the gearbox
J_m = motor inertia
J_l = load inertia

$$T_o = K_g T_m = K_g (J_m \dot{\omega}_m + J_l \frac{\dot{\omega}_l}{K_g})$$
$$= J_m K_g^2 \dot{\omega}_l + J_l \dot{\omega}_l$$
$$= \dot{\omega}_l (J_m K_g^2 + J_l)$$
$$\text{but } T_m = K_m I_m$$

then

$$I_m = \frac{T_m}{K_m}$$
$$= \frac{T_o}{K_m K_g}$$
$$= \dot{\omega}_l \frac{K_g^2 J_m + J_l}{K_m K_g}$$
$$= \dot{\omega}_l \frac{J_{eq}}{K_m K_g}$$

Note that $J_{eq} = K_g^2 J_m + J_l$ is the equivalent inertia seen at the output of the gearbox.

hence

$$\frac{\theta(s)}{V_{in}(s)} = \frac{1}{s\,(s\frac{R_m J_{eq}}{K_m K_g} + K_m K_g)}$$

with the given parameters, we have:

$$\frac{\theta(s)}{V_{in}(s)} = \frac{1}{s\ (0.0026\ s + 0.1081)}$$

2.3 CONTROL SYSTEM DESIGN

Clearly, the open loop position response of the DC motor is "unstable" due to the pole at the origin. A "proportional plus derivative" controller:

$$V_{in} = K_p(\theta_d - \theta) - K_d\dot{\theta}$$

is used so the output will track the desired angle. The feedback gain K_p and K_d are used to design the response of the system. Substituting the feedback law into the open loop transfer function we obtain:

$$\frac{\theta}{\theta_d} = \frac{K_p}{0.0026s^2 + (0.108 + K_d)s + K_p}$$

The required response is a system that has 200 msec. peak time and is critically damped. The desired characteristic polynomial is:

$$s^2 + 2\zeta w_o s + w_o^2 \quad t_p = \frac{\pi}{w_o\sqrt{(1-\zeta^2)}} = 0.2 \quad \zeta = 0.707$$

Solving for K_p and K_d results in:

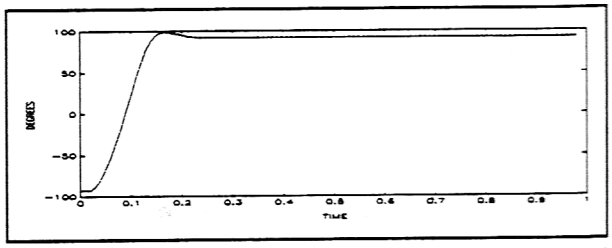
K_p = 0 .025 V/Deg and K_d = -0.0004 V.Sec/Deg

2.1.4 RESULTS

All control programs are implemented in Turbo C running on an IBM PC 486 compatible computer equipped with a 12 bit analog board. For details about the control programs, see the section entitled "Control Software".

The above gains are implemented in the controller running at 200 Hz. The step response of the system is shown in Figure SRV_R3. The system response closely matches the design. It is important to note however that the voltage saturates to Vmax (±5V) if the error is greater than (Vmax/Kp) degrees. The saturation causes a delay in the peak time as shown in the figure. For small changes in the command, the peak time is 100 msec and as the command gets larger, the peak time increases.

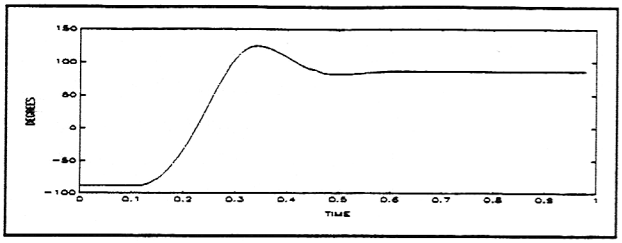
Changing the load also results in a variation in response as shown in Figure SRV_R4. Different gains are required for the new load. An adaptive controller may be designed that compensates for load variations.



SRV_R3 Step response of servo plant with low inertia load

CAUTION

High frequency voltage applied to a (any) motor will eventually damage the gearbox or the brushes. The most likely source for high frequency noise is derivative feedback. If the derivative feedback gain is too high, a noisy voltage will be fed to the motor. You should have a band-limited differentiator rather than a pure differentiator running in the feedback loop (See section 4.5). If you hear a "buzz" in the motor you are feeding high frequency noise to the motor and will damage it. **Turn the motor off immediately and reconsider your design!** Select a low pass filter frequency that eliminates the "buzz" or reduce the derivative gain. Always have an anti-aliasing filter connected to the input of the A/D. This could simply be a capacitor as shown in the wiring diagrams. The capacitor will filter out high frequency noise before it is processed.



SRV_R4 Step response of servo plant with high inertia load using feedback gains designed for low inertia case

A steady state error also occurs due to deadband and static friction. This error can be compensated for using an integrator in the loop:

$$V_{in} = K_p(\theta_d - \theta) - K_d\, s\, \theta - K_i \frac{(\theta_d - \theta)}{s}$$

The gain of the integrator should not be too high however since this will result in undesired overshoot and eventually instability. The integrator output should also be limited to a reasonable value, for example 1 volt.