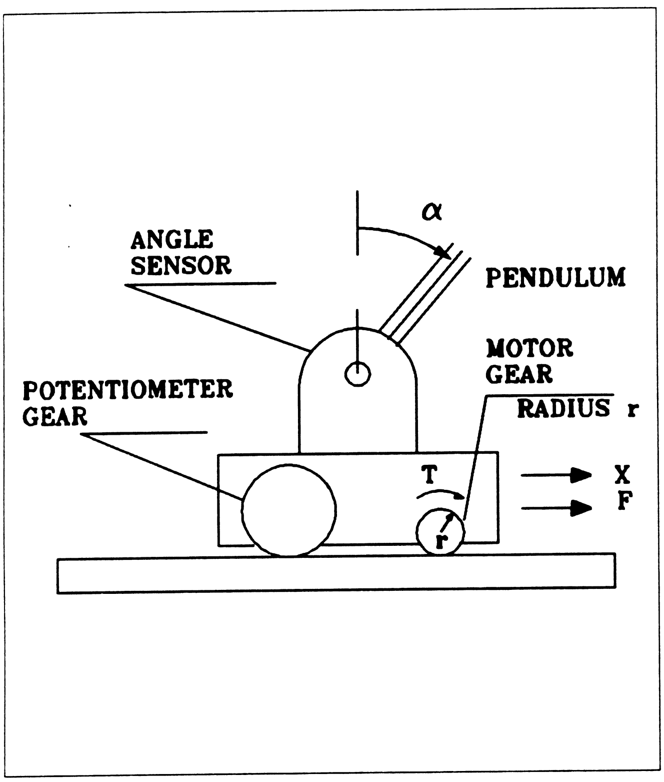
3.1.1 DESCRIPTION

The basic component for the linear motion experiments is the inverted pendulum cart and track shown in Figure SRV_L1 It consists of a cart which slides on a ground stainless steel shaft. The cart is equipped with a motor and a potentiometer. These are coupled to a rack and pinion mechanism to input the driving force to the system and to measure cart position respectively. The motor shaft is connected to a 0.5" diameter gear while the potentiometer shaft is connected to a 1.166" diameter gear. Both these gears mesh with the toothed rack. When the motor turns, the torque created at the output shaft is translated to a linear force which results in the cart's motion. When the cart moves, the potentiometer shaft turns and the voltage measured from the potentiometer can be calibrated to obtain the track position. The purpose of the feedback system is to control the position of the cart.



SRV_L 1 Inverted Pendulum Cart

3.1.2 MATHEMATICAL MODEL

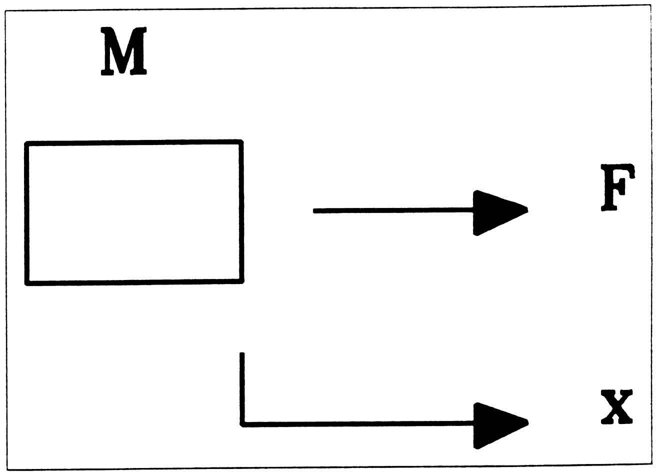
The differential equation for the cart with force as input is simply (see Figure SRV_L 2):

$$F = m\ddot{x}$$

the system however is driven by a DC motor whose equations are:

$$V = I_m R_m + K_m K_g \omega_l = I_m R_m + K_m K_g \frac{\dot{x}}{r}$$

where ω_l is the motor output angular velocity (after the gearbox), x' is the cart linear velocity and r is the radius of the output gear that meshes with the track.



SRV_L 2 Simplified model for linear positioning servo

The above equation results in:

$$I_m = \frac{V}{R_m} - K_m K_g \frac{\dot{x}}{r}$$

The torque generated at the output of the motor is:

$$\tau = K_m K_g I_m$$

which results in a force applied to the cart as

$$F = \frac{\tau}{r} = \frac{K_m K_g I_m}{r} = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}$$

resulting in

$$m\ddot{x} = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}$$

or

$$\frac{x}{V} = \frac{1}{s\left(\frac{m\,R\,r}{K_m K_g} s + \frac{K_m K_g}{r}\right)}$$

Note the similarity of this transfer function with the transfer function of the open loop rotary servo plant. Substituting the system parameter values we have:

$$\frac{x}{V} = \frac{1}{s(0.26s + 4.47)}$$

3.1.3 CONTROL SYSTEM DESIGN

The design proceeds as in the rotary servo design. The controller is of the form:

$$V = K_p(x_d - x) - K_d\dot{x}$$

resulting in a closed loop transfer function:

$$\frac{x}{x_d} = \frac{K_p}{0.26s^2 + (4.47 + K_d)s + K_p}$$

the desired response has a peak time of 0.5 seconds and is critically damped:

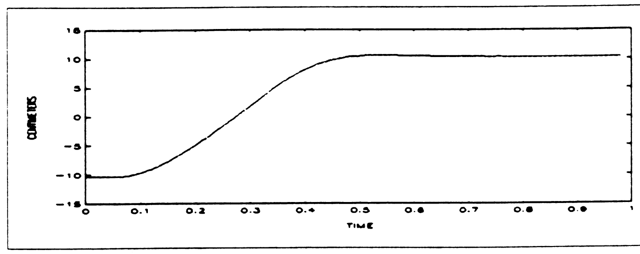
$$s^2 + 2\zeta w_o s + w_o^2 \quad t_p = \frac{\pi}{w_o\sqrt{(1-\zeta^2)}} = 0.25 \quad \zeta = 0.707$$

resulting in

Kp = 0.83 V/cm and Kd = 0.02 V sec/cm

3.1.4 RESULTS

The step response of the system using the above gains is shown in Figure SRV_L 3. The system behaves as expected but it is important to note that when the amplifier saturates, the dynamic response will slow down.



SRV_L 3 Step response of linear servo

CAUTION

High frequency voltage applied to a (any) motor will eventually damage the gearbox or the brushes. The most likely source for high frequency noise is derivative feedback. If the derivative feedback gain is too high, a noisy voltage will be fed to the motor. You should have a band-limited differentiator rather than a pure differentiator running in the feedback loop (See section 4.5). If you hear a "buzz" in the motor you are feeding high frequency noise to the motor and will damage it. Turn the motor off immediately and reconsider your design! Select a low pass filter frequency that eliminates the "buzz" or reduce the derivative gain. Always have an anti-aliasing filter connected to the input of the A/D. This could simply be a capacitor as shown in the wiring diagrams. The capacitor will filter out high frequency noise before it is processed.