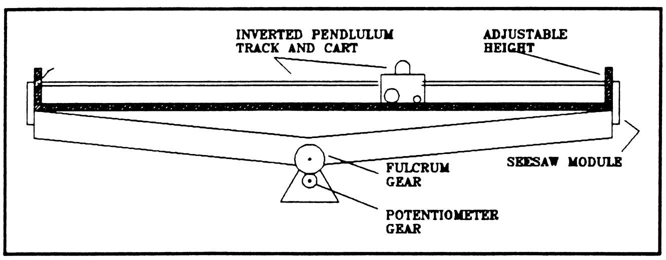
### 3.4.1 DESCRIPTION

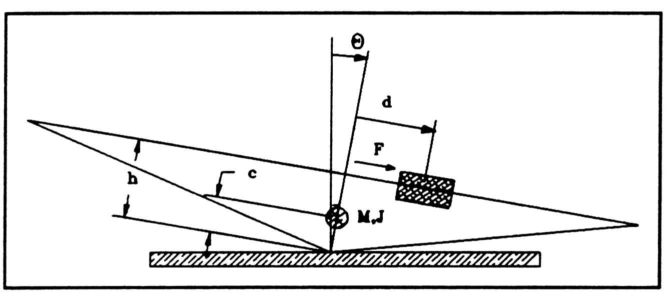
The Seesaw Module shown in Figure SS1 consists of two long arms hinged onto a support fulcrum. The axis is coupled to a potentiometer via a 3:1 gear ratio which is used to measures the angular deflection of the seesaw. The pendulum track and cart **(the pendulum itself is not used and should be removed)** can easily be placed on top of the seesaw resulting in the experimental setup for this lab. The objective of the experiment is to design a feedback control system that controls the position of the cart such that the seesaw remains horizontal.




SS 1 Seesaw Module with Inverted Pendulum track and cart

### 3.4.2 MATHEMATICAL MODEL

Consider the simplified model shown in Figure SS2.




SS 2 Simplified model for seesaw experiment

where

| | | |
|---|---|---|
| F | = | input force to the cart (N) |
| m | = | mass of the cart (Kg) |
| M | = | Mass of [seesaw + track] (kg) |
| c | = | distance of centre of gravity of [seesaw+track] from pivot point(m) |
| h | = | height of track from pivot point(m) |

*The states of the system are:*

| | | |
|---|---|---|
| $x$ | = | position of the cart relative to the centre of the track (m) |
| $\dot{x}$ | = | velocity of the cart(m/sec) |
| $\theta$ | = | angle of the seesaw relative to the vertical (rad) |
| $\theta$ | = | angular velocity of the seesaw(rad/sec) |

Note the directions defined for these variables in Figure SS2.

The nonlinear model is derived using MAPLE (symbolic manipulation language). The kinetic and potential energies of each element in the system are individually derived and then the non-linear differential equations are obtained as:

$$m\ddot{x} + m h \ddot{\theta} - m x \dot{\theta}^2 - m g \sin(\theta) = F$$

$$(J + m h^2 + 2 m x \dot{x} + m x^2)\ddot{\theta} + m h \ddot{x} - m g h \sin(\theta) - m g x \cos(\theta) - M g c \sin(\theta) = 0$$

The linear model is derived as:

$$\begin{bmatrix}\dot{x}\\ \dot{\theta}\\ \ddot{x}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\dfrac{m h g}{J} & -g\dfrac{M h c - J}{J} & 0 & 0\\ \dfrac{m g}{J} & \dfrac{M g c}{J} & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \alpha\\ \dot{x}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \dfrac{m h^2 + J}{J m}\\ -\dfrac{h}{J}\end{bmatrix} F$$

Using system parameters and converting to voltage input we have:

$$\begin{bmatrix}\dot{x}\\ \dot{\theta}\\ \ddot{x}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1.45 & 9.18 & -17.2 & 0\\ 10.44 & 4.38 & 2.5 & 0\end{bmatrix}\begin{bmatrix}x\\ \theta\\ \dot{x}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 3.85\\ -.56\end{bmatrix} V$$

Which is the desired representation.

### 3.4.3 CONTROL SYSTEM DESIGN

Using MATLAB (lqr design) we obtain the optimal feedback gain K which minimizes the quadratic performance index:

$$J = \int (x'Qx + rV^2)\, dt$$

with

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 3000 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and $r = 1$

resulting in

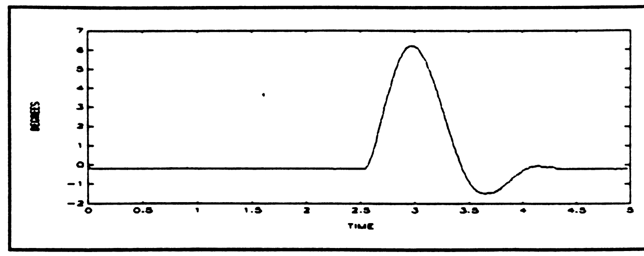
K = [96  114 9.3 40.7] for units in V/rad and V/m.

or

K = [0.96  2.0 .09 .7] for units in V/deg and V/cm

### 3.4.4 RESULTS

Figure SS3 shows the response of the seesaw angle to a disturbance. The disturbance was a tap to the seesaw that caused it to swing to approximately 6 degrees. The seesaw tilts back and settles in the horizontal position within 2 seconds.



SS 3 Disturbance response of seesaw angle