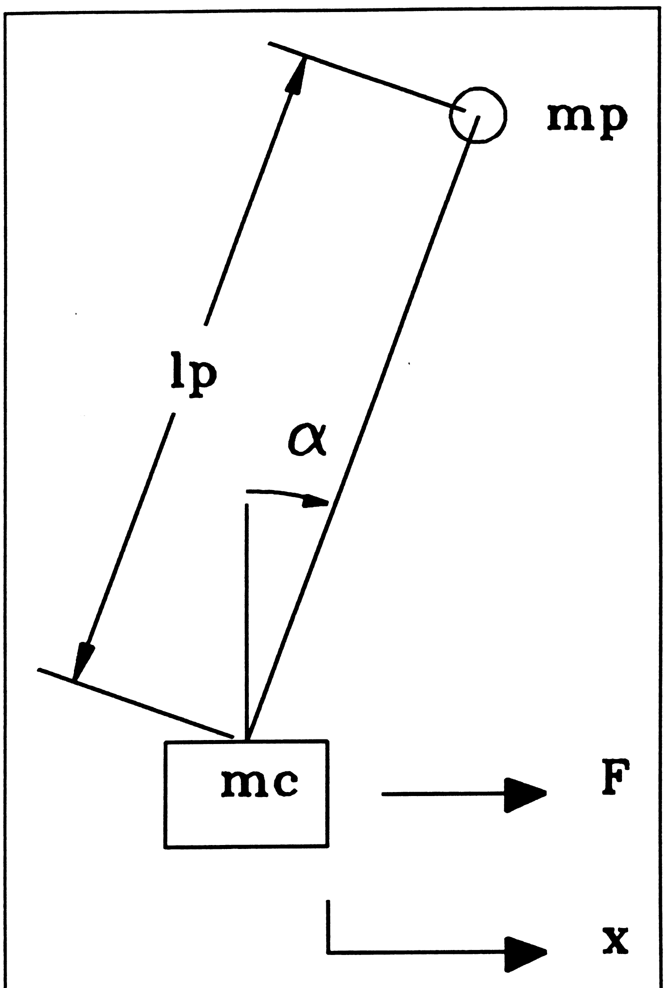
3.3.1 DESCRIPTION

A rod is mounted on the cart used in the previous experiments whose axis of rotation is perpendicular to the direction of motion of the cart. A potentiometer mounted on the axis of rotation allows you to measure the angle of the rod with the vertical axis. The objective of this experiment is to design a controller that would stabilize the rod and keep the cart in a desired position.

3.3.2 MATHEMATICAL MODEL

Consider the simplified model in Figure IP1. Note that l_p is half the actual length of the pendulum ($l_p = 0.5\ L_p$).



IP 1 Simplified model of inverted pendulum experiment

The differential equations are derived like the Rotary Inverted Pendulum resulting in the following:

$$(m_p+mc)\,\ddot{x} + m_p\,\ddot{\alpha}\,l_p\cos(\alpha) - m_p\,\dot{\alpha}^2\,l_p\sin(\alpha) = F$$
$$m_p\,l_p\cos(\alpha)\,\ddot{x} - m_p\,l_p\sin(\alpha)\,\dot{\alpha}\,\dot{x} + m_p\,\ddot{\alpha}\,l_p^2 - m_p\,g\,l_p\sin(\alpha) = 0$$

and the linearized equation:

$$\begin{bmatrix}\dot{x}\\ \dot{\alpha}\\ \ddot{x}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{m_p g}{m_c} & 0 & 0\\ 0 & \dfrac{(m_p+m_c)g}{m_c\,l_p} & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \alpha\\ \dot{x}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \dfrac{1}{m_c}\\ -\dfrac{1}{m_c l_p}\end{bmatrix} F$$

where

F	= input force to the cart (N)
m_p	= mass of rod (Kg)
m_c	= mass of the cart (Kg)
l_p	= centre of gravity of rod (m) (half of full length)

To convert to voltage input we had derived the relationship (section 3.1.2):

$$F = \frac{K_m K_g}{Rr}V - \frac{K_m^2 K_g^2}{Rr^2}\dot{x}$$

substituting this into the matrix equation we have:

$$\begin{bmatrix}\dot{x}\\ \dot{\alpha}\\ \ddot{x}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -4.5 & -16.8 & 0\\ 0 & 46.9 & 55.3 & 0\end{bmatrix}\begin{bmatrix}x\\ \alpha\\ \dot{x}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 3.8\\ -12.4\end{bmatrix} V_{in}$$

which is the desired representation.

3.3.3 CONTROL SYSTEM DESIGN

Using MATLAB (lqr design) we obtain the optimal feedback gain K for the feedback law:

$$V_{in} = -(k_1 (x-x_d) + k_2 \alpha + k_3 \dot{x} + k_4 \dot{\alpha})$$

such that the closed loop system:

$$A_c = A\text{-}bK$$

minimizes the quadratic performance index:

$$J = \int (x'Qx + rV_{in}^2)\, dt$$

with

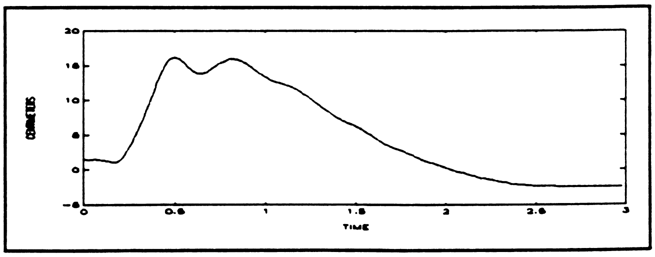
$$Q = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \textit{and} \quad r = 0.0003$$

The optimal gain is given by:

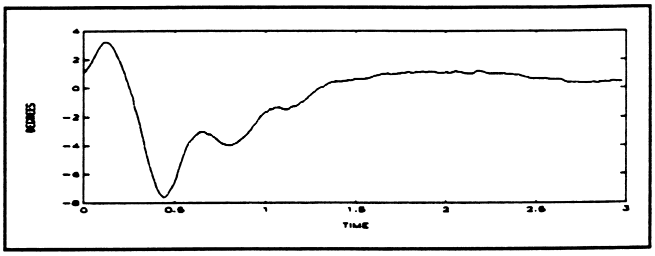
K = [-31 -141 -34 -14] for units in V/m and V/rad.
or
K = [-.31 -2.45 -.34 -.24] for units in V/cm and V/deg.

3.3.4 RESULTS

Figures IP2 and IP3 show the disturbance response of the stabilized system. The pendulum is tapped to about 4 degrees. This results in the cart moving towards the fall to about 15 cm which results in the pendulum tipping in the opposite direction to about -8 degrees. The system balances within 3 seconds.



IP 3 Disturbance response of cart position to a tap to the pendulum



IP 2 Disturbance response of pendulum angle to a tap to the pendulum

(Note that the two traces are not obtained at the same time and are not from the same trial)

> CAUTION
>
> To zero the angle measurement you should hold the pendulum vertical(with the motor turned off) and hit the letter 'z' from the main menu of the controller. This takes the present measurement as zero. Do this before you start the controller.
>
> **Always start the controller with the pendulum held vertical!**