Applied Sliding Mode Control

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Outline

1. Introduction
2. Motivating Example:
   (a) Linear control
   (b) Variable structure control
3. Chattering Phenomena
4. SMC based on observer
5. SMC based on high-gain observer
6. SMC of time-delay systems
7. Applications
8. Conclusion

Introduction

- Automatic control applications:
  - aerospace;
  - robotics;
  - consumer electronics;
  - industrial process control;
  - power systems;
  - biomedical;
  - etc.
- Benefits of automatic control:
  - improves transient and steady-state performance;
  - reduces the effects of uncertainties and disturbances;
  - reduces energy consumption ...

Introduction

- Some control approaches:
  - linear: PIDs, state feedback, etc;
  - linear robust: H_∞, QFT, etc;
  - adaptive;
  - neural networks;
  - fuzzy logic;
  - learning control;
  - sliding mode control (SMC) or
  - variable structure control (VSC) ,

Introduction

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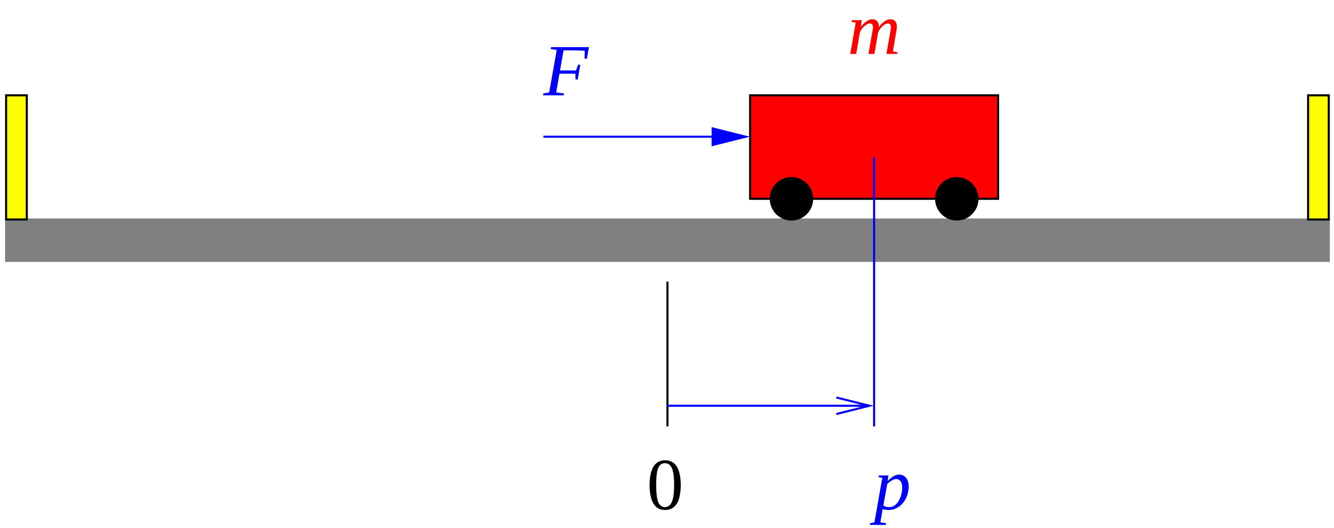
  - and if nothing works, ...

Introduction

- Some control approaches:
  - linear: PIDs, state feedback, etc;
  - linear robust: H_∞, QFT, etc;
  - adaptive;
  - neural networks;
  - fuzzy logic;
  - learning control;
  - sliding mode control (SMC) or
  - variable structure control (VSC) ,
  - and if nothing works, then try *voodoo*.

Motivating Example

- Simple mechanical system:



- Dynamic model:

$$\frac{d^2 p}{dt^2} = \frac{1}{m} F \, .$$

Motivating Example

- State-space model:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F \,,$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \,,$$

  where:

  - State: $x := \begin{bmatrix} p \\ \dot{p} \end{bmatrix}$,
  - Input: $u := \begin{bmatrix} F \end{bmatrix}$,
  - Output: $y := \begin{bmatrix} p \end{bmatrix}$.

Motivating Example

- Linear control:
  - proportional (P) : no damping;
  - proportional + derivative (PD) : damped oscillations;
  - proportional + integral + derivative (PID) : disturbance elimination.
- PD control is equivalent to state feedback:

$$u(t) = K_p p_{ref}(t) - Kx(t) \, ,$$

with gain matrix $K := \begin{bmatrix} K_p & K_d \end{bmatrix}$.

- Problem: closed-loop transfer function is sensitive to m:

$$G_f(s) := \frac{p(s)}{p_{ref}(s)} = \frac{K_p}{ms^2 + K_d s + K_p} \, .$$

Motivating Example

- Variable structure control (VSC):
  - based on state feedback;
  - damps oscillations;
  - rejects disturbances;
  - immune to parametric uncertainties.
- Control laws:

$$u = \begin{cases} u^+(x,t)\,, & \text{if } \sigma(x) > 0\,, \\ u^-(x,t)\,, & \text{if } \sigma(x) < 0\,, \end{cases}$$

or

$$u = -\rho(x,t)\,\mathrm{sgn}(\sigma(x))\,.$$

Motivating Example

- Sliding surface:

$$\sigma(x) = Sx = 0\,.$$

- In this case:

$$\sigma(x) = \dot{p} + \lambda p\,.$$

- When $\sigma(x) = 0\,, \quad \forall t \geq t_1 \geq 0\,,$ the state is governed by:

$$\dot{p} + \lambda p = 0\,,$$

- which has the solution:

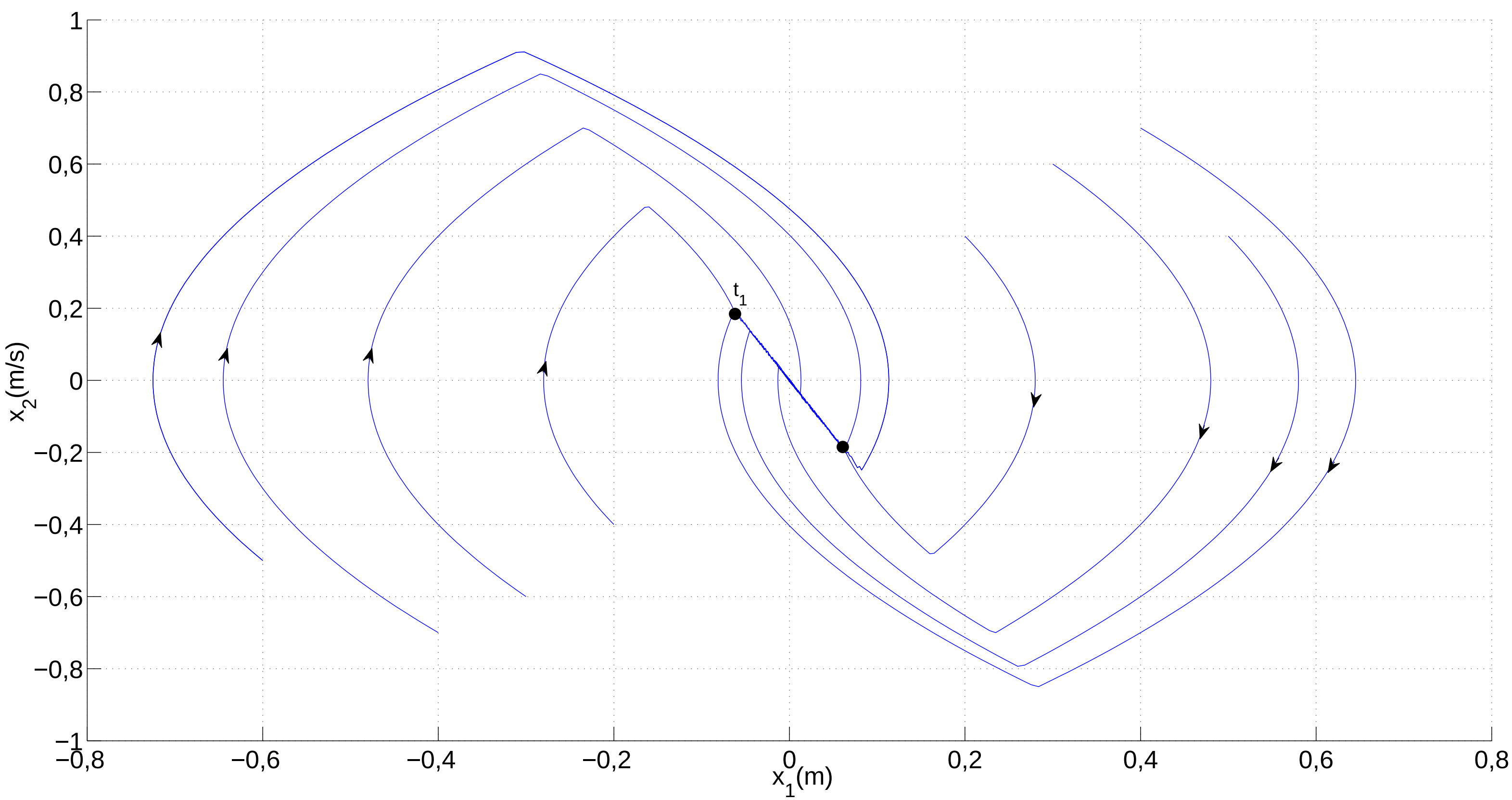
$$p(t) = \mathrm{e}^{-\lambda(t-t_1)}p(t_1)\,, \qquad \forall t \geq t_1 \geq 0\,,$$

- that is immune to parameter uncertainties or disturbances.
- This is the invariance property of SMC!

Motivating Example

- Phase portrait:

$$u = -\text{sgn}(\sigma(x))\,, \qquad \sigma(x) = x_1 + \frac{1}{3}x_2\,.$$



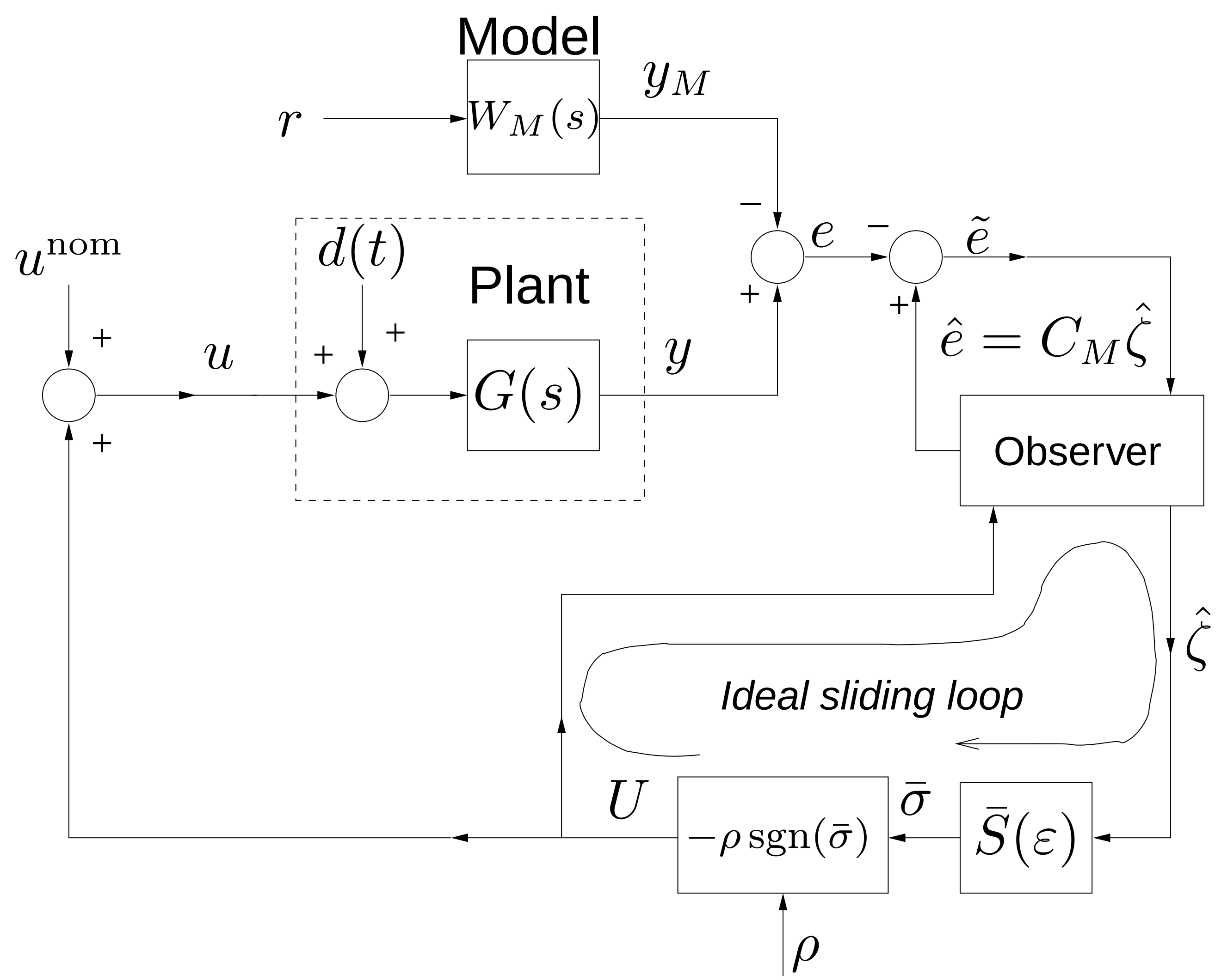
Chattering Phenomena

- Ideal sliding mode: infinite switching frequency.
- Chattering:
  - Imperfections cause finite switching frequency:
    - time delays;
    - hysteresis;
    - etc.
  - May lead to:
    - power losses;
    - mechanical wear;
    - noise;
    - tracking errors;
    - other undesirable effects.
- Some remedies: (Utkin, Guldner & Shi 2009).

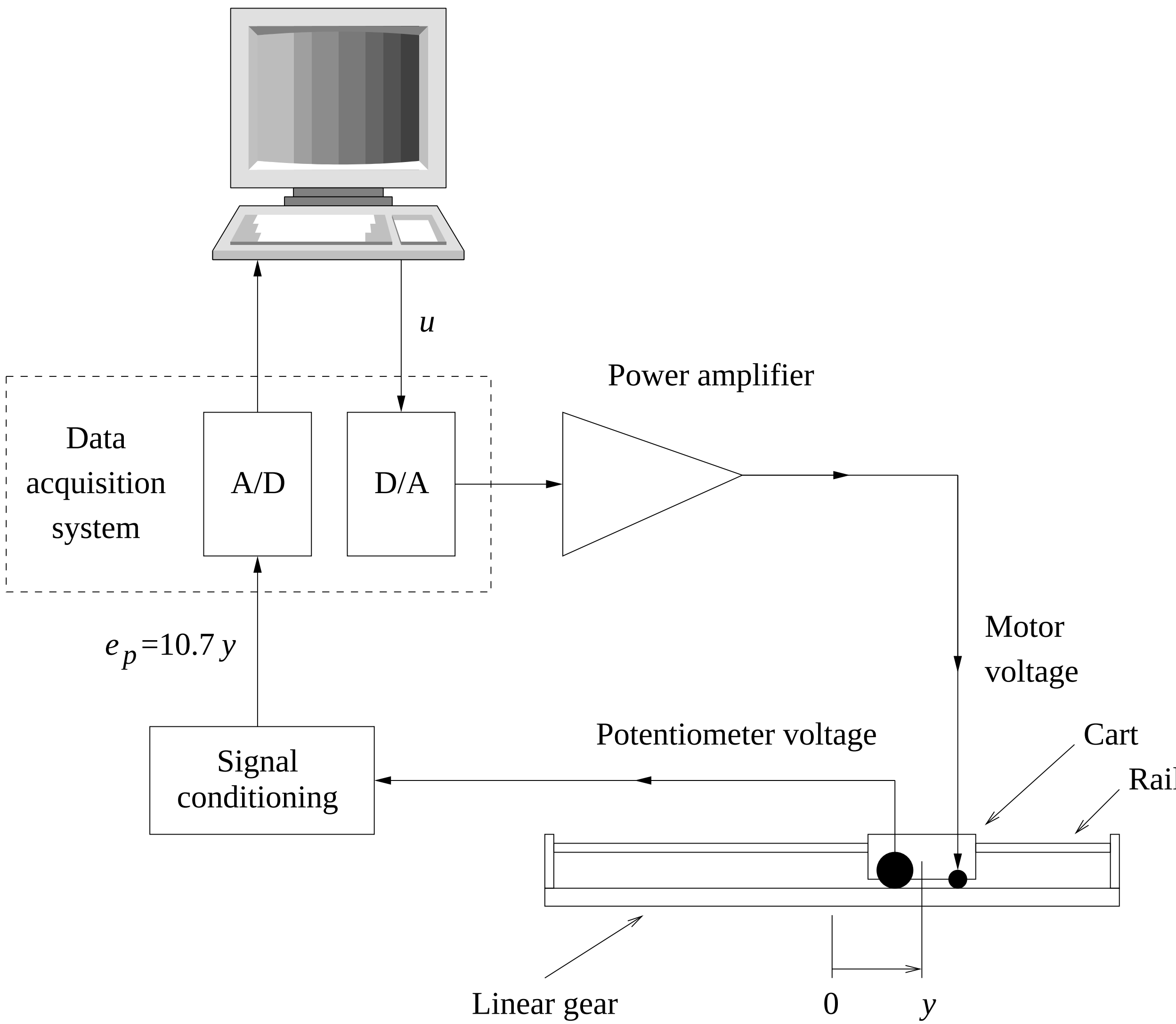
SMC Based on Observer

- State observer to avoid *chattering* in VSC (Bondarev, Bondarev, Kostyleva & Utkin 1985), (Utkin et al. 2009).
- Output-feedback SMC:
  - *Variable structure model-reference adaptive control* (VS-MRAC) (Hsu, Araújo & Costa 1994);
  - High-gain observer (HGO) robust to uncertainties designed for output-feedback VSC (Oh & Khalil 1997), (Cunha, Costa, Lizarralde & Hsu 2009);
  - Exact differentiators (Shtessel, Edwards, Fridman & Levant 2014, Hsu, Nunes, Oliveira, Peixoto, Cunha, Costa & Lizarralde 2011), ...

SMC Based on High-Gain Observer



SMC Based on High-Gain Observer



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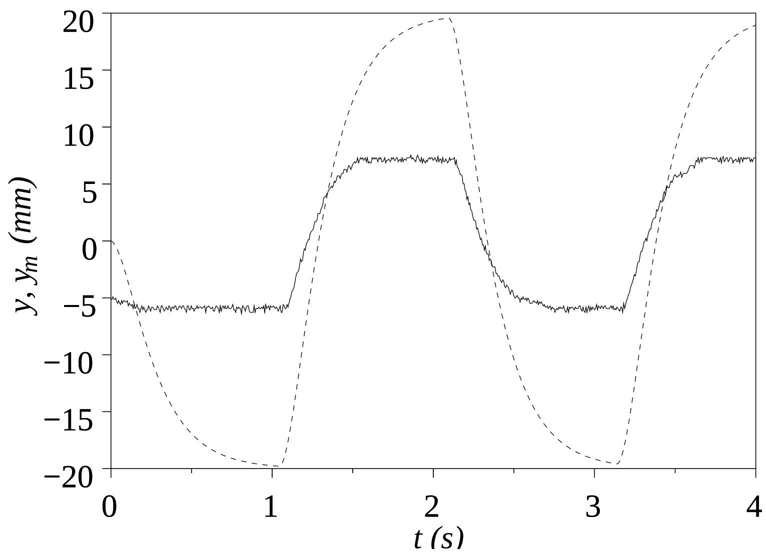
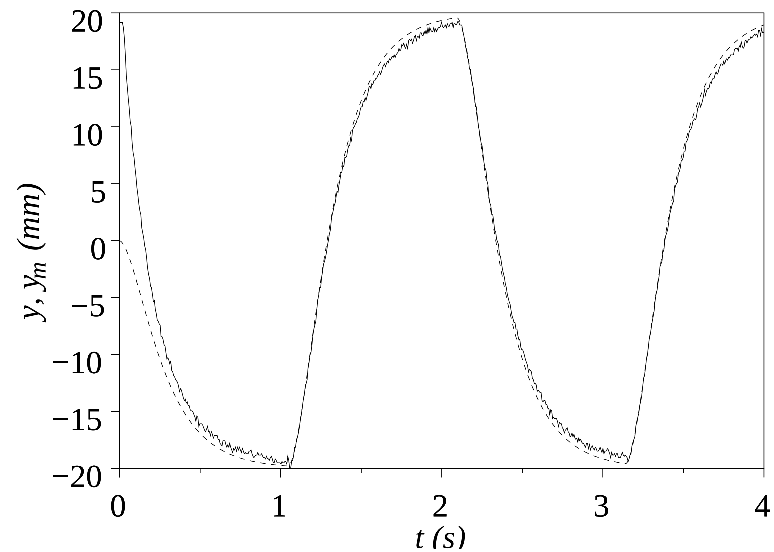
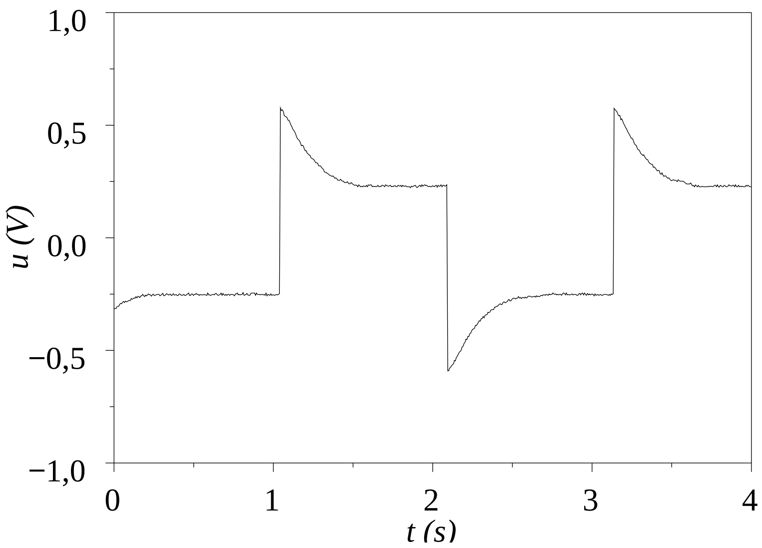


Linear control

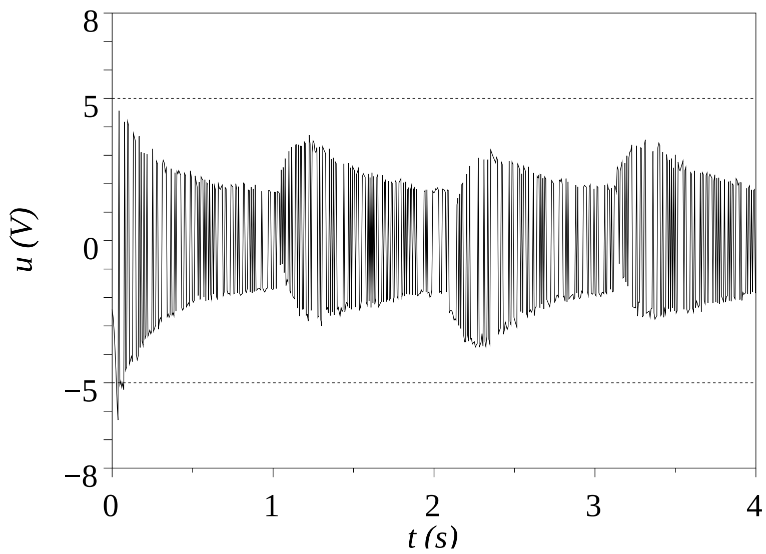


HGO + VSC

SMC Based on High-Gain Observer



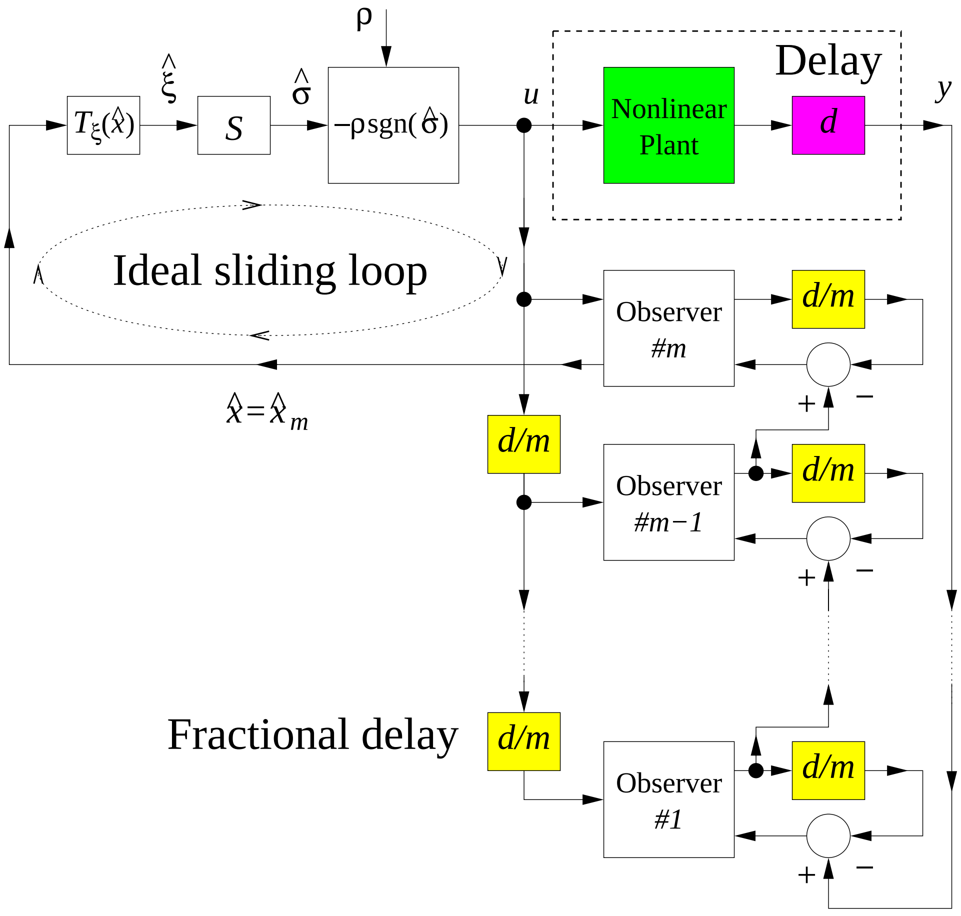
Linear control



HGO + VSC

SMC for Time-Delay Systems

- Cascade observers + VSC (Coutinho, Oliveira & Cunha 2014):



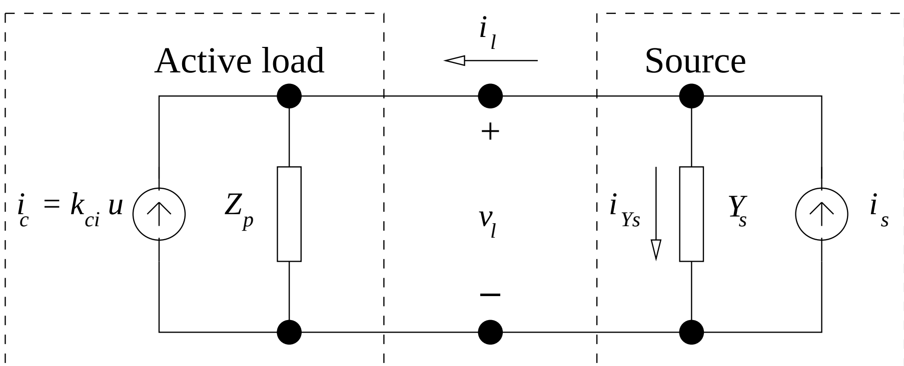
Applications

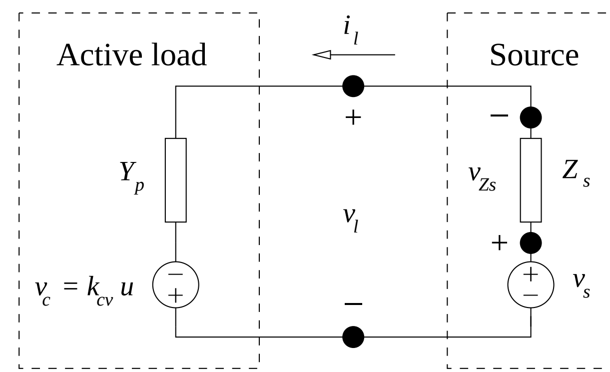
1. Control of electrical impedance/admittance:
   - Example: admittance control.
2. Marine control systems:
   - Experiments: unmanned surface vehicle (USV) control.
3. Fault tolerant control (FTC):
   - Example: trailer chain.

Impedance/Admittance Control

- Impedance/admittance control of an active load (Cunha & Costa 2016);
- Model-reference control approach;
- Model reference with unstable poles & nonminimum phase zeros is allowed: unlike usual MRAC!

Impedance/Admittance Control





Impedance control:

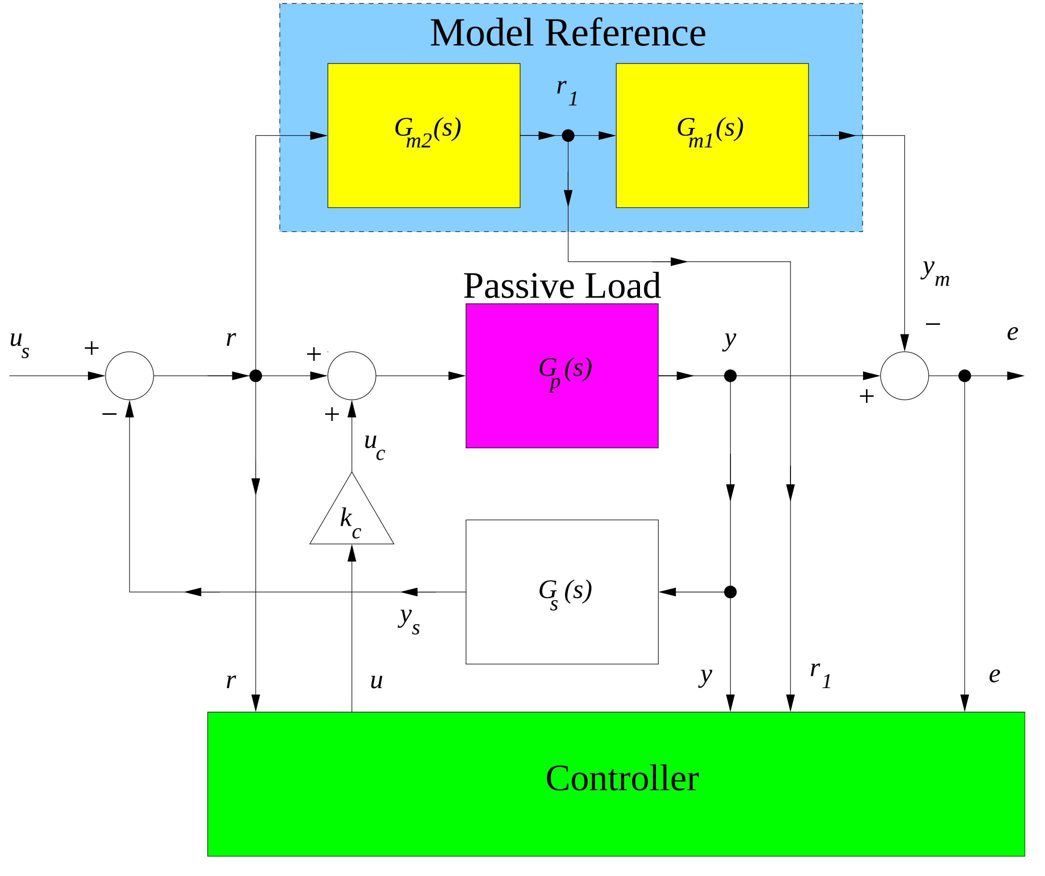
$$Z_l(s) = \frac{v_l(s)}{i_l(s)}$$

Admittance control:

$$Y_l(s) = \frac{i_l(s)}{v_l(s)}$$

Impedance/Admittance Control

- Model reference: $G_m(s) = G_{m1}(s)G_{m2}(s)$.



Impedance/Admittance Control

- Model-reference adaptive control (MRAC):

$$u(t) = \theta^T(t)\omega(t)\,,$$

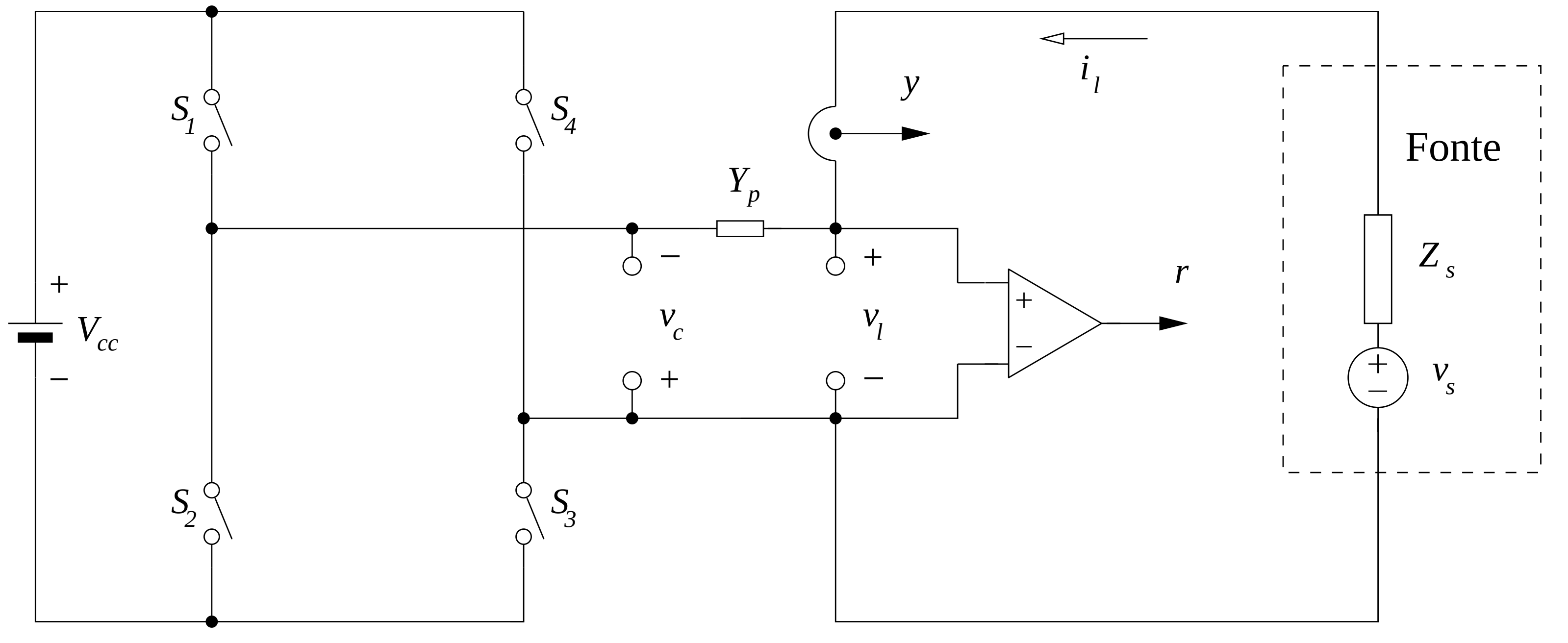
$$\dot{\theta}(t) = \Gamma e(t)\omega(t)\,.$$

- Variable structure model-reference adaptive control (VS-MRAC):

$$u = -\rho(t)\,\mathrm{sgn}(e)\,.$$

Impedance/Admittance Control

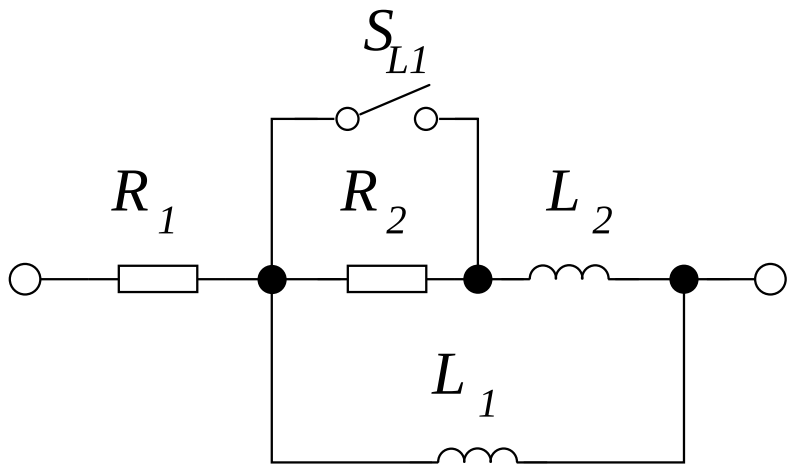
- H-bridge realization of the active load:



- MRAC: pulse-width modulated (PWM) control signal;
- VS-MRAC: drives the power switches directly.

Example: Admittance Control

- Passive load:



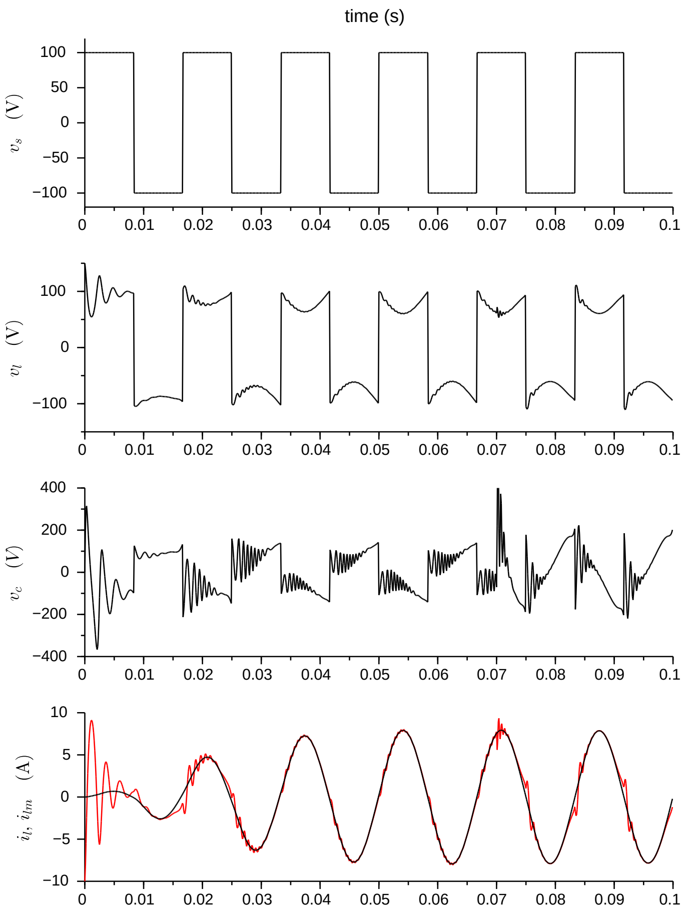
- Reference model:

$$G_{m2}(s) = k_m \frac{s^2(s + 2\pi f_c)}{\left[s^2 + 2\zeta(2\pi f_r)\, s + (2\pi f_r)^2\right]^2}$$

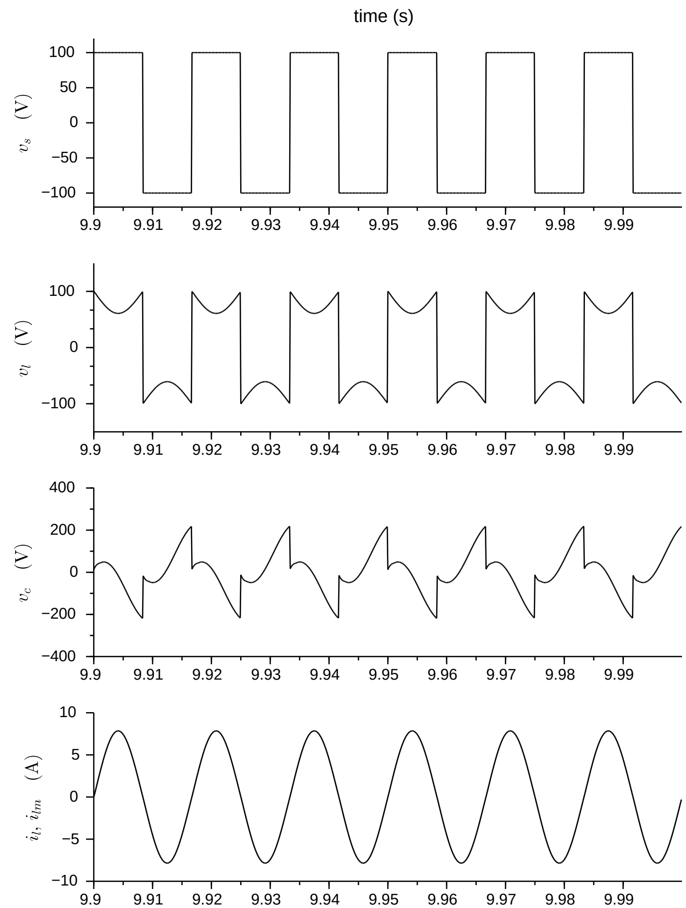
$$G_{m1}(s) = \frac{1}{s + 2\pi f_c}$$

  - $\zeta = 0.2\,, \qquad f_r = 60\,\mathrm{Hz}\,, \qquad k_m = 2\,\mathrm{kS\,rad^2 s^2}\,,$
  - $f_c = 300\,\mathrm{Hz}\,.$

Example: Admittance Control

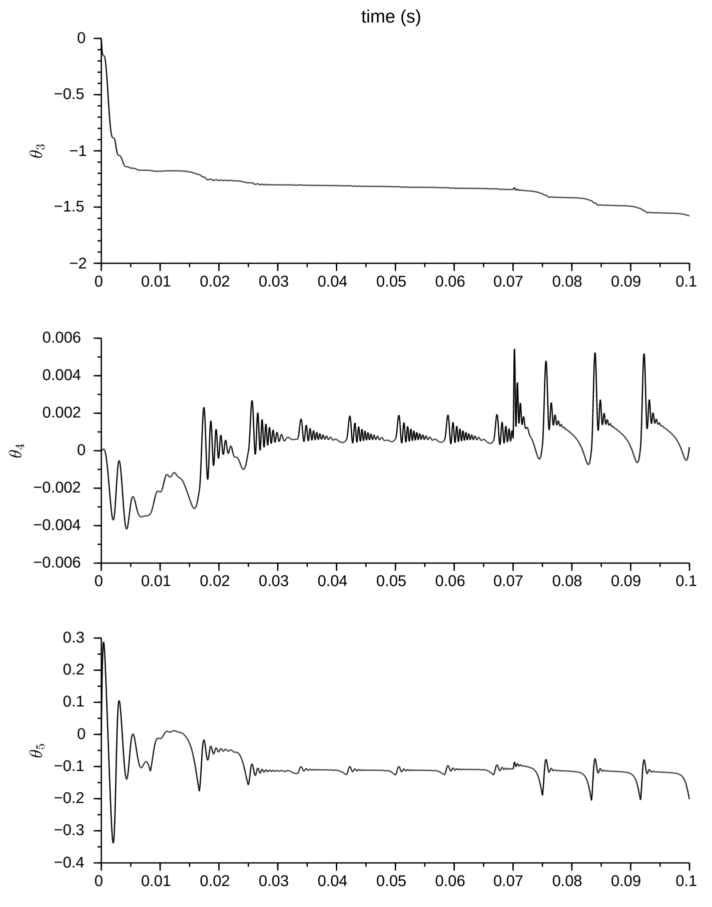


MRAC: transient



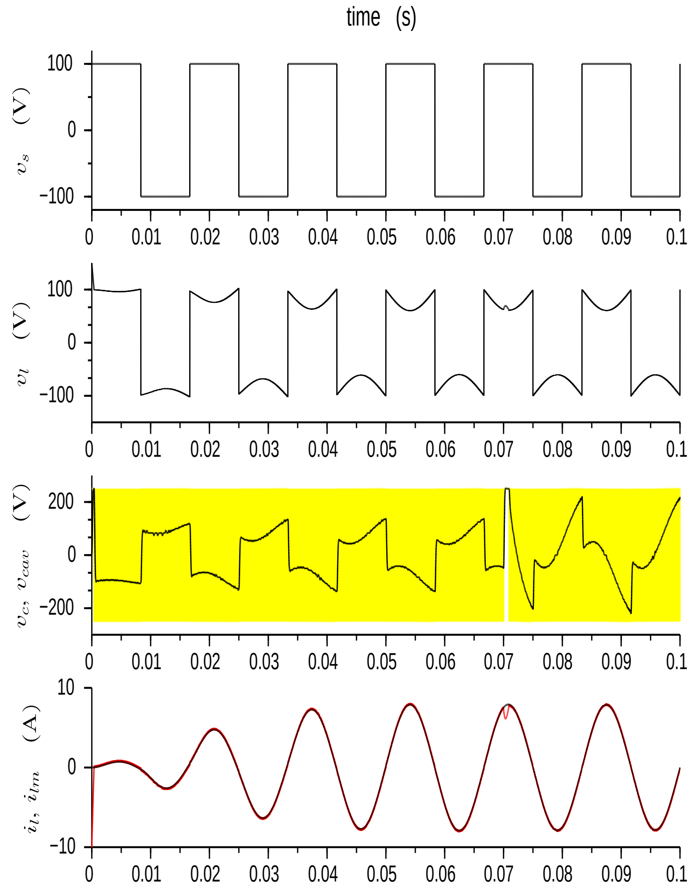
MRAC: steady-state

Example: Admittance Control



MRAC: parameters

Example: Admittance Control



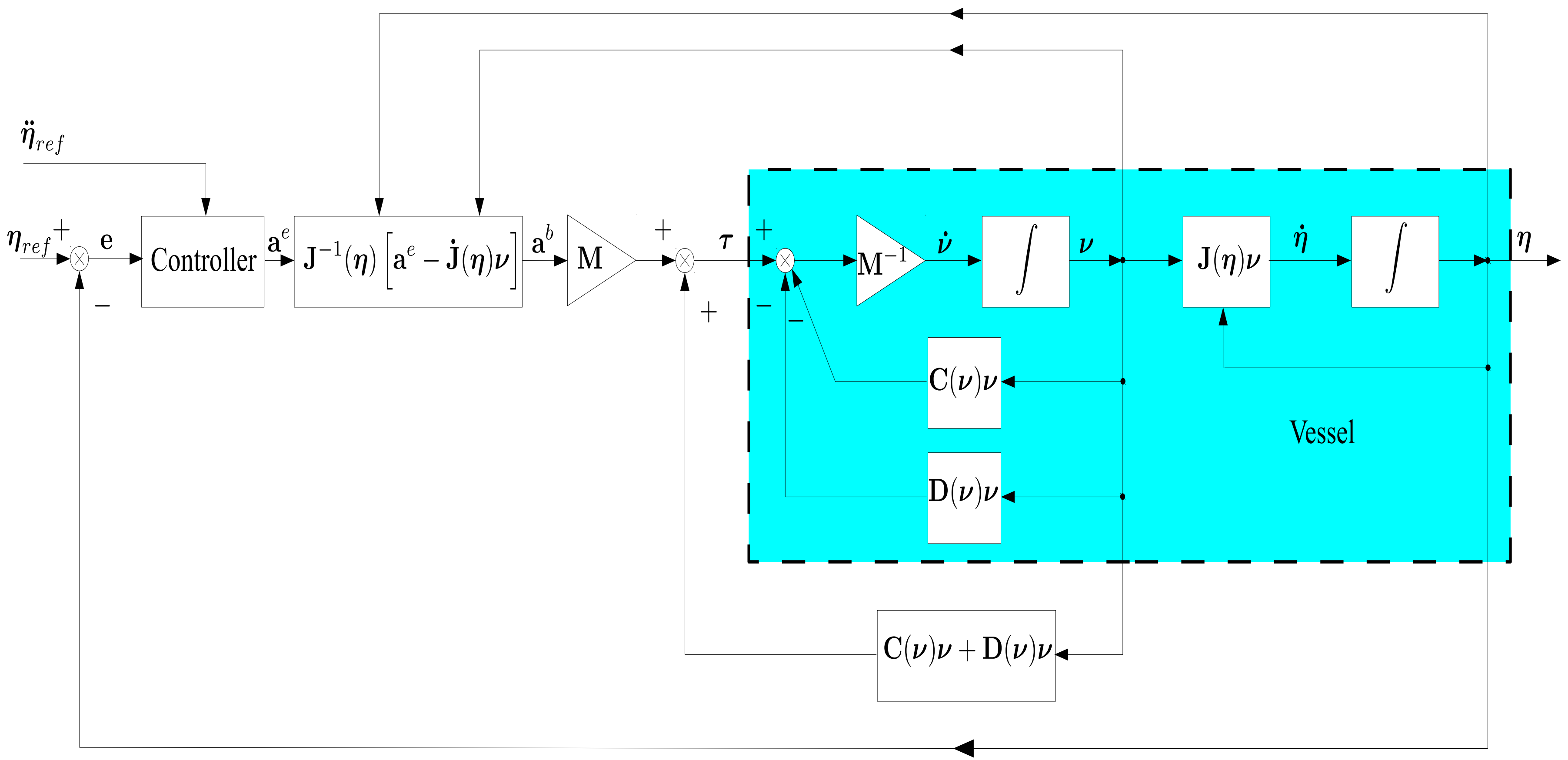
VS-MRAC signals

Marine Control Systems

- Characteristics of marine systems:
  - Hydrodynamics is unknown or uncertain;
  - Large parametric changes (e.g.: load mass);
  - Environmental disturbances: currents, waves and winds.
- SMC is advantageous!
- Applications:
  - State-feedback VSC for remotely operated underwater vehicles (ROVs), (Yoerger, Newman & Slotine 1986);
  - Output-feedback VS-MRAC for ROVs (Cunha, Costa & Hsu 1995);
  - VSC trajectory tracking of unmanned surface vessels (USVs), (Cheng, Yi & Zao 2007), (Rosario & Cunha 2017).

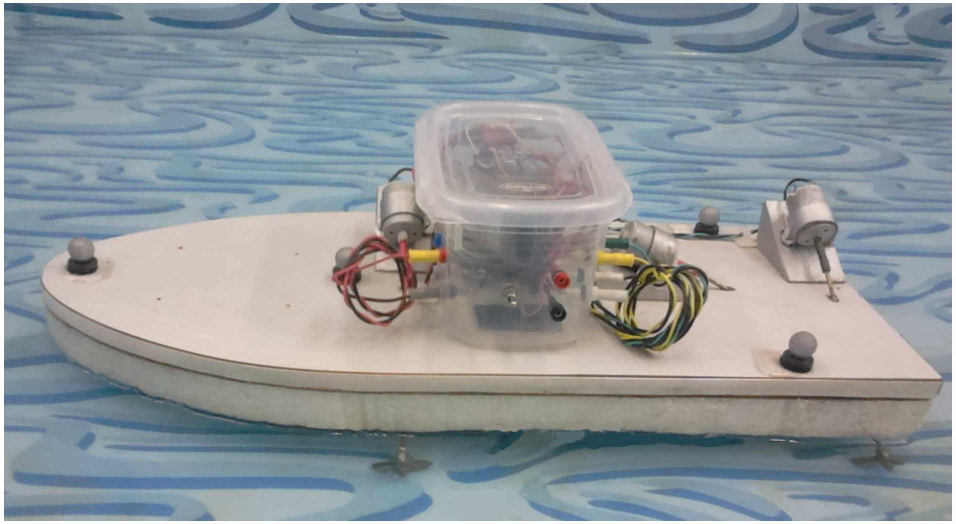
Control of an USV

- Feedback linearization:



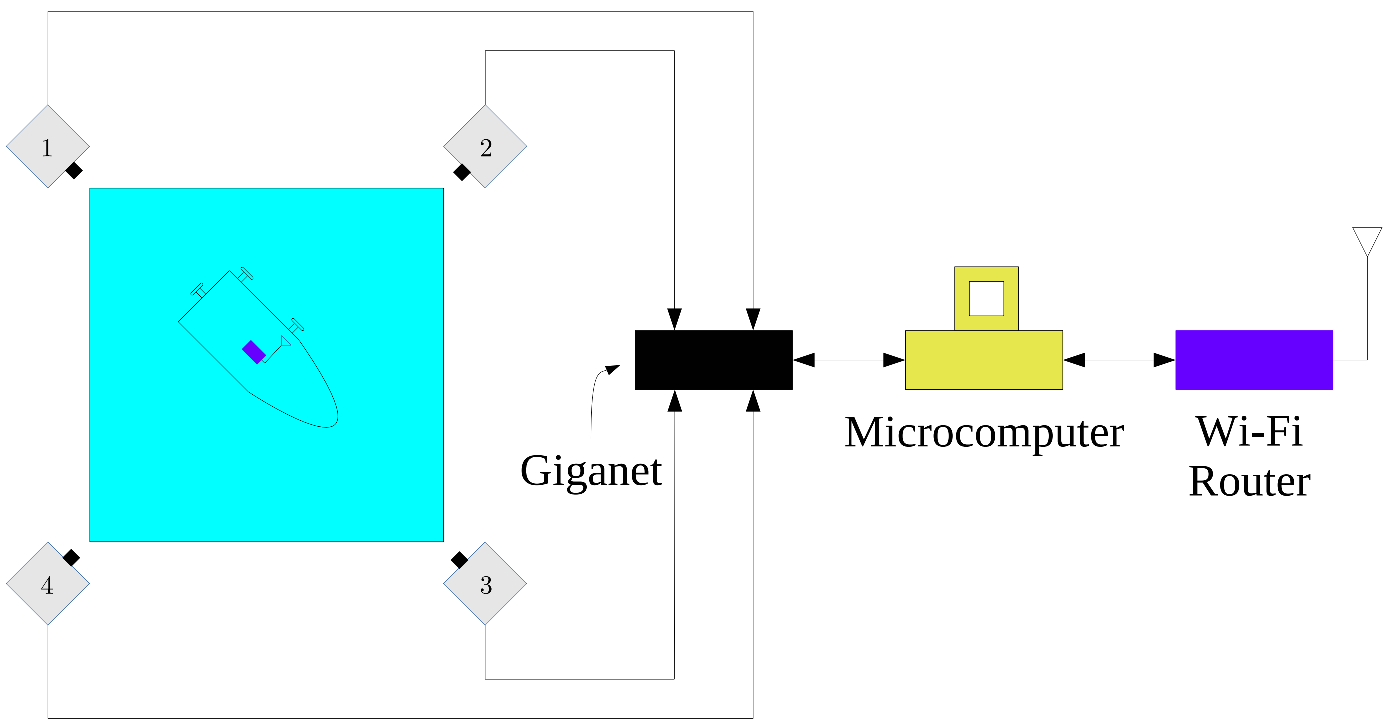
Control of an USV

- Small boat:
  - Maximum forward speed: 0.26 m/s ;
  - Length: 0.48 m ;
  - Mass: 1.4 kg ;
  - 3 thrusters with DC motors;
  - Commanded by an Arduino + Wi-Fi.



Control of an USV

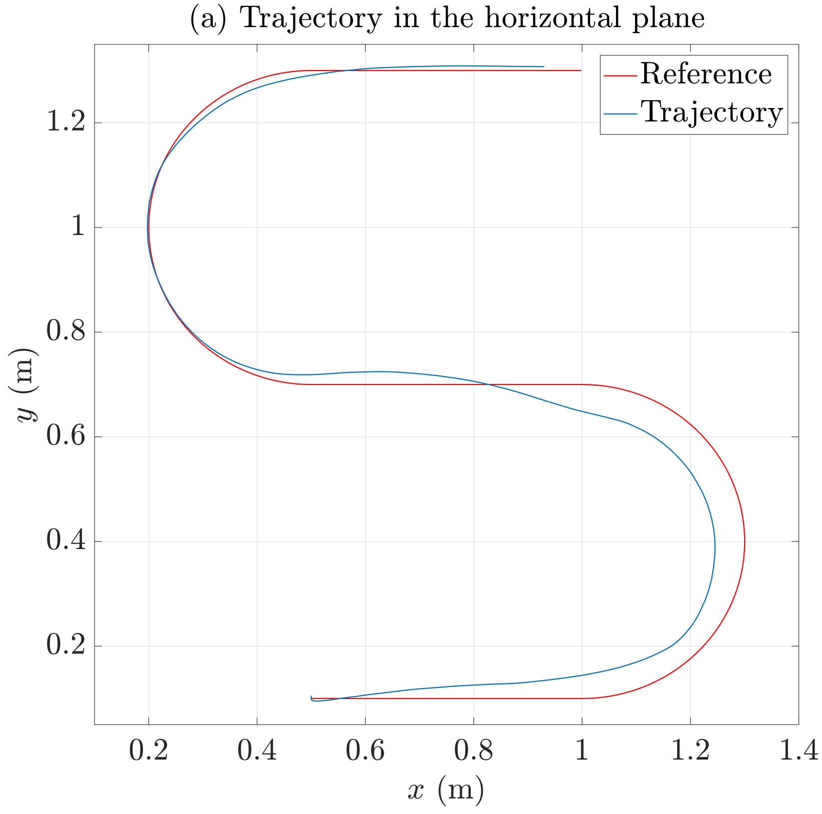
- Position and attitude measurement: 4 Vicon MX cameras;
- Sampling frequency up to 1 kHz;
- Accuracy better than 1 mm.

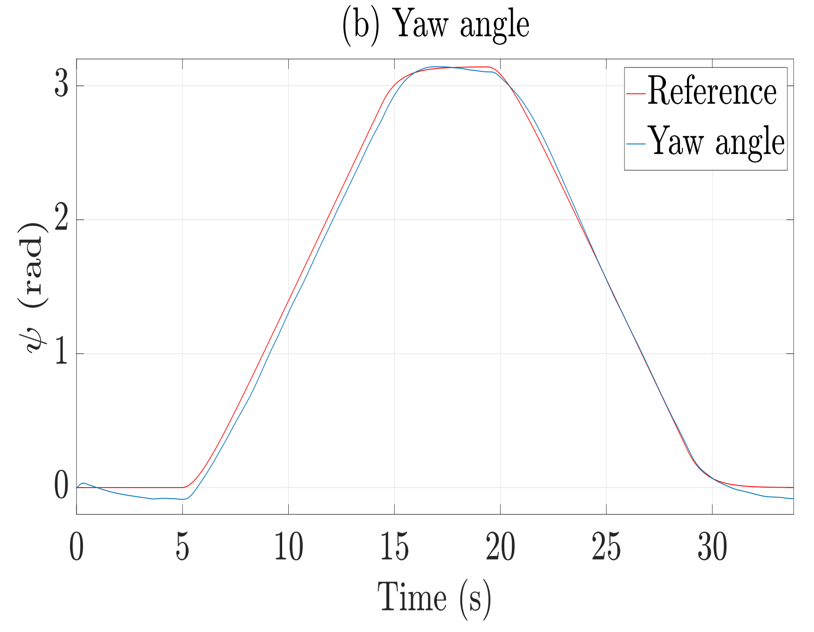


Control of an USV

- Zigzag trajectory simulate a scan;
- Velocity: 0.1 m/s;
- Tests:
  1. PD at $f_s = 30\,$Hz;
  2. VSC at $f_s = 30\,$Hz;
  3. VSC at $f_s = 150\,$Hz.

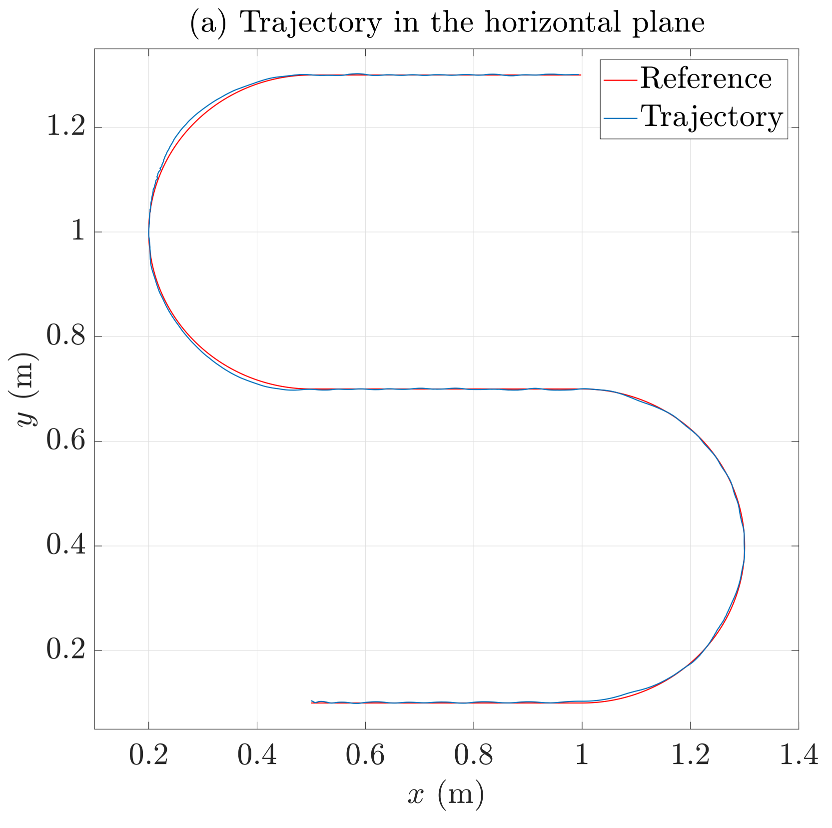
Control of an USV

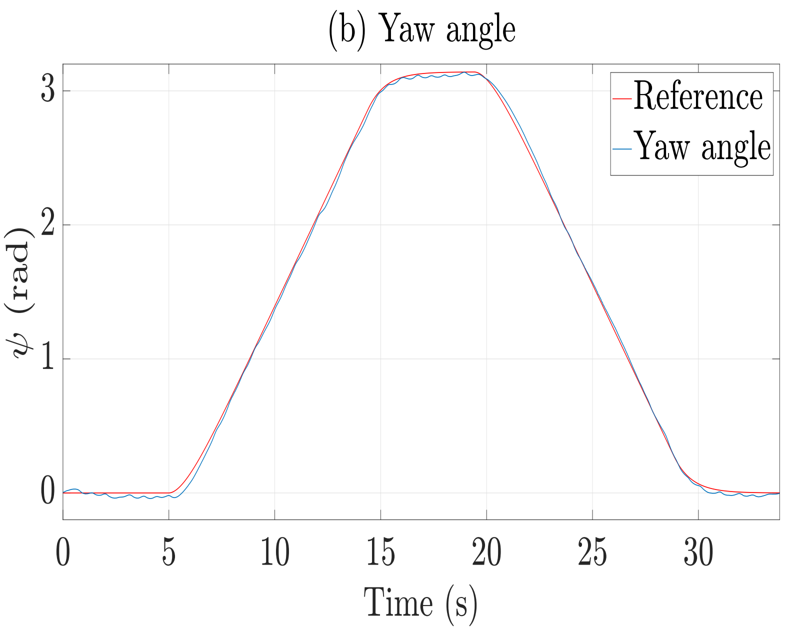




PD at $f_s = 30\,\text{Hz}$.

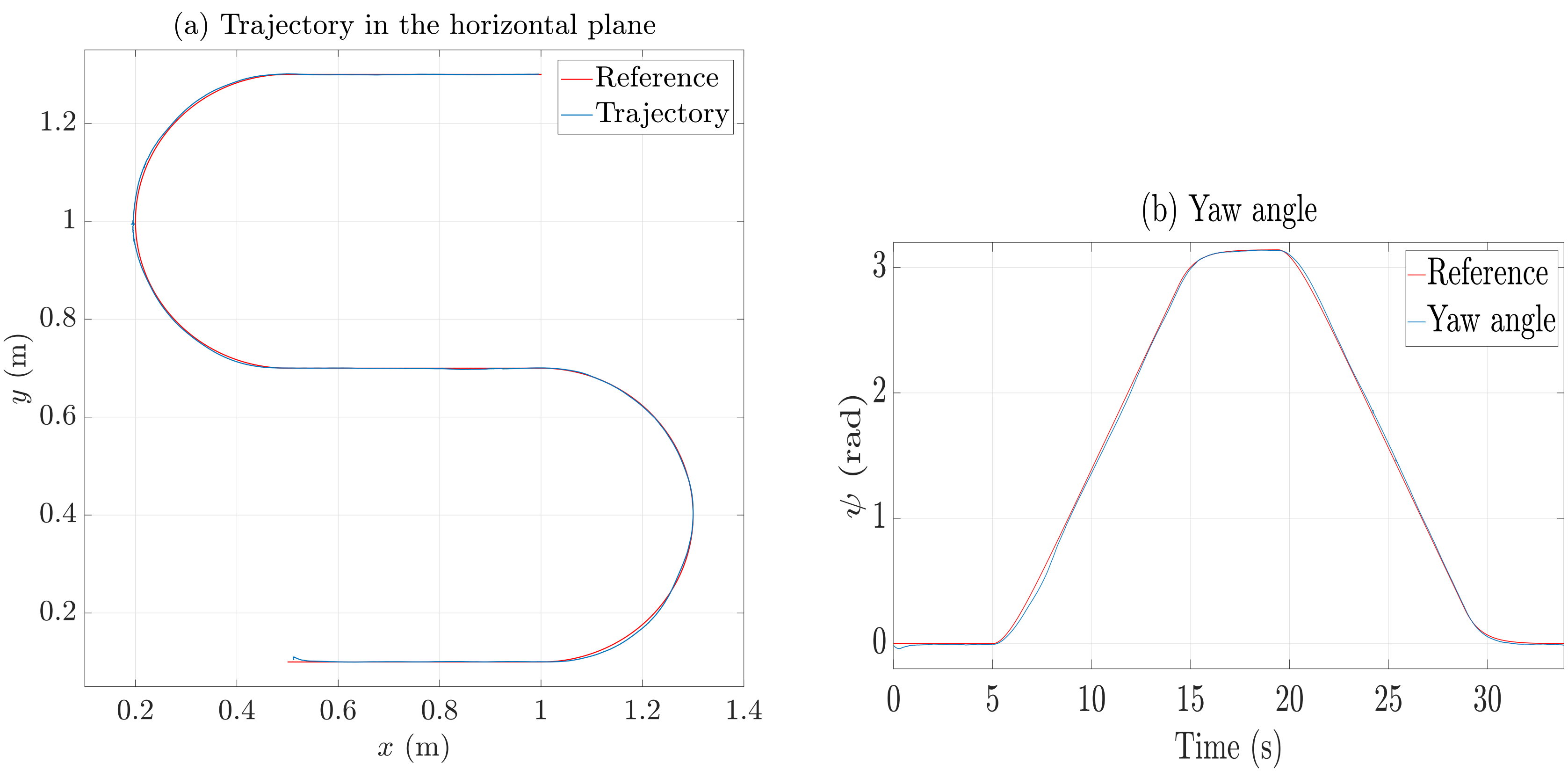
Control of an USV





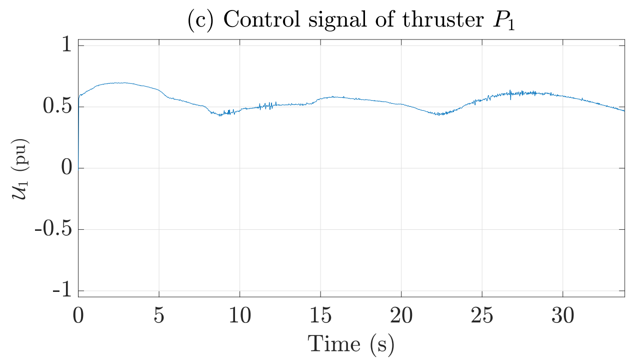
VSC at $f_s = 30\,\text{Hz}$.

Control of an USV

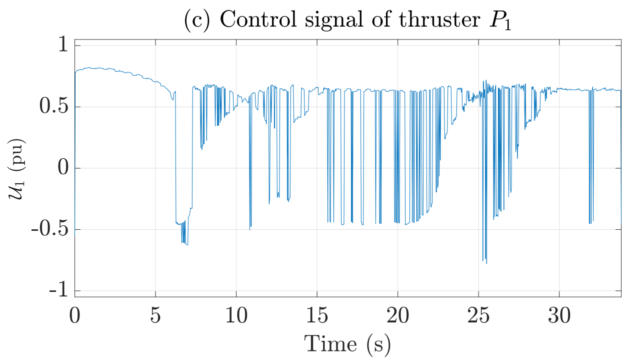


VSC at $f_s = 150\,\text{Hz}$.

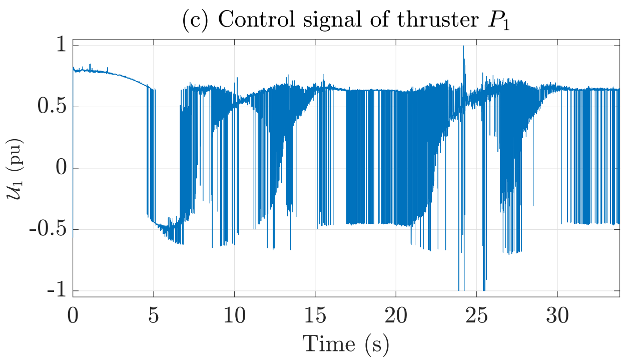
Control of an USV



PD at $f_s = 30\,$Hz;



VSC at $f_s = 30\,$Hz;



VSC at $f_s = 150\,$Hz.

Fault Tolerant Control

- Consider the system in *regular form*:

$$\begin{aligned}\dot{x}_1 &= A_{11}x_1 + A_{12}y_p\,,\\ \dot{y}_p &= A_{21}x_1 + A_{22}y_p + K_p(t)\left[u_p + d_f(x_1, y_p, t)\right]\,,\end{aligned}$$

- Output (measured) : $y_p \in \mathbb{R}^m\,,$
- Other state variables (not measured) : $x_1 \in \mathbb{R}^{n-m}\,.$

Fault Tolerant Control

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- Output (measured) : $y_p \in \mathbb{R}^m$,
- Other state variables (not measured) : $x_1 \in \mathbb{R}^{n-m}$.
- Actuator faults modeled by (Cunha, Costa, Hsu & Oliveira 2015):
  - Time-variant control distribution matrix $K_p(t)$ and/or
  - Input disturbance $d_f(x_1, y_p, t)$.

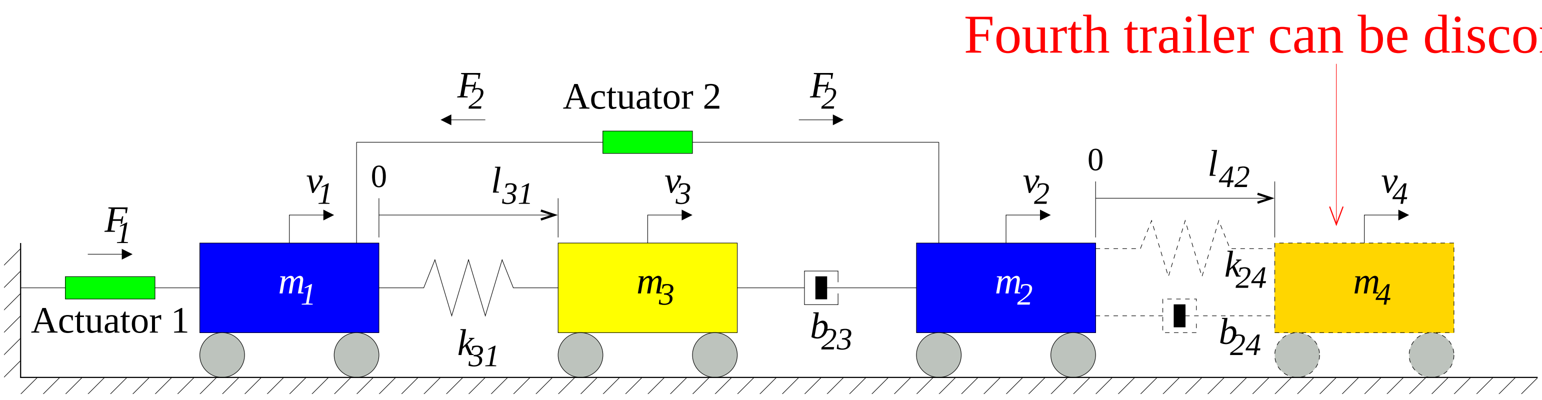
Fault Tolerant Control

- Unit vector control (UVC) stabilizes the system if (Cunha et al. 2015):
  - Matrix A_{11} is Hurwitz (minimum phase assumption);
  - Exist $P = P^T > 0$ and $Q = Q^T > 0$ such that

$$K_p^T(t)P + PK_p(t) - Q \geq 0\,, \quad \forall t \geq 0\,.$$

Fault Tolerant Control

- Example: Trailer Chain



$$y_p = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad u_p = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Fault Tolerant Control

- Without fourth trailer: $x_1 = \begin{bmatrix} l_{31} \\ v_3 \end{bmatrix}$
- Fourth trailer connected: $x_1 = \begin{bmatrix} l_{31} \\ v_3 \\ l_{42} \\ v_4 \end{bmatrix}$
- Uncertain parameters: $$\begin{aligned} 0.5 \le m_3 \le 1.5 \quad & (\mathrm{kg}) \\ 2.5 \le b_{23} \le 6 \quad & (\mathrm{Ns/m}) \\ 25 \le k_{31} \le 35 \quad & (\mathrm{N/m}) \end{aligned}$$

Fault Tolerant Control

- Model Matching :
  - Reference model: $\dot{y}_M(t) = -\gamma_M y_M(t) + r(t)\,.$
  - $r(t)$ is piecewise continuous & uniformly bounded.
  - Velocity error: $e(t) = y_p(t) - y_M(t) \to 0\,.$

Fault Tolerant Control

- Unit vector control: $u = u^{\mathrm{nom}} - \rho \dfrac{e}{||e||}\,.$

- Modulation function: $\rho = \delta + c_2 \|y_p\| + c_3 \|r\| + c_4\, \bar{x}_1(t)\,.$

- The signal $\bar{x}_1(t) \geq \|g_y(t) * y_p(t)\|\,,$

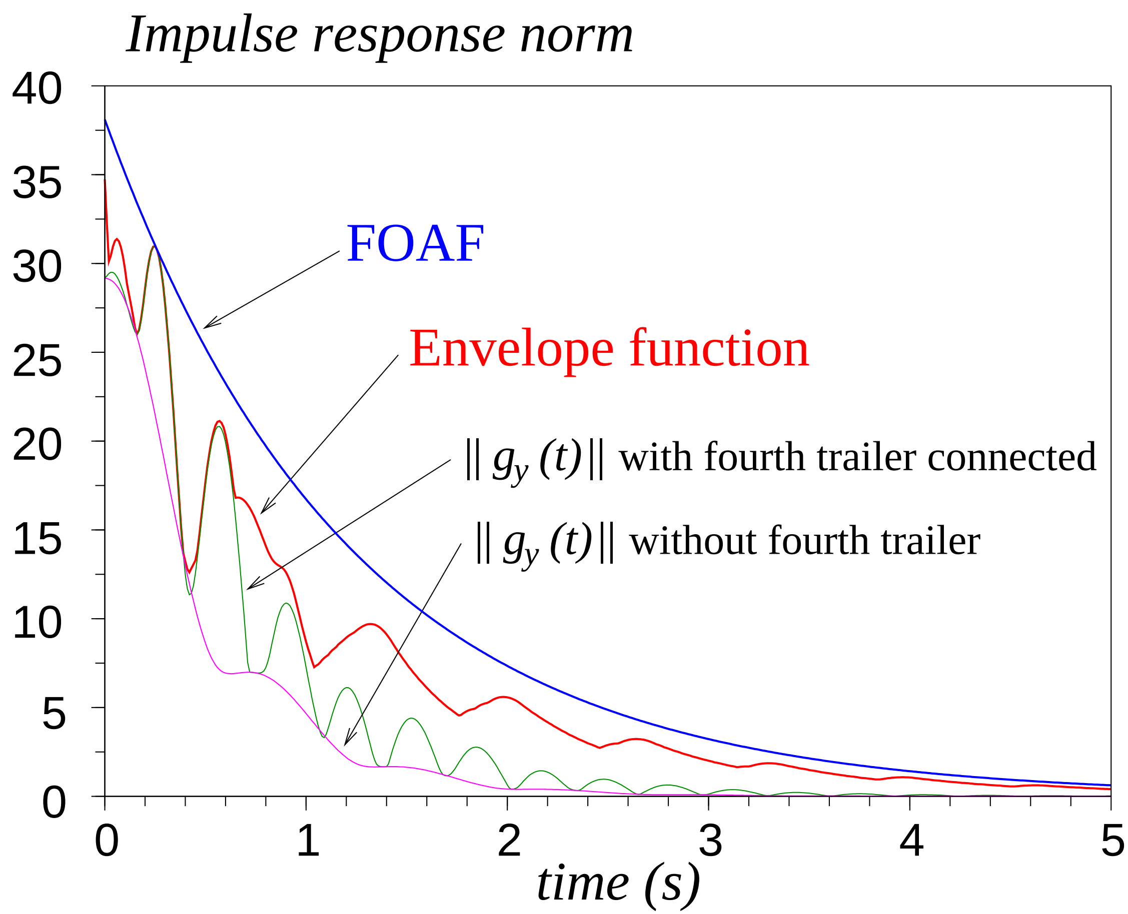
  with $g_y(t) = A_{21} \exp(A_{11} t) A_{12}\,,$

  is estimated by the first order approximation filter (FOAF) (Cunha, Costa & Hsu 2008):

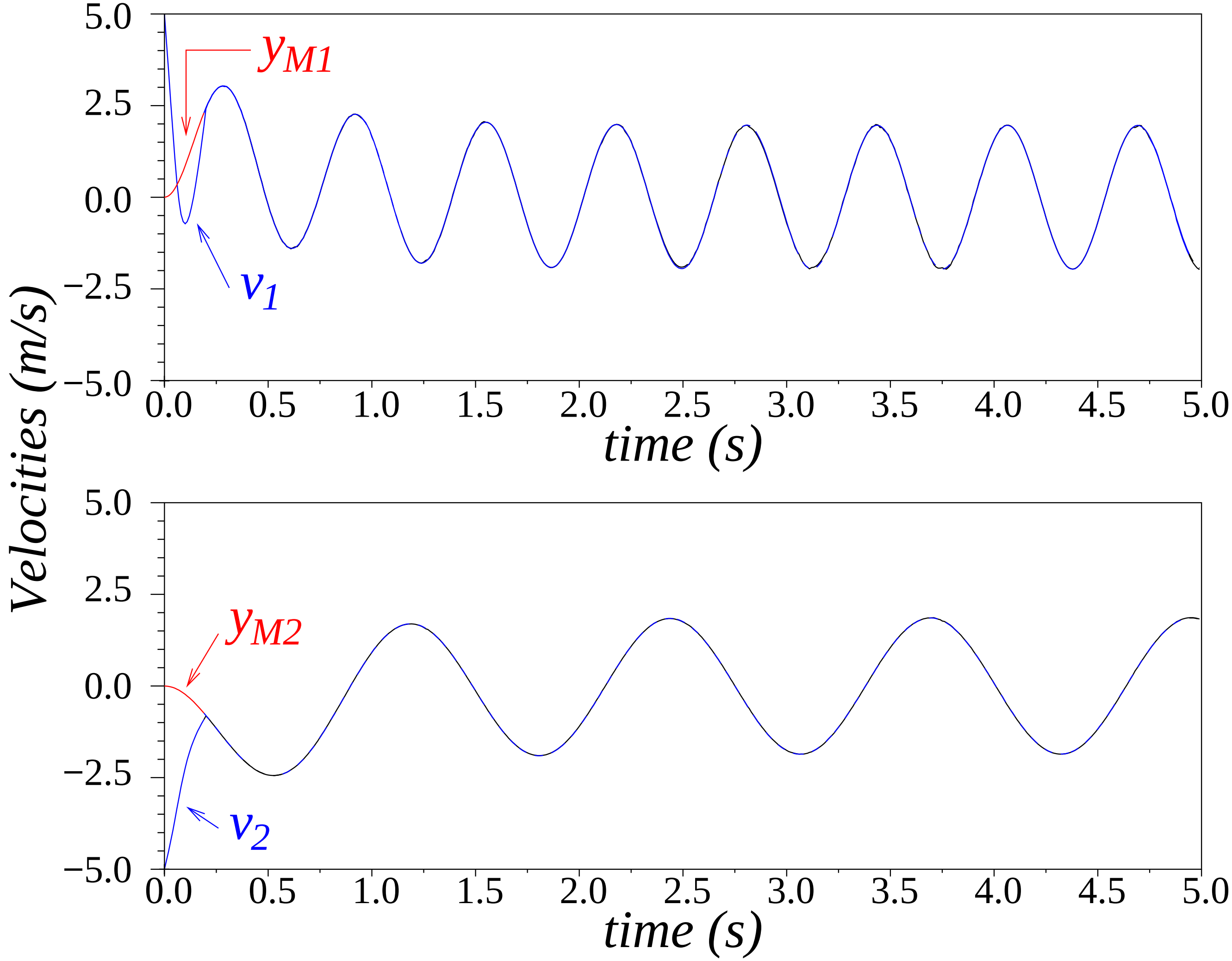
$$\dot{\bar{x}}_1(t) = -\gamma_1 \bar{x}_1(t) + c_5 \, \|y_p(t)\| \,.$$

Fault Tolerant Control

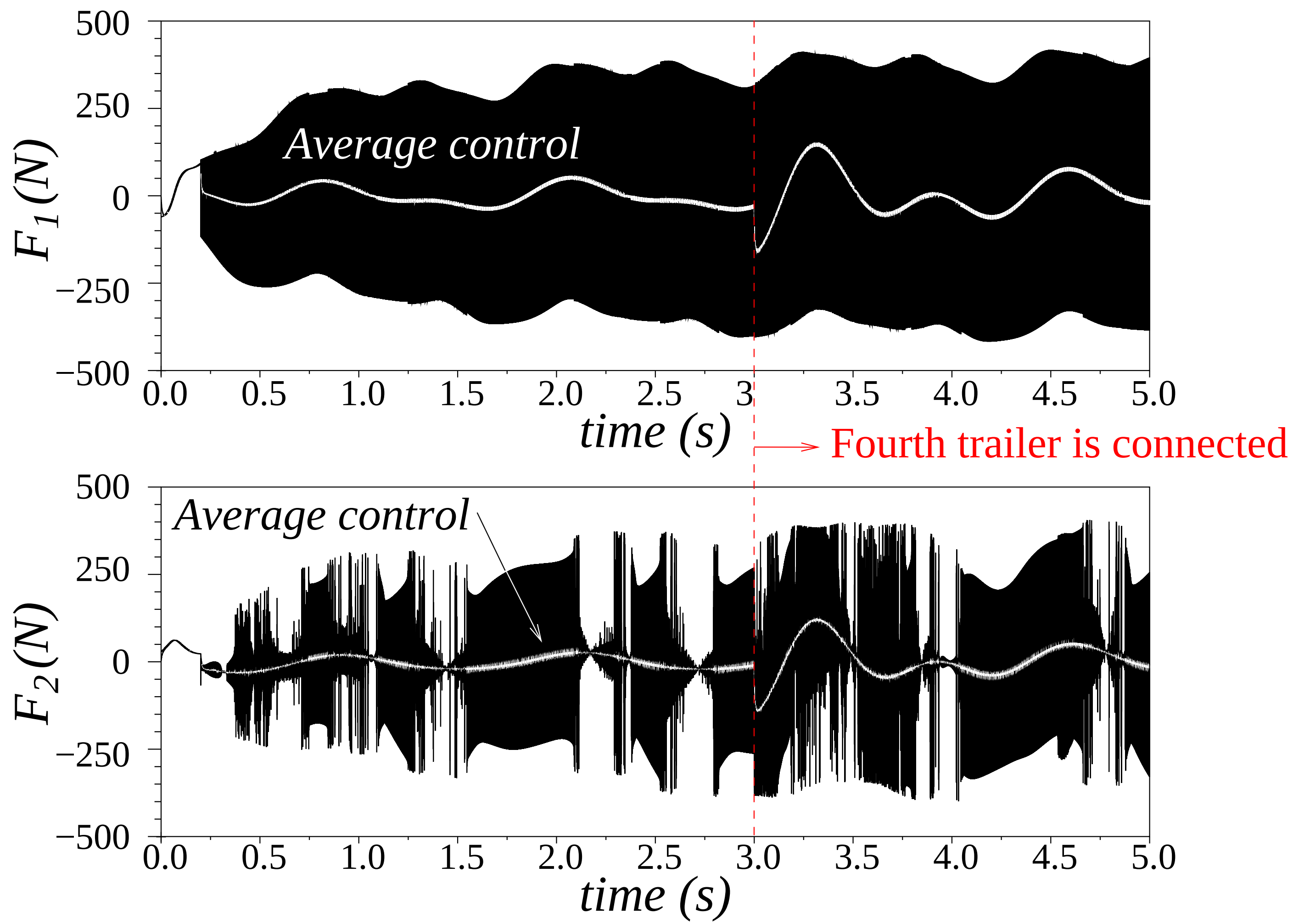
- FOAF:



Fault Tolerant Control



Fault Tolerant Control



Conclusion

- SMC:
  - Robust to parametric uncertainties;
  - Rejects disturbances;
  - Invariance property guarantees good performance.
- VSC:
  - Suitable for switched actuators such as solenoid valves and power electronics devices;
  - Simple realization.
- Disadvantages:
  - Requires high-frequency switching;
  - Chattering;
  - Sensitive to measurement noise.

Current Works

- Control of a vessel pushing an underactuated floating load;
- Cooperative control of USVs;
- SMC of partial differential equations (PDEs) (Molina & Cunha 2017);
- Adaptive SMC:
  - Extended equivalent control approach (Oliveira, Cunha & Hsu 2016);
  - Monitoring function (Yan, Hsu & Xiuxia 2006) approach (Oliveira, Melo, Hsu & Cunha 2017);
- Uncertain time-delay systems.

Suggested Books

- Classic book: (Utkin 1978).
- More recent books: (Utkin 1992), (Edwards & Spurgeon 1998) and (Utkin et al. 2009).
- Book with up to date techniques: (Shtessel et al. 2014).

Contact

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