



Programa de Pós-Graduação em Engenharia Eletrônica - UERJ

# Disciplina: Controle por Computador



Aula: **Parte I: Sistemas Lineares, Resposta Impulsiva, Convolução e Propriedades**

Professor: José Paulo Vilela Soares da Cunha

Turma 01 – 2021/2

Rio de Janeiro, 24 de agosto de 2021.



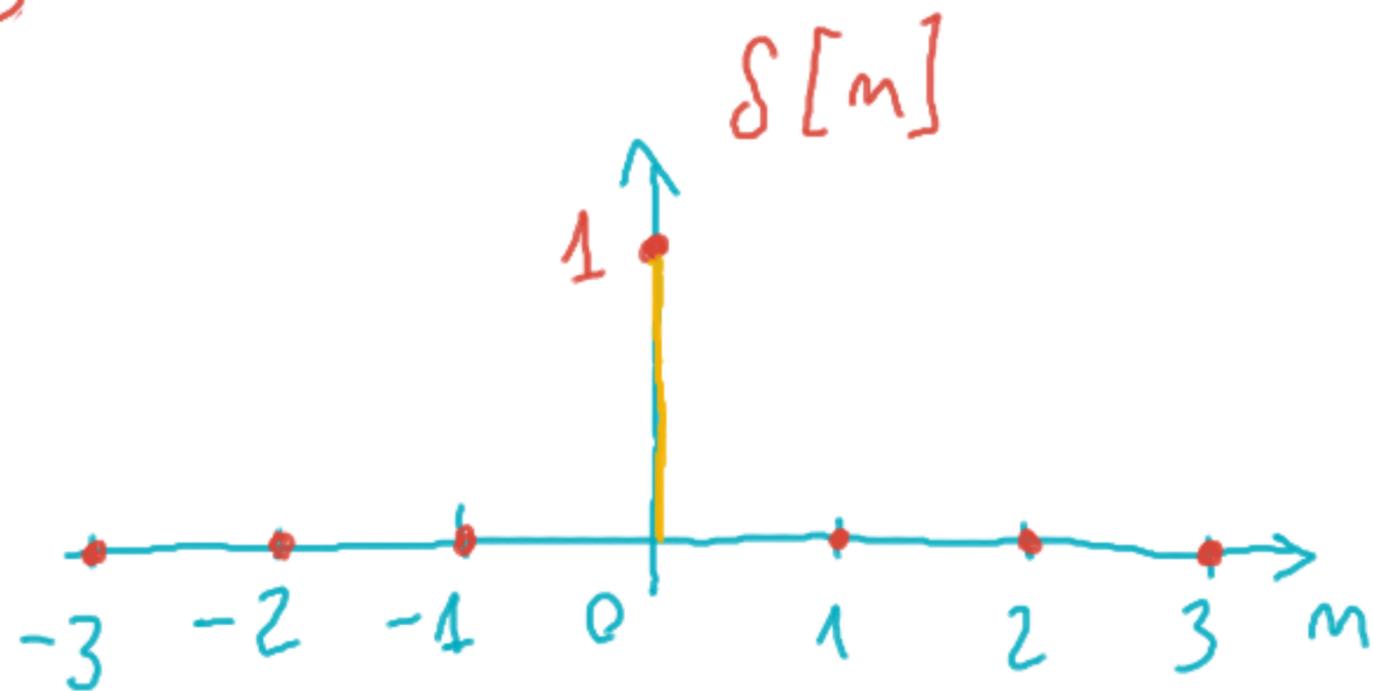
## Referências

- Oppenheim, A. V., Schafer, R. W. & Buck, J. R. (1999). Discrete-Time Signal Processing, 2<sup>nd</sup> ed., Prentice-Hall. (\*)
- Oppenheim, A. V. & Schafer, R. W. (2013). Processamento em Tempo Discreto de Sinais, 3<sup>a</sup> ed., Pearson Brasil.

(\*) Organizado para as Seções 2.3 e 2.4 da 2<sup>a</sup> Edição.

# IMPULSO UNITÁRIO

$$\delta[m] = \begin{cases} 1, & \text{se } m=0, \\ 0, & \text{se } m \neq 0. \end{cases}$$



SEU USO PARA REPRESENTAR SEQUÊNCIAS ARBITRÁRIAS:

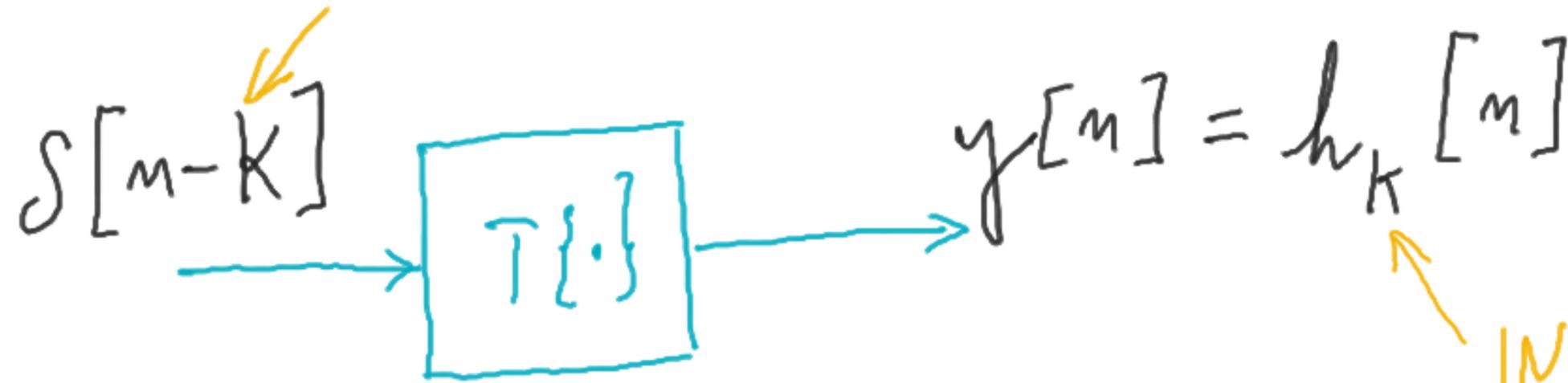
$$x[m] = \sum_{k=-\infty}^{+\infty} x[k] \delta[m-k]$$

# SISTEMA EM TEMPO DISCRETO



$$y[n] = T\{x[n]\}$$

# RESPOSTA IMPULSIVA



$$h_k[n] := T\{s[n-k]\}$$

INSTANTE DE  
APLICAÇÃO DO  
IMPULSO

SE  $T\{\cdot\}$  FOR LINEAR:

$$\begin{aligned} y[n] &= T\{x[n]\} = T\left\{\sum_{k=-\infty}^{+\infty} x[k] s[n-k]\right\} = \\ &= \sum_{k=-\infty}^{+\infty} x[k] T\{s[n-k]\} = \sum_{k=-\infty}^{+\infty} x[k] h_k[n] \end{aligned}$$

SE  $T\{\cdot\}$  FOR LINEAR E INVARIANTE NO TEMPO (LTI):

$$h_k[n] = h_0[n-k] := h[n-k]$$

ENTÃO:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] h[n-k] = x[n] * h[n]$$

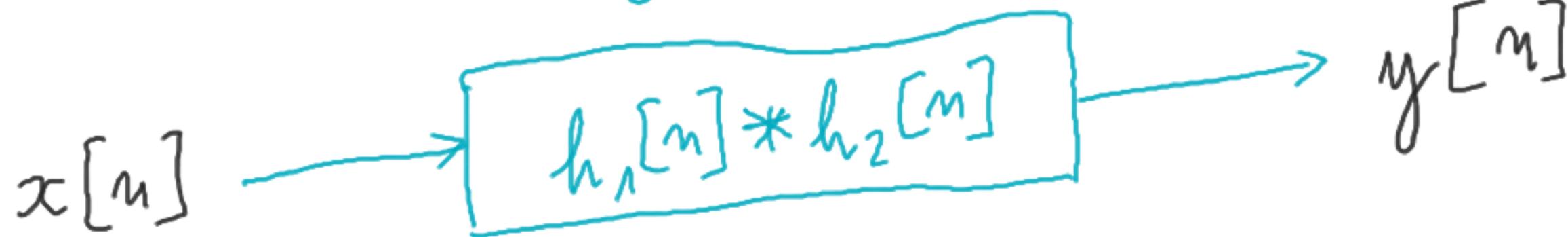
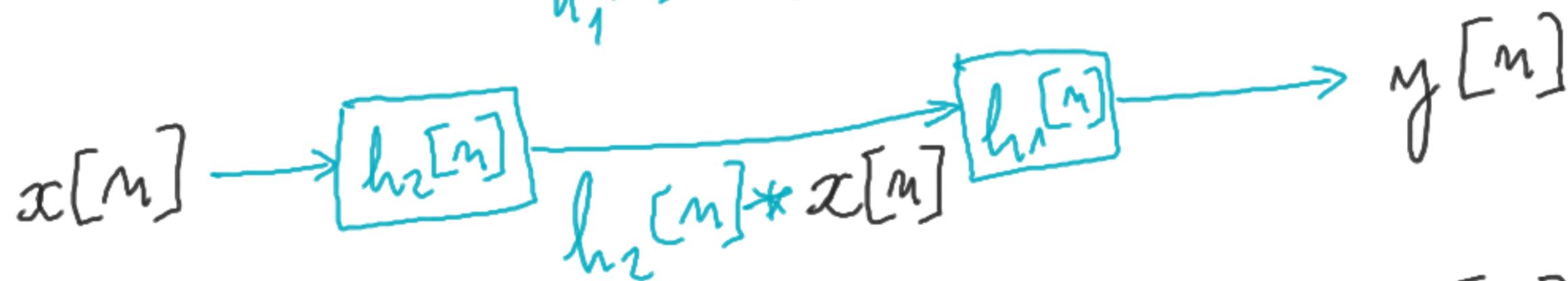
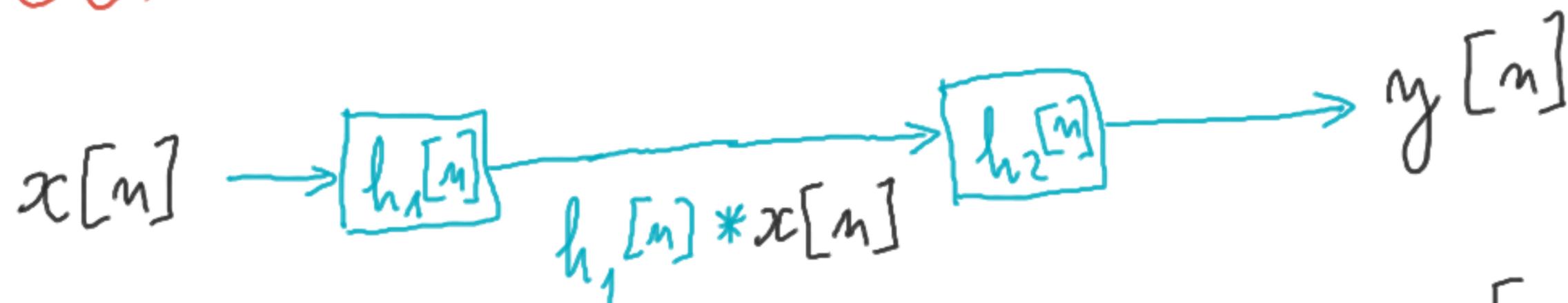
↑  
CONVOLUÇÃO

DEFINIÇÃO:  $h[n] := T\{s[n]\}$

↑  
IMPULSO APLICADO EM  $n=0$

# PROPRIEDADES DE SISTEMAS LTI

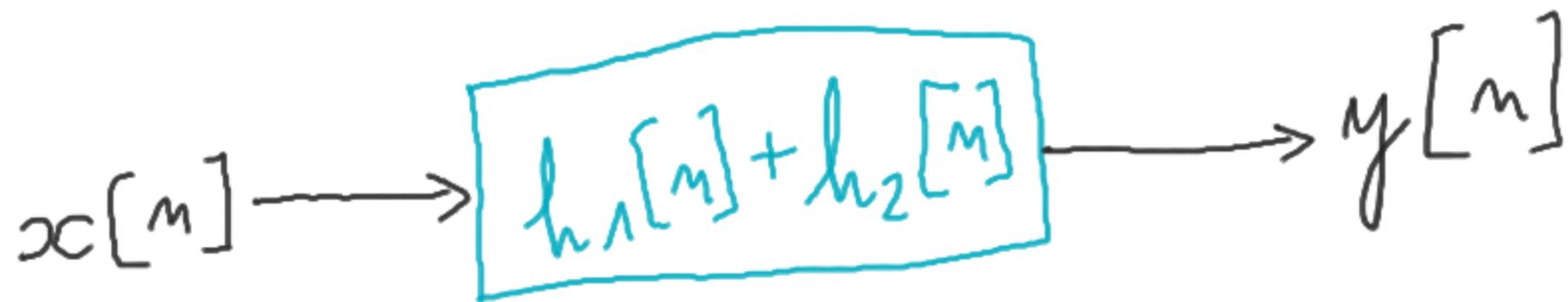
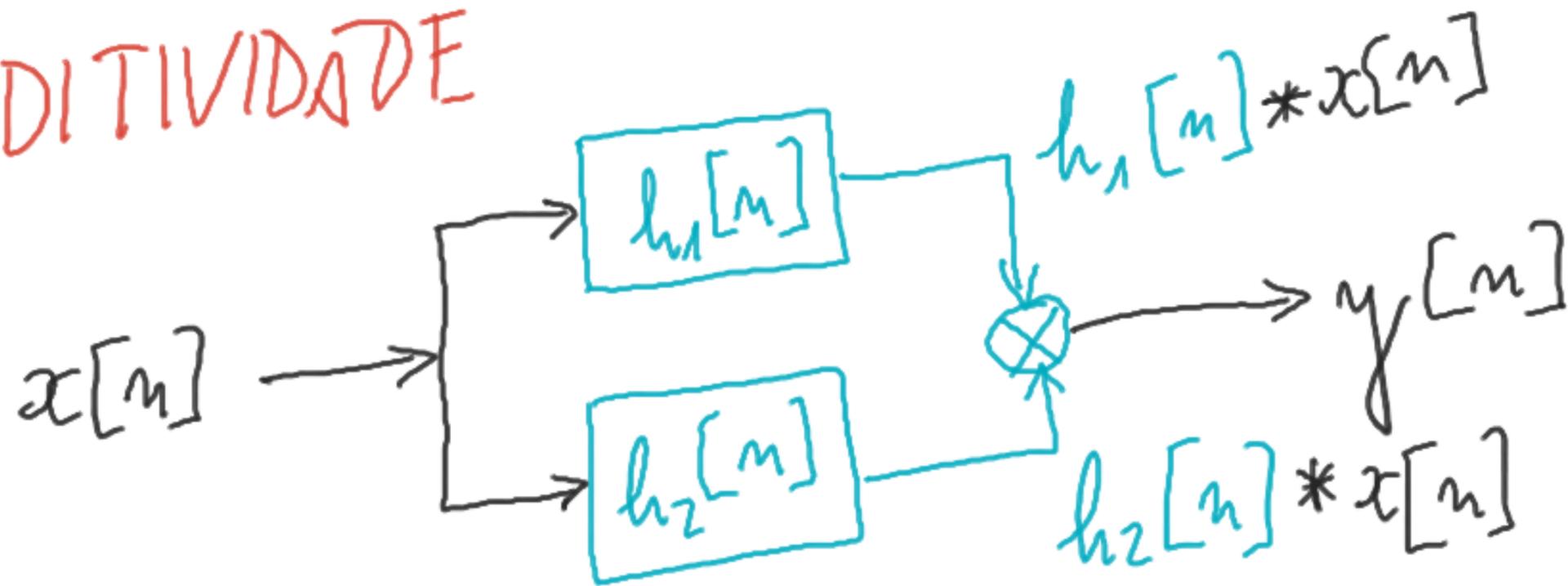
## ① COMUTATIVIDADE



EQUIVA-  
LENTES

# PROPRIEDADES DE SISTEMAS LTI

## ② ADITIVIDADE



EQUIVA-  
LENTES



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# Disciplina: Controle por Computador



**Aula: Parte II: Equações a Diferenças**

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- Oppenheim, A. V., Schafer, R. W. & Buck, J. R. (1999). Discrete-Time Signal Processing, 2<sup>nd</sup> ed., Prentice-Hall. (\*)
- Oppenheim, A. V. & Schafer, R. W. (2013). Processamento em Tempo Discreto de Sinais, 3<sup>a</sup> ed., Pearson Brasil.
- Franklin, G. F., Powell, J. D. & Workman, M. L. (1990). Digital Control of Dynamic Systems, 2<sup>nd</sup> ed., Addison-Wesley, Seções 2.1 e 2.2.

(\*) Organizado para a Seção 2.5 da 2<sup>a</sup> Edição.

# EQUAÇÕES A DIFERENÇAS

- CLASSE IMPORTANTE DE SISTEMAS LTI;

$$y[n] + a_1 y[n-1] + \dots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

- EQUAÇÃO CAUSAL:

$$y[n] = \underbrace{b_0 x[n] + \dots + b_M x[n-M]}_{\text{ENTRADA}} - \underbrace{a_1 y[n-1] - \dots - a_N y[n-N]}_{\text{SAÍDA - TERMOS RECURSIVOS}}$$

Ex. 1:  $y[n] = 0,5 y[n-1] + x[n]$

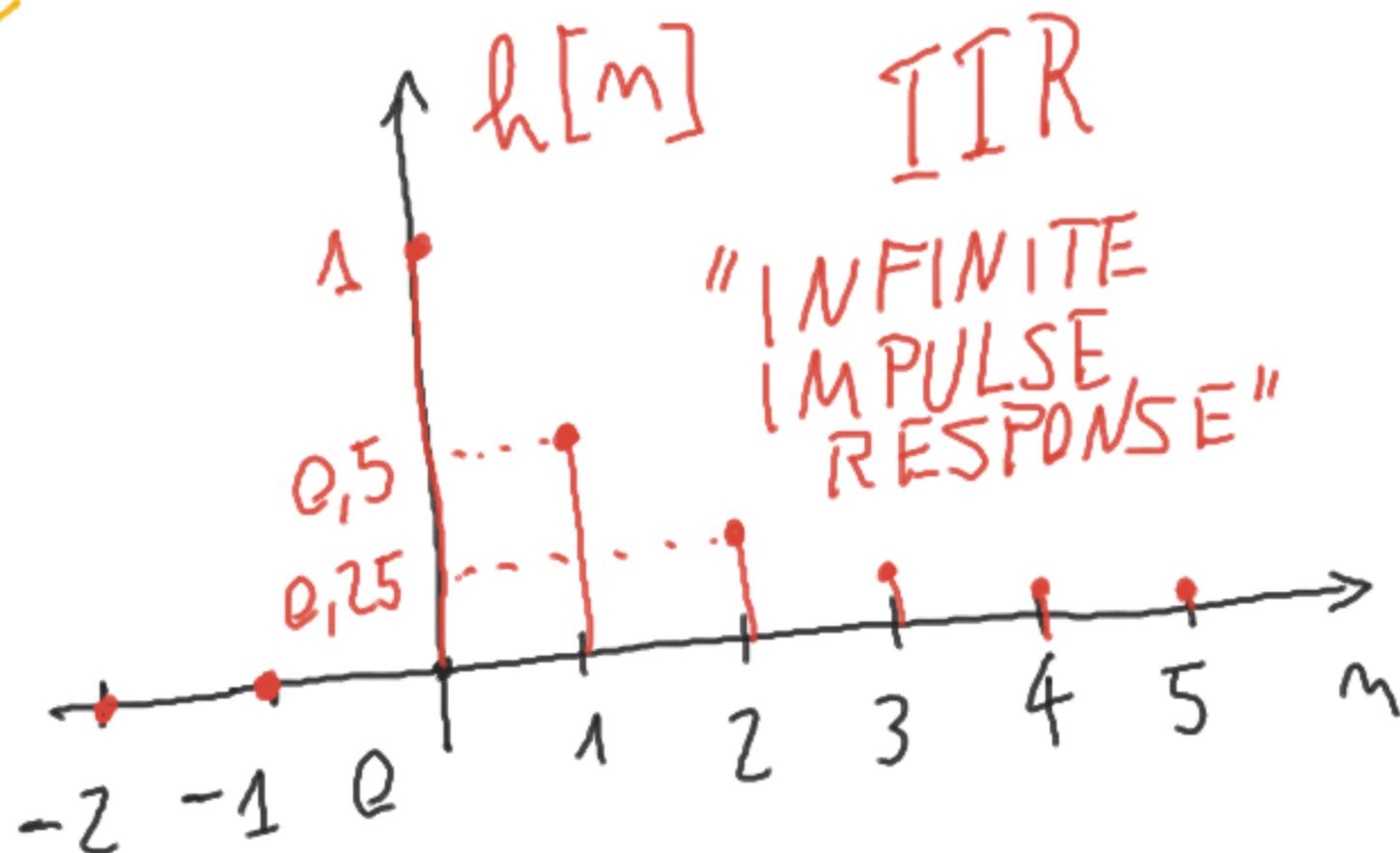
RESPOSTA IMPULSIVA:  $x[n] = \delta[n]$

↑ TERMO RECURSIVO

$$x[n] = \delta[n]$$

$y[-1] = 0$  ← CONDIÇÃO INICIAL NULA

$n$	$x[n]$	$y[n]$
-1	0	0
0	1	1
1	0	0,5
2	0	0,25
3	0	0,125
...	...	...

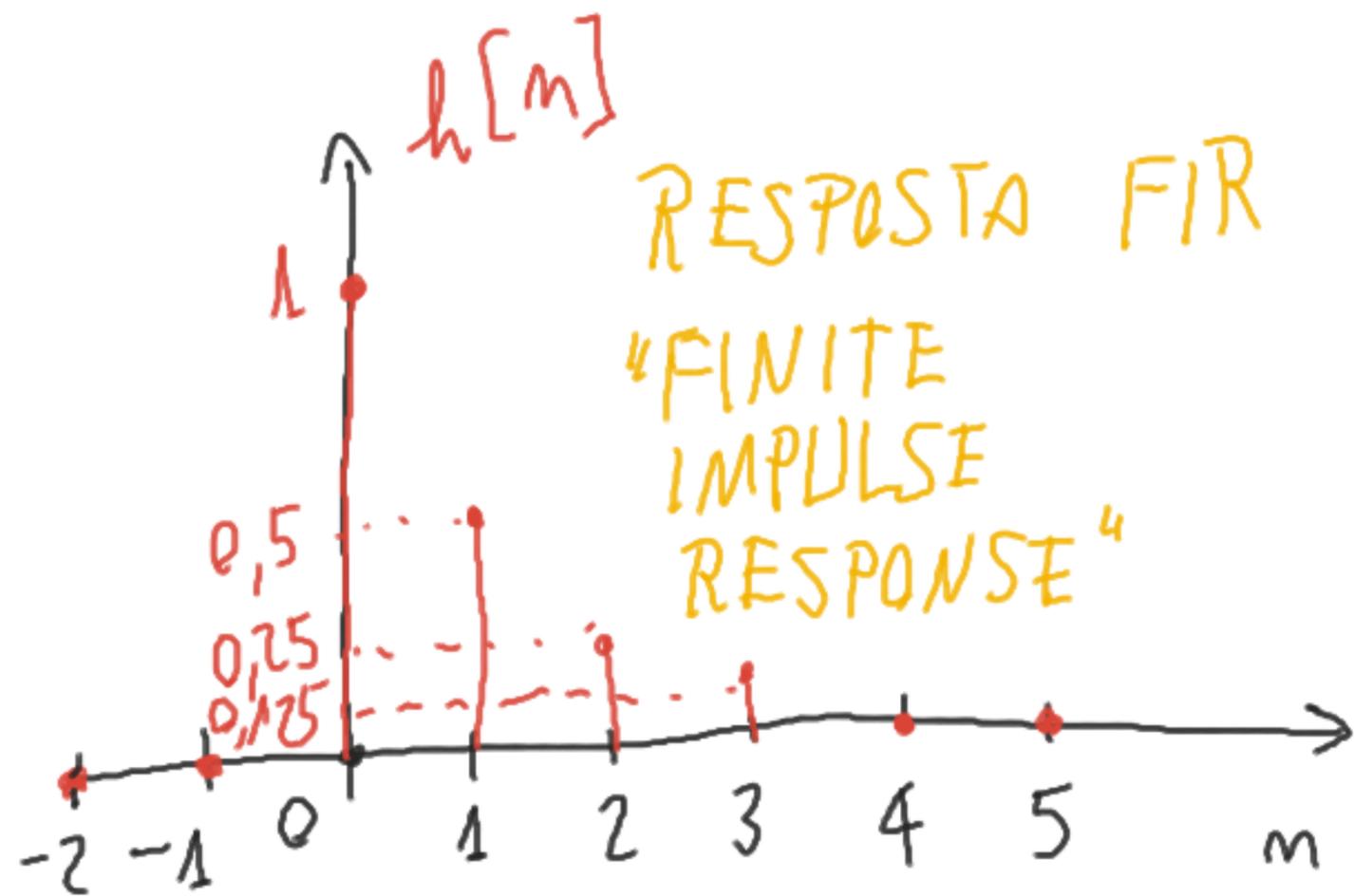


Ex. 2:  $y[n] = x[n] + 0,5x[n-1] + 0,25x[n-2] + 0,125x[n-3]$

RESPOSTA IMPULSIVA:  $x[n] = \delta[n]$

- É NÃO RECURSIVA
- DISPENSA CONDIÇÕES INICIAIS

n	x[n]	y[n]
-1	0	0
0	1	1
1	0	0,5
2	0	0,25
3	0	0,125
4	0	0,0625
5	0	0,03125
6	0	0,015625
7	0	0,0078125
8	0	0,00390625
9	0	0,001953125
10	0	0,0009765625
11	0	0,00048828125
12	0	0,000244140625
13	0	0,0001220703125
14	0	0,00006103515625
15	0	0,000030517578125
16	0	0,0000152587890625
17	0	0,00000762939453125
18	0	0,000003814697265625
19	0	0,0000019073486328125
20	0	0,00000095367431640625
21	0	0,000000476837158203125
22	0	0,0000002384185791015625
23	0	0,00000011920928955078125
24	0	0,00000059604644775390625
25	0	0,000000298023223876953125
26	0	0,0000001490116119384765625
27	0	0,00000007450580596923828125
28	0	0,000000037252902984619140625
29	0	0,0000000186264514923095703125
30	0	0,00000000931322574615478515625
31	0	0,000000004656612873077392578125
32	0	0,0000000023283064365386962890625
33	0	0,00000000116415321826934814453125
34	0	0,000000005820766091346724222265625
35	0	0,0000000029103830456733621111328125
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38	0	0,0000000003637978807091702638916015625
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100	0	0,00000000000000000000000000078886090522100956919552029349594436835937515639461440986328125



NUMA EQUAÇÃO A DIFERENÇAS  
NÃO RECURSIVO:

$$y[n] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

$$b_0 = h[0]$$

$$b_1 = h[1]$$

⋮

$$b_M = h[M]$$