# Transformations and Combinations of Adaptive and Sliding Mode Control

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September 11-14, 2022 (Seminar Version October 5, 2022)

# 1. Overview

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- Multivariable Binary-MRAC
- Global exact tracking with HOSM
- Adaptive Unit Vector Control with transient and steady-state specifications
- Prediction error-based chattering alleviation under unmodeled dynamics

# 2. Lyapunov Control Synthesis

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- Ambrosino, Celentano, and Garofalo (1984) introduced the term **Variable Structure MRAC** using only input and output measurements.

# 2.2 A 1965 survey(L.P.Grayson, Automatica)

| Technique | Plant | Procedure | Resulting System | Literature |
|---|---|---|---|---|
| 7 | $\dot{x}=Ax+Bu$ where $A$ and $B$ are unknown, but constant. | A model $\dot{y}=\mathscr{A}y+\mathscr{B}r$ is selected to be asymptotically stable, and $u$ chosen so that controller parameters $D$ and $E$ approach $A$ and $B$ and $e=x-y\to 0$. | A parameter identifying scheme. It has adaptive properties. | Extension of technique by RANG [12] LASALLE and RATH [13] |
| 8 | $\dot{x}=A(t)x+B(t)u$ where $A(t)$ and $B(t)$ are unknown, but are within known bounds. | An asymptotically stable, linear, time-invariant model of the form $\dot{y}=\mathscr{A}y+\mathscr{B}r$ is selected, and $u$ chosen to make $e=x-y\to 0$ faster than $e_1$ of $\dot{e}_1=ae_1$. | A nonlinear controller with relays. The system has adaptive properties. | GRAYSON [14, 15, 16] Extensions by HIZA and LI [17] MONOPOLI [18] |
| 9 | $\dot{x}=A(t)x+B(t)u$ where $A(t)$ and $B(t)$ are unknown, but are within known bounds. | A controller of the form $\dot{y}=\mathscr{A}y+\mathscr{B}u+De$ is chosen, where $e=x-y$, and $\mathscr{A}$, $\mathscr{B}$ and $D$ are chosen so that $x\to y$. | A nonlinear controller with relays. The system has adaptive properties. | GRAYSON [14] |
| 10 | $\dot{x}=f(x, u)$ where $f(0, 0)=0$ | Choose $u(x)$ such that the system is asymptotically stable, and $\varphi(u)=\int_0^\infty G(x, y)dt$ | Linear or nonlinear controllers may result. Overall system is optimal. | AL'BREKHT [19] BOYANOVITCH [20] AOKI [21] |

# 3. MRAC background

# 3.1 Lyapunov based MRAC design

A pioneering work in Lyapunov based design for adaptive control was published in 1966 by Parks

362 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-11, NO. 3, JULY, 1966

Liapunov Redesign of Model Reference Adaptive Control Systems

PATRICK C. PARKS

Lyapunov design has set the stage for modern adaptive control theory (see (Ioannou and J. Sun, 1996)).

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- State variable filters $(\omega_1,\ \omega_2 \in \mathbb{R}^{n-1})$

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$$\begin{aligned} \dot{\omega}_1 &= \Lambda\omega_1 + gu \\ \dot{\omega}_2 &= \Lambda\omega_2 + gy \end{aligned}$$

- Regressor vector: $\omega^T = [{\omega_1}^T \ {\omega_2}^T \ y \ r]$
- Adaptive parameter vector: $\theta^T = [\theta_1^T \theta_2^T \theta_3 \theta_4]$

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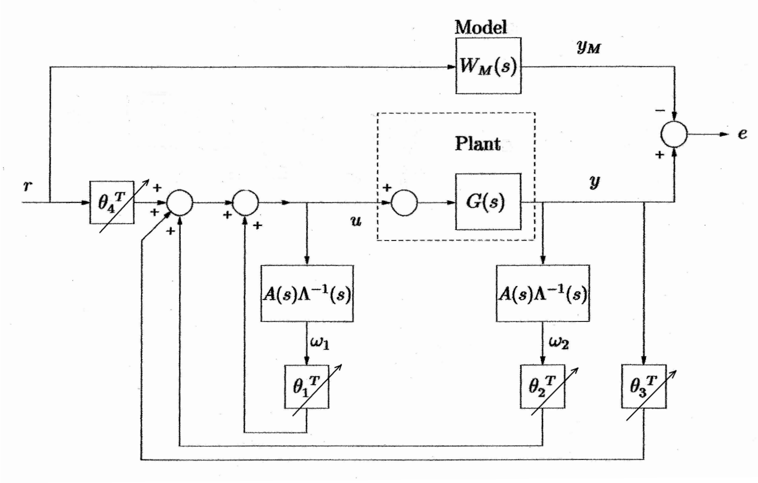
- $e_1 = h^T e$ for some $h \in \mathbb{R}^{3n-2}$
- $\{A, b, h\}$ is a *nonminimal* realization of the SPR reference model $W_M(s)$

### The (simplified) Kalman-Yakubovitch-Popov Lemma

$G(s) = C(sI - A)^{-1}B$ is strictly positive real iff $\exists P = P^T > 0,\ Q > 0$ such that

$$
\begin{aligned}
PA + A^TP &= -2Q \\
PB &= C^T
\end{aligned}
$$

# 3.3.1 MRAC block diagram



# 3.3.2 Adaptive laws, $n^* = 1$

Now, choose candidate Lyapunov function $V$ and adaptive law for $\dot{V} \leq 0$

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**Adaptation law (SISO, $n^* = 1$)**

- Adaptation law: $\dot{\theta} = -sign(K_p)\Gamma \omega e_1; \quad \Gamma = \Gamma^T > 0$

- or $\dot{V} = -e^T Q e + ~~e^T P b \rho^* [\tilde{\theta}^T \omega]~~ - ~~\rho^* \tilde{\theta}^T \omega e_1~~$;
- Thanks to the KYP Lemma:

$$\dot{V} = -e^T Q e \leq 0 \quad \text{(semidefinite negative)}$$

## 3.3.3 Hidden difficulties of semi-definite $\dot{V}$

With $V(e, \tilde{\theta}) > 0$ but $\dot{V} = -e^T Q e \leq 0$ (semi-definite) one can conclude:

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In fact,

**The adaptation transient can be extremely slow or oscillatory.**
**Still a challenge in adaptive control!**

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- The Reference Model cannot be SPR.
- Solution for adaptive control:
  Monopoli's augmented error
- Adaptive algorithm analysis and synthesis much more complicated!

# 4. From MRAC to VS-MRAC

# 4.1 Brief history

Early papers

STATE FEEDBACK: (Devaud and Caron, 1975), (Zinober, El-Ghezawi, and Billings, 1982)

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2. Bartolini and Zolezzi, 1988: "The V.S.S. Approach to the Model Reference Control of Nonminimum Phase Linear Plants".

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From MRAC to VS-MRAC with $n^* = 1$

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- ...but now using only output feedback!.

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- Conclude $\dot{V} < -e^T Qe < 0$ using KYP Lemma

SPR Lemma makes the "magic" of allowing sign-indefinite terms to be dominated!

# 4.3 The (output feedback) VS-MRAC, $n^* = 1$

## Compact form control

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\begin{aligned}
u &= -\rho(\omega)\,\mathrm{sign}(e_1) \\
\rho &= \left[ \sum_{1}^{2n} \bar{\theta}_i |\omega_i| \ + \delta \right]
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$$\begin{aligned} u &= -\rho(\omega)\text{sign}(e_1) \\ \rho &= \left[\sum_{1}^{2n} \bar{\theta}_i |\omega_i| + \delta\right] \end{aligned}$$

$\rho$ is called "gain" or "modulation" function of the relay function sign(.), with arbitrary design constant $\delta > 0$.

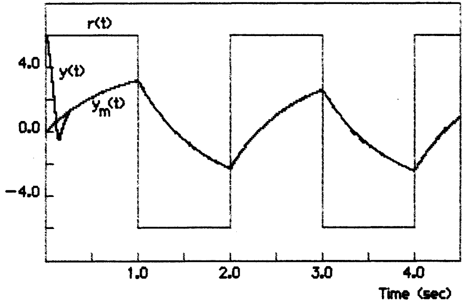
**Main Result: Global tracking, $n^* = 1$**

- $\|e(t)\| \to 0$ with at least an exponential rate, independent of the excitation $r(t)$;
- The output error $y(t) - y_M = e_1(t) = h^T e$ becomes zero after finite time $t_1 \geq t_0$, in sliding mode.

# 4.3.2 Simulation results

Uncertain nonlinear time-varying plant
(Hsu and R. R. Costa, 1989)

$$\begin{aligned}\dot{x}_1 &= [1 + a(t)]x_2 \\ \dot{x}_2 &= sin\ x_1 - 2sin\ x_2 + d(t) + u \\ \dot{y}_m &= -2y_m + r(t); \\ y &= 6x_1 + x_2\end{aligned}$$

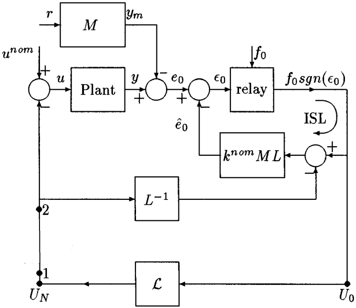


# 4.4 VS-MRAC, $n^* > 1$, prediction error

An augmented error was also proposed (Hsu, 1990) for the VS-MRAC, inspired by the MRAC works of

- (Monopoli, 1974) and
- (Goodwin and Mayne 1987)

# 4.4.1a Block diagram, $n^* > 1$, $N := n^* - 1$



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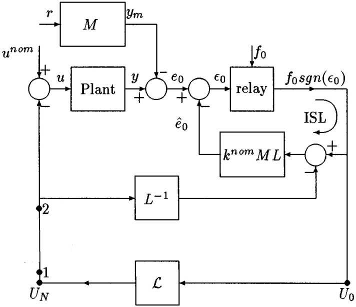


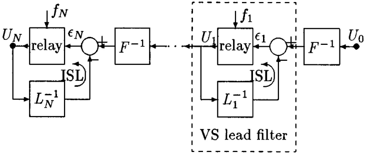

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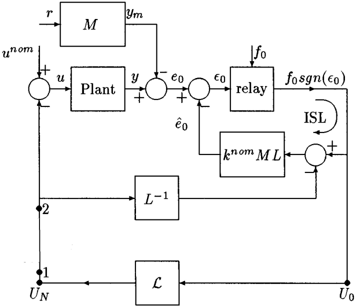


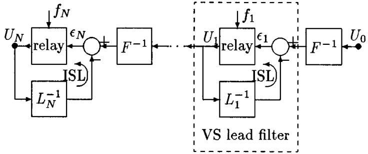


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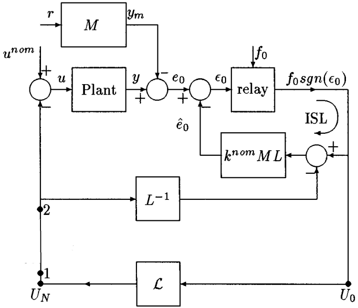


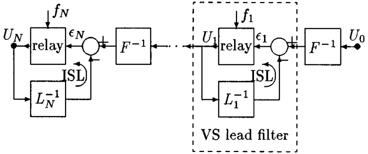


$$\mathcal{L} \approx L$$

- $L(s) = (s + \alpha_i) \ldots (s + \alpha_N);$

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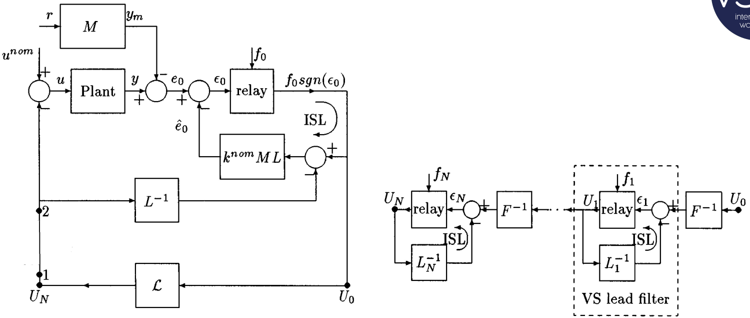
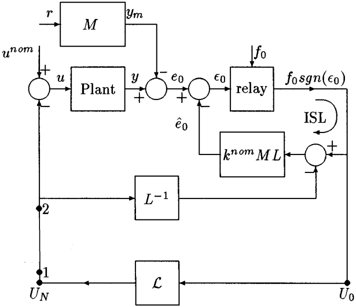
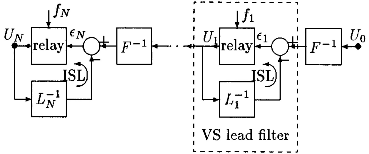



$$\mathcal{L} \approx L$$

- $L(s) = (s + \alpha_i) \ldots (s + \alpha_N)$;
- $F^{-1} = 1/(\tau s + 1)$ is an averaging filter;

# 4.4.1a Block diagram, $n^* > 1$, $N := n^* - 1$




$$\mathcal{L} \approx L$$

- $L(s) = (s + \alpha_i) \dots (s + \alpha_N)$;
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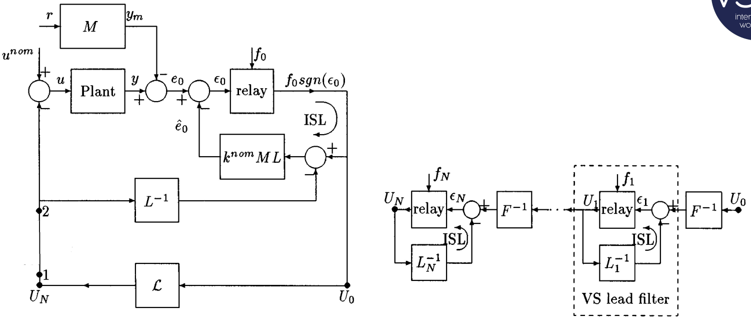


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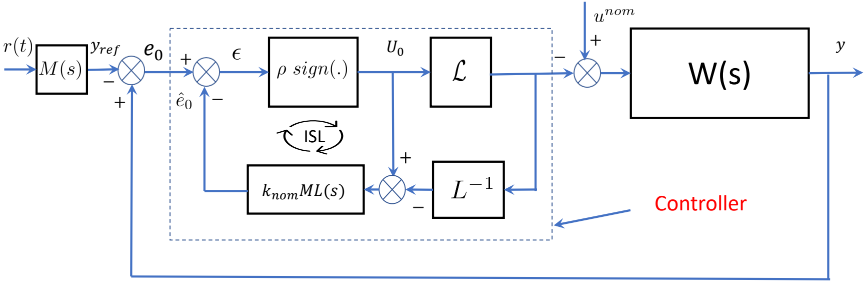
- $ML \in$ SPR allows an "Ideal Sliding Loop" (ISL)
- $\mathcal{L}$ is a cascade of VS-lead filters.

# 4.4.1b "Controller form" diagram, $n^* > 1$

**The VS-MRAC**



# 4.4.2 Stability Theorem

## Global tracking for $n^* > 1$

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(Hsu, Araújo, and R. R. Costa, 1993; Hsu, Lizarralde, and Araújo, 1997)

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### Ideal Sliding Modes (ISM): no finite frequency chattering

All auxiliary errors $\epsilon_i$ tend to zero in finite time as ideal sliding modes.

**Remark:** For $\epsilon_N$, special nonrestrictive conditions are needed (Hsu, 1997), (Oliveira, Hsu, and Nunes, 2021)

# 4.5 VS-MRAC, $n^* > 1$, with HGO

- Instead of a cascade of VS-lead filters, it is possible to use a High Gain Observer for the VS-MRAC.
  (J. P. V. S. Cunha, R. R. Costa, Lizarralde, and L. Hsu, 2009).

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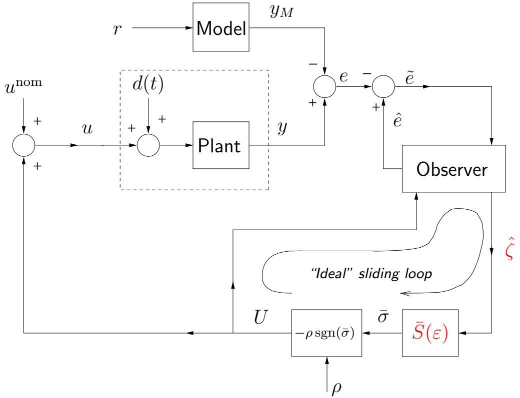
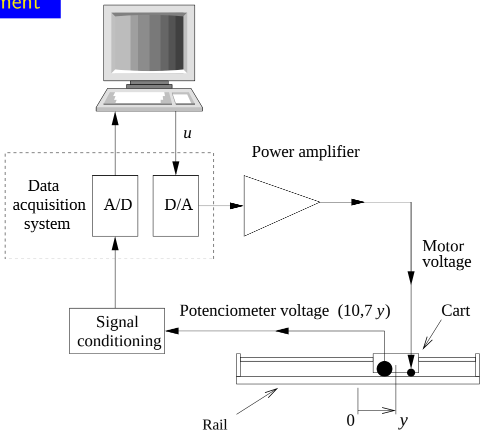
- The HGO allows an Ideal Sliding Loop around the relay function, even when the plant has unmodeled dynamics.
- So, the controller is expected to be less prone to chattering.
- The controller is also free of control peaking.

# 4.5.1 VS-MRAC with HGO

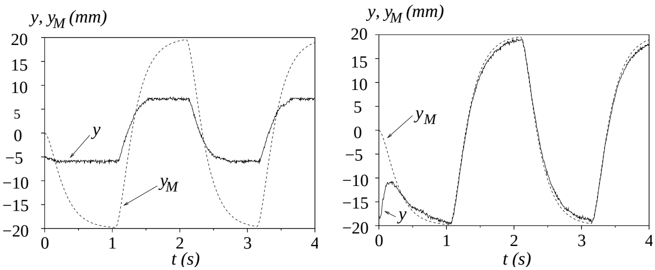
Peaking-free control with Ideal Sliding Mode (ISM) via HGO



Experiment


u
Power amplifier
Data
acquisition
system
A/D
D/A
Motor
voltage
Potenciometer voltage (10,7 y)
Cart
Signal
conditioning
Rail
0
y

HGO VS-MRAC cart position control




**Nominal linear control** **HGO + VSC + SVF**

**Nominal cart mass** **Augmented cart mass**

# 4.5.1 VS-MRAC with HGO (cont.)

**Main conclusion:**

A global peaking-free VS-MRAC was developed using high gain observer (HGO).

# 4.6 UV-MRAC: multivariable and nonlinear plants

Output Feedback SMC of multivariable systems was considered by several authors, e.g., Edwards and Spurgeon (1998), Emelyanov, Korovin, Nersisian, and Nisenzon (1992), and Chien, K.-C. Sun, and Wu (1996), Saaj, Bandyopadhyay, and Unbehauen (2002) (discrete-time systems), Oh and Khalil, 1995; Oh and Khalil, 1997 (High-gain observers).

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- The VS-MRAC was generalized for multivariable and nonlinear plants using Unit-Vector (UV) control.
- The output-feedback controller was named Unit-Vector-Model Reference Adaptive Control (UV-MRAC)

(Hsu, J. P. V. S. Cunha, R. R. Costa, and Lizarralde, 2002; J. P. V. S. Cunha, Hsu, R. R. Costa, and Lizarralde, 2003), (Hsu, Peixoto, J. P. V. S. Cunha, R. R. Costa, and Lizarralde, 2006)

# 4.6.1 UV-MRAC $n^* = 1$

## Problem statement

- Plant

$$\dot{x}_p = A_p x_p + \phi(x_p, t) + B_p u \,, \qquad y = C_p x_p$$

$$x_p, \phi \in \mathbb{R}^n, \quad y, u \in \mathbb{R}^m$$

- Linear subsystem transfer function matrix:

$$G(s) = C_p (sI - A_p)^{-1} B_p$$

- High frequency gain matrix: $K_p = C_p B_p$

# 4.6.1 UV-MRAC $n^* = 1$ (cont.)

## Special assumptions

(A1) $S_p$ is known such that $-K_p S_p$ is Hurwitz

(A2) $\phi(x_p, t)$: piecewise continuous in $t$ and locally Lipschitz in $x_p$

(A3) $\|\phi(x_p, t)\| \leq k_x \|x_p\| + \varphi(y, t)\,, \quad k_x, \varphi \geq 0$ are known

### Unit Vector control law

$$u = u^{nom} - S_p\, \rho \frac{e}{\|e\|}$$

Modulation (or variable gain) function:

$$\rho = \delta + c_1\|\omega\| + c_2\|r\| + c_3\|e\| + \hat{\phi}(t)$$

(output feedback law)

**Main result ($n^* = 1$)**

- The UV-MRAC system is globally exponentially stable.

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- The UV-MRAC system is globally exponentially stable.
- Moreover, if $\delta > 0$, the output error $e(t)$ becomes zero after some finite time.

# 4.6.2 UV-MRAC $n^* > 1$

## Problem statement

- Plant: $y,\, u \in \mathbb{R}^m$

$$\begin{aligned} \dot{x}_p &= A_p x_p + \phi(x_p, t) + B_p u \\ y &= C_p x_p \end{aligned}$$

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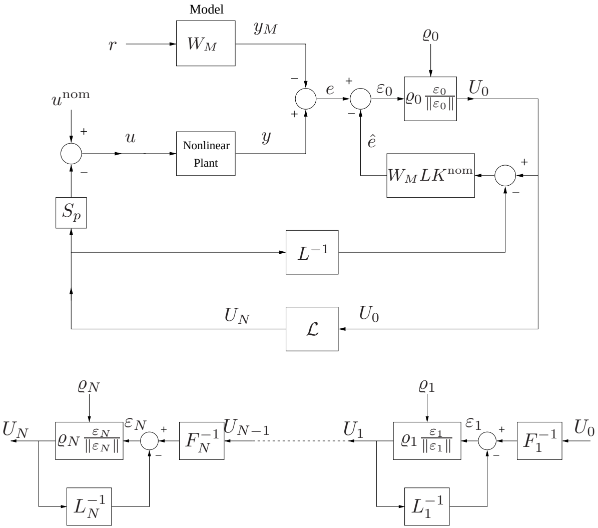
$$G(s) = C_p(sI - A_p)^{-1} B_p$$

- High frequency gain matrix $K_p = C_p A_p^{n^*-1} B_p$ is nonsingular (uniform relative degree $n^*$)

# 4.6.3 UV-MRAC Block Diagram, $n^* > 1$



# 4.6.4 FOAF for state norm-bound

Consider a square $m \times m$ system in normal form :

$$\begin{aligned} \dot{\eta} &= A_{11}\eta + A_{12}y, \\ \dot{y} &= A_{21}\eta + A_{22}y + K_p[u + d(x,t)], \end{aligned} \tag{1}$$

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Then, a norm-bound $\hat{\eta}(t)$ for $\eta(t)$ can be obtained with a FOAF (First Order Approximation Filter):

$$\hat{\eta}(t) := \frac{c_f}{s+\gamma_f}\|y(t)\|, \quad c_f,\ \gamma_f > 0 \tag{2}$$

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$$\|\eta(t)\| \leq \hat{\eta}(t) + \pi_\eta(t), \qquad (3)$$

where $\pi_\eta(t)$ is an exponentially decaying term.

# 4.6.4 FOAF for state norm-bound (cont.)

**Global output feedback SMC**

FOAFs are instrumental in designing global FOSM or HOSM controllers.

(J. P. V. S. Cunha, R. R. Costa, and L. Hsu, 2008; J. P. V. S. Cunha, Hsu, R. R. Costa, and Lizarralde, 2003)

# 5. From Theory to Practice

# Applications

The VS-MRAC was successfully applied to a number of practical problems.

# 5.1 ROV Dynamic Positioning

Remotely Operated underwater Vehicles (ROV) are widely used in underwater oil exploration and many other industrial, military and scientific activities.

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# 5.1.1 Dynamic Positioning of an ROV

The Passive Arm gives the ROV pose by direct kinematics

# 5.1.1 Dynamic Positioning of an ROV (cont.)

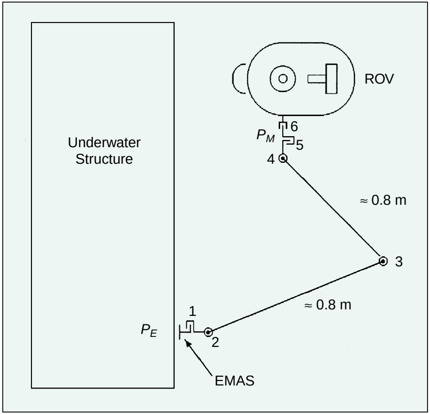



***Figure 1.*** *Passive arm.*

# 5.1.1 Dynamic Positioning of an ROV (cont.)

The ROV-Passive Arm system in experimental test

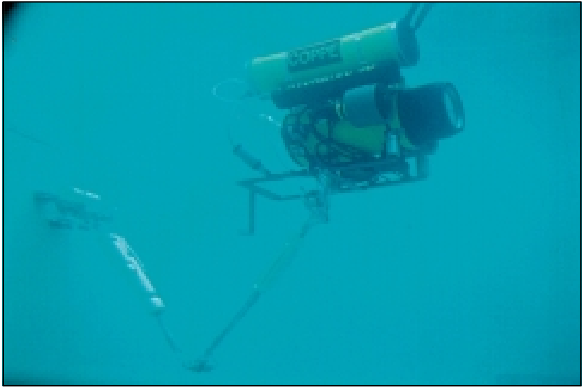



***Figure 3.*** *The passive arm installed on the MKII ROV.*

P-PI (*Proportional-Proportional Integral*) linear Control

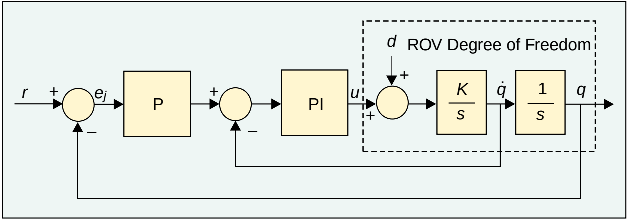
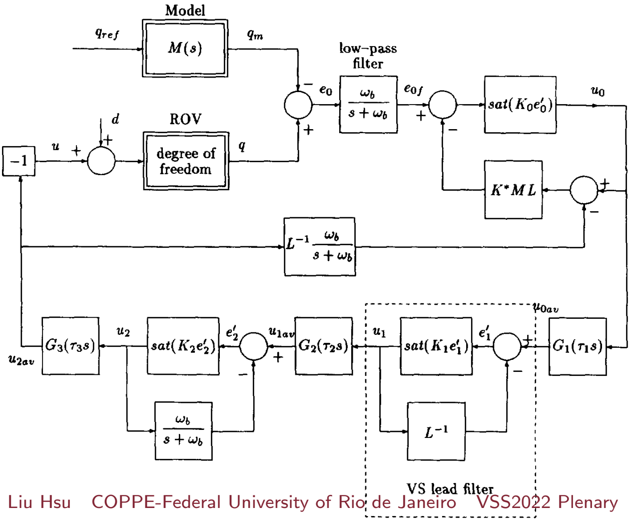



***Figure 8.*** *Block diagram of the P-PI.*

(IEEE RAM 2000)

# 5.1.1 Linear vs SMC algorithms (cont.)

VS-MRAC ($n^* = 3$) as applied for ROV DP (Note the noise filter)



# 5.1.2 Benchmark DP motion

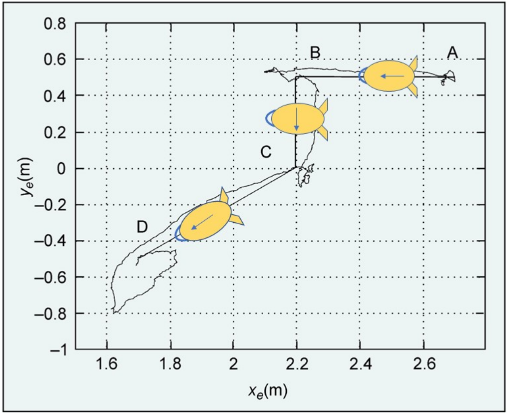



***Figure 10.*** *Trajectory tracking tests with the P-PI control algorithm applied to a large ROV. Horizontal $x_e y_e$ plane view.*

# 5.1.3 P-PI result with a 350Kg ROV (Tatuí-I) P-PI

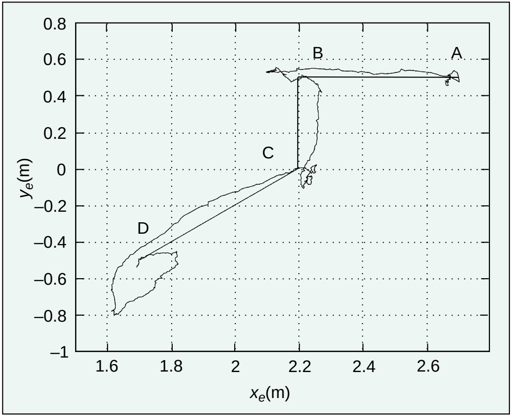



***Figure 10.*** *Trajectory tracking tests with the P-PI control algorithm applied to a large ROV. Horizontal $x_e y_e$ plane view.*

(IEEE RAM 2000)

# 5.1.4 VS-MRAC result with ROV Tatuí-I

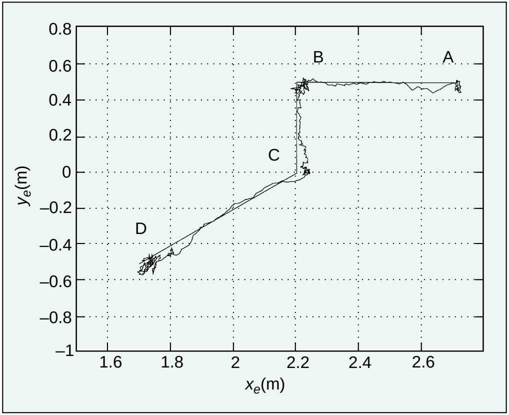



***Figure 11.*** *Trajectory tracking tests with the VS-MRAC control algorithm applied to a large ROV. Horizontal $x_e y_e$ plane view.*

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- A decentralized VS-MRAC was implemented on a PUMA 560 manipulator
- The results were better than those in the existing literature

# 5.2.1 Manipulator equations

Equations of $n$-link rigid manipulator in joint space

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \Gamma \tag{4}$$

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A nonlinear system!!

# 5.2.1 Manipulator equations (cont.)

- Goal: design a suitable control to ensure small joint tracking error

$$\tilde{q} = q - q_d \tag{5}$$

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$$\tilde{q} = q - q_d \tag{5}$$

- Only nominal robot parameters were available.

# 5.2.2 Manipulator VS-MRAC design

- The error system is reduced to $n$ disturbed and coupled double integrators:

$$\ddot{\tilde{q}}_i = u_i + d_i(q, \dot{q}, q_d, \dot{q}_d, u) \tag{6}$$

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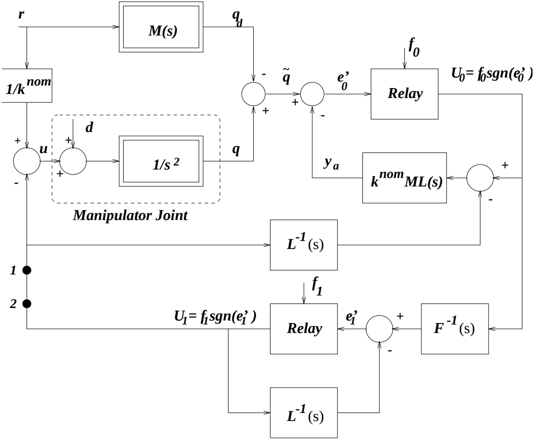
- Thus, the VS-MRAC for $n^* = 2$ can be applied to each joint.
- Stability analysis invokes Frobenius-Perron's Theorem, to account for the residual control couplings among the scalar subsystems.

# 5.2.3 Manipulator VS-MRAC per joint



# 5.2.4 VS-MRAC results on a PUMA 560

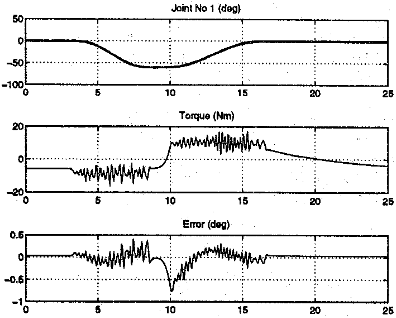



**Figure 2: Joint $N°1$, a) $q$ and $q_d$ in degrees, b) control signal $\Gamma$ in $Nm$ and c) tracking errors in degrees.**

# 5.2.4 VS-MRAC results on a PUMA 560 (cont.)

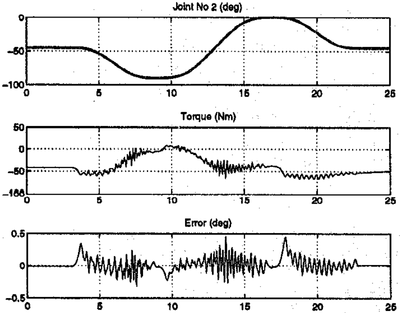



**Figure 3:** Joint $N°2$, a) $q$ and $q_d$ in degrees, b) control signal $\Gamma$ in $Nm$ and c) tracking errors in degrees.

# 5.2.4 VS-MRAC results on a PUMA 560 (cont.)

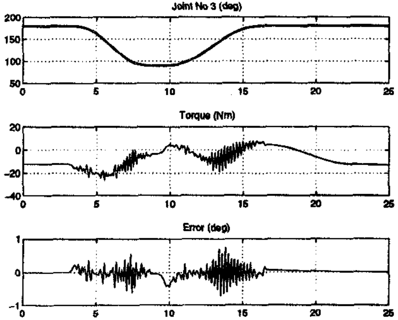



Figure 4: Joint $N°3$, a) $q$ and $q_d$ in degrees, b) control signal $\Gamma$ in $Nm$ and c) tracking errors in degrees.

# 5.3 Other Applications

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- One example (2012) is about satellite formation control (Lee and S. N. Singh, 2012).

# 6. Binary MRAC with Passivation

# 6.1 Motivation

- MRAC: continuous control signal but lacks robustness and can present bad adaptation transient.

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- B-MRAC: a bridge between them, combining their desirable properties and avoiding their drawbacks.

- The B-MRAC consists of conventional MRAC modified by parameter projection with high adaptation gain (Hsu and R. R. Costa, 1991; Hsu and R. R. Costa, 1994).

# 6.2 Passivity framework

Consider multivariable plants.

- The Lyapunov based multivariable MRAC requires the SPR passivity condition

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- A new generalized passivity requires the weaker WSPR condition.

- WSPR only requires $K_p$ to have Positive Diagonal Jordan form (PDJ).

# 6.2 Passivity framework (cont.)

WSPR condition

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WSPR condition

The system satisfies the WSPR condition if besides $P,\ Q$, there exists $W$ SPD, such that

$$A^T P + PA = -Q, \quad (7)$$

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# 6.2 Passivity framework (cont.)

WSPR condition

The system satisfies the WSPR condition if besides $P,\ Q$, there exists $W$ SPD, such that

$$A^T P + PA = -Q, \tag{7}$$

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Note that $W$ is not used for the control design. Only its existence is required!

(Barkana, Teixeira, and Hsu, 2006)
(Yanque, Nunes, R. R. Costa, and H., 2012)
(Hsu, Teixeira, R. R. Costa, and Assunção, 2015)

# 6.2 Passivity framework (cont.)

### Passifying multiplier $L$

There exists a lower triangular passifying multiplier $L$ such that the PDJ condition holds for the modified output error

$$e_L = Le.$$

# 6.2 Passivity framework (cont.)

- The B-MRAC adaptation law is given by

$$\dot{\theta} = \text{Proj}[-\gamma \Omega e_L]$$

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$$M_\theta > \|\theta^*\|$$

- and the control law is

$$u(t) = \Omega^T(t)\theta(t).$$

# 6.3 From B-MRAC to Unit Vector Control

When $\gamma \to \infty$, the B-MRAC law tends to the UVC law

$$u = -M_\theta \|\omega\| \frac{e_L}{\|e_L\|}.$$

# 6.4 Example: Adaptive visual tracking

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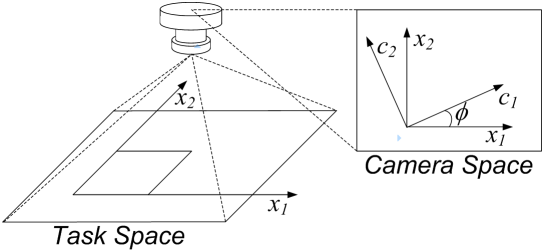


Figure: Representation of the camera-robot system

# MRAC control with passivation and $\gamma = 5$

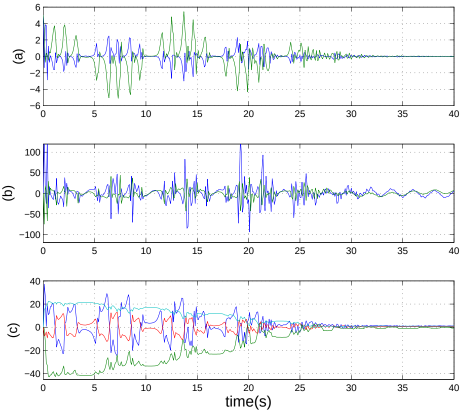


Figure: Behavior of the MRAC control with passivation and $\gamma = 5$:
(a) Tracking errors $e$; (b) Plant control signals $u$; (c) Adaptive parameters

# B-MRAC control without passivation and $\gamma = 5$

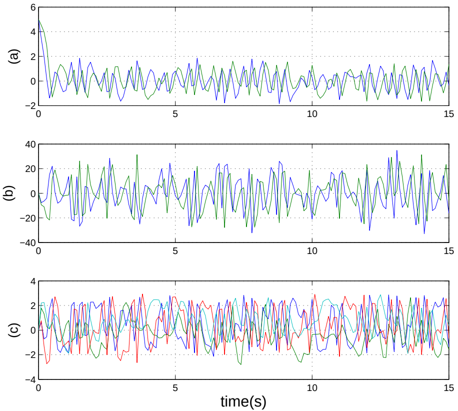


Figure: Behavior of the B-MRAC control without passivation and $\gamma = 5$: (a) Tracking errors $e$; (b) Plant control signals $u$; (c) Adaptive parameters

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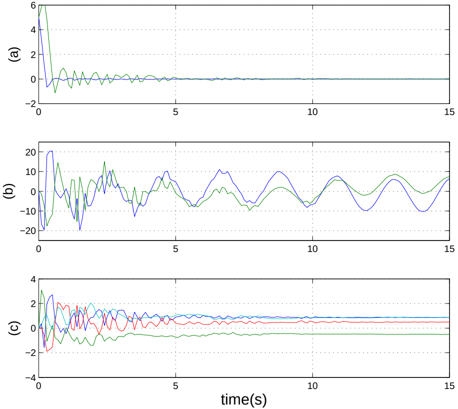


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 5$:
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# B-MRAC control with passivation and $\gamma = 20$

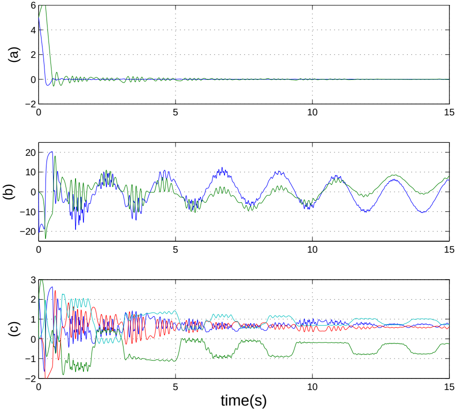


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 20$:
(a) Tracking errors $e$; (b) Plant control signals $u$; (c) Adaptive parameters

# UVC without passivation

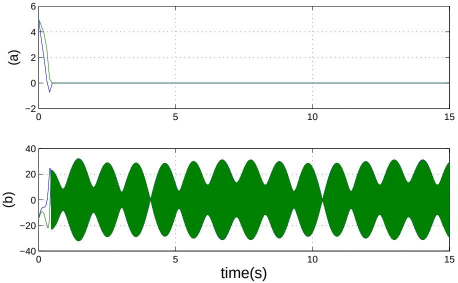


Figure: UVC without passivation: (a) Tracking errors $e$; (b) Plant control signals $u$

# B-MRAC control without passivation and $\gamma = 100$

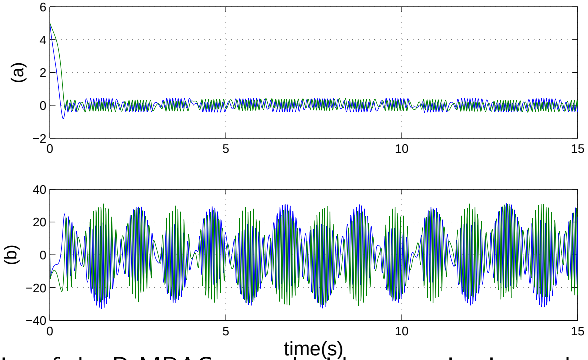


Figure: Behavior of the B-MRAC control without passivation and $\gamma = 100$: (a) Tracking errors $e$; (b) Plant control signals $u$

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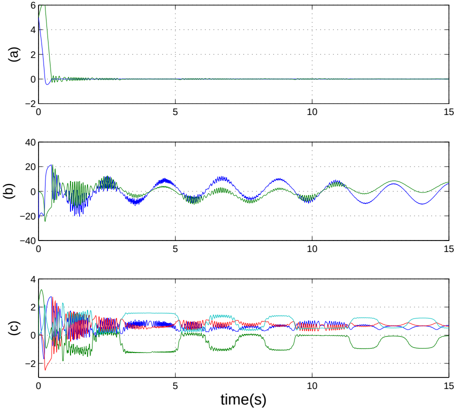


Figure: Behavior of the B-MRAC control with passivation and $\gamma = 100$: (a) Tracking errors $e$; (b) Plant control signals $u$; (c) Adaptive parameters

# 7. Global Exact Tracking with HOSM

# 7.1 Main objective and idea

Consider an uncertain linear systems:

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Consider an uncertain linear systems:

- Main Objective:
  - Propose an SMC scheme for global stability and asymptotic exact tracking
  - Output feedback is required
- Main Idea:
  - Implement a VS-MRAC combining a standard lead filter and an RED-based lead filter.
  - RED: Robust Exact Differentiator (Levant, 1998)

# 7.1.2 An ideal VS-MRAC for plants with $n^* > 1$

- Consider an operator $L(s)$ so that the case of $n^* > 1$ is reduced to the simple case of $n^* = 1$ according to the block diagram

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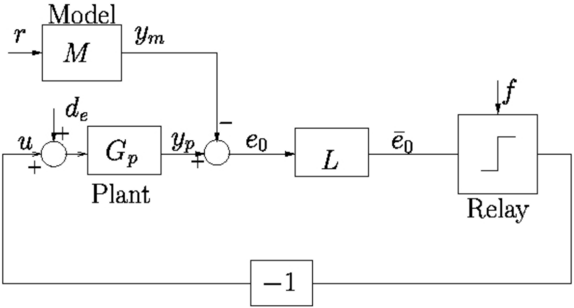

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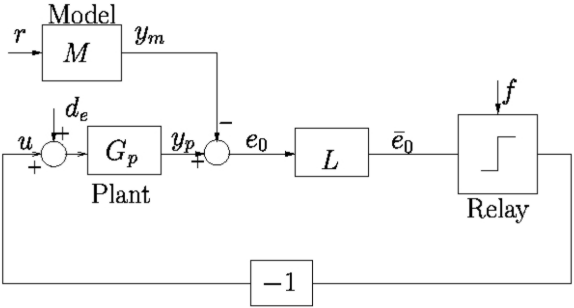


- Then, $\bar{e}_0 = L(s)e_0 \to \bar{e}_0 = k^* M L(s)[u + \bar{U}]$ and the relative degree from $u$ to $\bar{e}_0$ is one.

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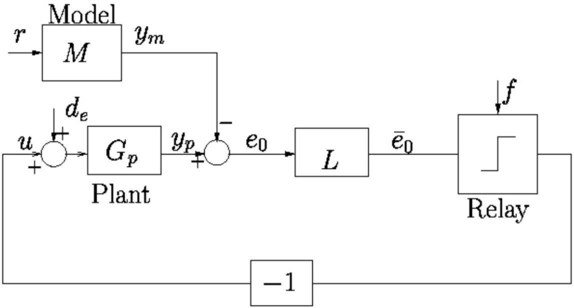


- Then, $\bar{e}_0 = L(s)e_0 \rightarrow \bar{e}_0 = k^* M L(s)[u + \bar{U}]$ and the relative degree from $u$ to $\bar{e}_0$ is one.
- The problem is how to implement the non-causal operator $L(s)$.

# 7.3 Global output feedback exact tracking

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  - Global stability
  - Residual tracking error (and chattering)
- Use RED-based lead compensation
  - Exact estimate of $\bar{e}_0$
  - Local stability
  - Asymptotic convergence of the tracking error to zero

# 7.3.1 GRED/VS-MRAC Control Scheme

**A global RED (GRED) compensation**
(Nunes, Hsu, and Lizarralde, 2009)

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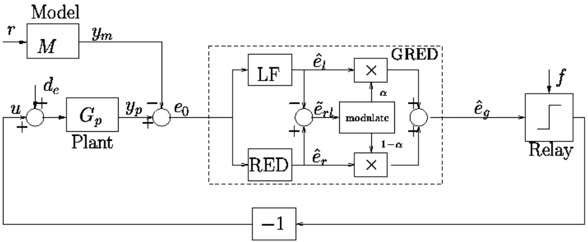

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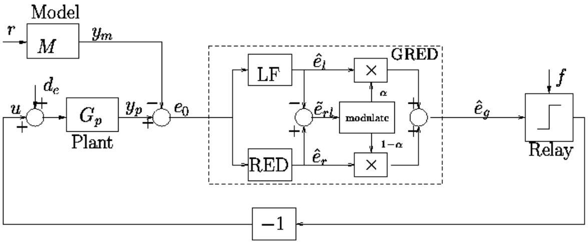


- Convex combination: $\hat{e}_g = \alpha(\tilde{e}_{rl})\hat{e}_l(t) + [1 - \alpha(\tilde{e}_{rl})]\,\hat{e}_r(t)$

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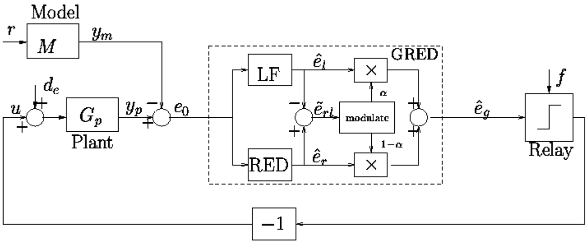


- Convex combination: $\hat{e}_g = \alpha(\tilde{e}_{rl})\hat{e}_l(t) + [1 - \alpha(\tilde{e}_{rl})]\, \hat{e}_r(t)$
- After finite-time, the RED takes over ($\alpha = 0$)

# 7.3.2 Plant $n^* = 3$ + unmodeled dynamics

- Tracking error $e_0(t)$

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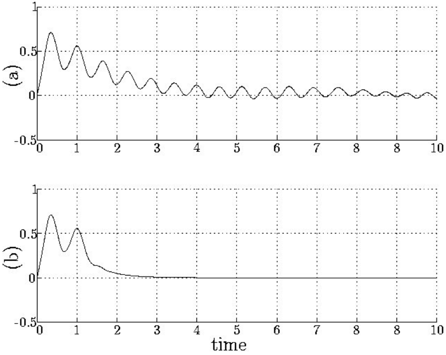

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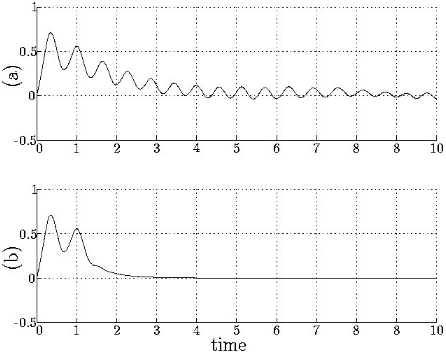


(a) LF/VS-MRAC: residual tracking error (chattering)

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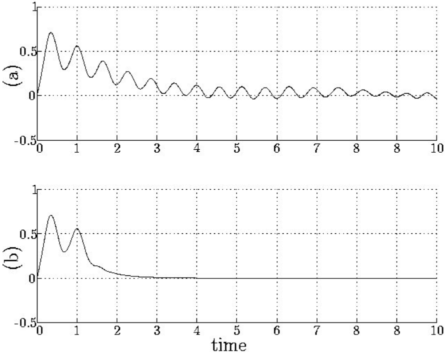
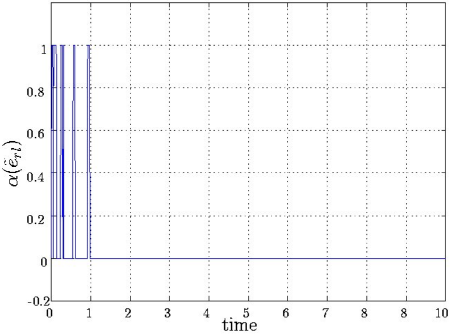



(a) LF/VS-MRAC: residual tracking error (chattering)

(b) GRED/VS-MRAC: global asymptotic tracking

# 7.3.2 Plant $n^* = 3$ + unmodeled dynamics (cont.)

- Weighted Switching Function




- GRED for multivariable plants in (Nunes, Peixoto, Oliveira, and Hsu, 2014).

# 7.7 Experimental Results

- GRED/VS-MRAC applied to a servomechanism (SRV-02) for single-link angular positioning (Quanser Consulting).

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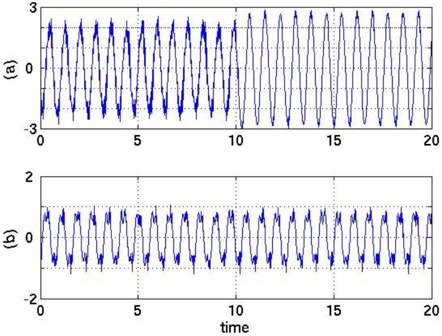
- GRED/VS-MRAC applied to a servomechanism (SRV-02) for single-link angular positioning (Quanser Consulting).
- Objective: show that the arm can follow a reference signal without significant chattering
- Control Signal:
    - Modulation Function: $f = 5$ (Maximum input voltage)
    - Boundary Layer ($\Delta$)

# 7.7.1 Experimental Results: Comparison

- Tracking error $e_0(t)$

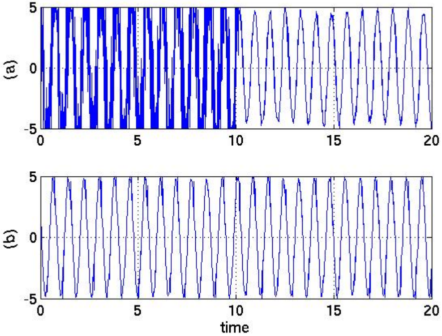



(a) LF/VS-MRAC ($t \in [0, 10] \rightarrow \Delta = 15$, $t \in (10, 20] \rightarrow \Delta = 25$)

(b) GRED/VS-MRAC ($\Delta = 15$): smaller tracking error (1,4%)

# 7.7.1 Experimental Results: Comparison (cont.)

- Control signal $u(t)$:




(a) LF/VS-MRAC ($t \in [0, 10] \rightarrow \Delta = 15$, $t \in (10, 20] \rightarrow \Delta = 25$)

(b) GRED/VS-MRAC ($\Delta = 15$): much reduced chattering

# 8. Adaptive Unit-Vector Control with transient and steady-state specifications

# 8.1 Problem statement

Consider (MIMO) systems in *regular form*

$$\dot{\eta} = A_{11}\eta + A_{12}\sigma + d_1(x,t)\,, \tag{9}$$

$$\dot{\sigma} = A_{21}\eta + A_{22}\sigma + d_2(x,t) + B_2 u\,, \tag{10}$$

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- Output $\sigma \in \mathbb{R}^m$
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- Input $u \in \mathbb{R}^m$
- Output $\sigma \in \mathbb{R}^m$
- Zero-dynamics state $\eta \in \mathbb{R}^{n-m}$
- Matched disturbance $d_2 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^m$

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$$
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- Input $u \in \mathbb{R}^m$
- Output $\sigma \in \mathbb{R}^m$
- Zero-dynamics state $\eta \in \mathbb{R}^{n-m}$
- Matched disturbance $d_2 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^m$
- Unmatched disturbance $d_1 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n-m}$

# 8.1 Problem statement (cont.)

- **Assumptions**

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**(A1)** Minimum phase from $u$ to $\sigma$: $A_{11}$ is Hurwitz.

# 8.1 Problem statement (cont.)

- **Assumptions**

**(A1)** Minimum phase from $u$ to $\sigma$: $A_{11}$ is Hurwitz.

**(A2)** $S_p \in \mathbb{R}^{m\times m}$ is known so that $-K_p$ is Hurwitz, where

$$K_p := B_2 S_p \tag{11}$$

is the *effective high-frequency gain* (HFG).

# 8.1 Problem statement (cont.)

- **Assumptions**

**(A1)** Minimum phase from $u$ to $\sigma$: $A_{11}$ is Hurwitz.

**(A2)** $S_p \in \mathbb{R}^{m\times m}$ is known so that $-K_p$ is Hurwitz, where

$$K_p := B_2 S_p \tag{11}$$

is the *effective high-frequency gain* (HFG).

**(A3)** $d_1(x,t)$ and $d_2(x,t)$ are locally Lipschitz in $x$, p.w.c. in $t$, and satisfy

$$\|d_1(x,t)\| \leq \bar{d}_1 < \infty\,, \quad \|d_2(x,t)\| \leq \bar{d}_2 < \infty\,, \quad \forall x \in \mathbb{R}^n\,, \quad \forall t \in \mathbb{R}^+\,, \tag{12}$$

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**(A4)** $\bar{d}_1 \geq 0$ and $\bar{d}_2 \geq 0$ are *unknown*.

# 8.1 Control performance specifications

**Definition (Performance Specifications)**

(1) $\|\sigma(t)\| \leq \|\sigma(0)\| + \Delta$, $\forall t \in [0\,, T)$, and

(2) $\|\sigma(t)\| \leq \varepsilon$, $\forall t \geq T$,

# 8.1 Control performance specifications (cont.)

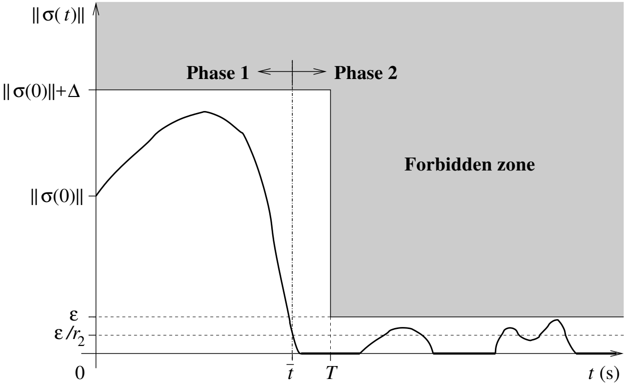


**FIGURE 1** Performance specifications on $\|\sigma(t)\|$.

# 8.2 Monitoring-based adaptive UVC

- How?: Monitoring-based adaptive Unit Vector Control (UVC).

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- In Phase 2 a class $\mathcal{K}_\infty\mathcal{L}$ function $\beta_2(k, t - \bar{t})$ guarantees the specified steady state.
- Two constants, $r_1, r_2 > 1$ are design constants, which can adjust the frequency of switchings.

## 8.2.1 The algorithm

Table: Adaptive sliding mode controller for the system (**??**)–(**??**).

| | |
|---|---|
| Unit vector control law | $u(t) = S_p U(t)\,, \qquad U(t) = -\rho(t)\frac{\sigma(t)}{\\|\sigma(t)\\|}$ |

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| $\rho$: modulation function | $\rho(t) = \rho_0(t) + \hat{d}(t)$ |
| Adaptive law | $\hat{d}(t) = \beta(k, t - \bar{t}) = \begin{cases} \beta_1(k)\,, & \text{if} \quad t < \bar{t}\,, \\ \beta_2(k, t - \bar{t})\,, & \text{if} \quad t \geq \bar{t}\,. \end{cases}$ |

# 8.2.1 The algorithm (cont.)

Table: Adaptive sliding mode controller for the system (**??**)–(**??**).

| | |
|---|---|
| In Phase 1 | $t_{k+1} := \min\limits_{y>t_k} \begin{cases} \lVert\sigma(t)\rVert = \lVert\sigma(0)\rVert + \Delta\left(1 - 1/r_1^k\right) \\ \text{or} \\ t = T\left(1 - 1/r_1^k\right) \quad \text{and} \quad \lVert\sigma(t)\rVert > \varepsilon/r_2 \end{cases}$ |

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| Phase 1 to 2 | $\bar{t} = \begin{cases} 0\,, & \text{if } \\|\sigma(\bar{0})\\| \leq \varepsilon/r_2\,, \\ t < T \ : \ \\|\sigma(t)\\| = \varepsilon/r_2\,, & \text{otherwise}\,. \end{cases}$ |

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| In Phase 2 | $t_{k+1} := \min\limits_{t>t_k} \begin{cases} \left\{\|\sigma(t)\| = \varepsilon\left(1 - 1/r_2^{k-j+1}\right)\right\}\,, & \text{if it exists}\,, \\ +\infty\,, & \text{otherwise}\,. \end{cases}$ |

# 8.4 Main result

**Fixed-time stability with guaranteed performance**

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## Fixed-time stability with guaranteed performance

- Fixed-time practical stabilization or tracking is achieved.

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- Fixed-time practical stabilization or tracking is achieved.
- The specified transient and steady-state behaviors are guaranteed.

(Hsu, Oliveira, J. P. V. S. Cunha, and Yan, 2019)

## Barrier function

A barrier function can be used instead of a monitoring function in Phase 2.
(Rodrigues, Hsu, Oliveira, and Fridman, 2022)

# 8.5 Tracking Control of a Surface Vessel

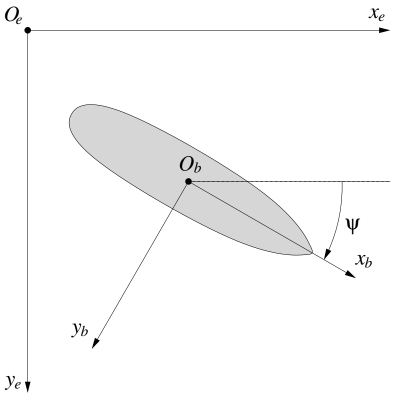


**FIGURE 2** Top view of the vessel and coordinate systems.

# 8.5 Tracking Control of a Surface Vessel (cont.)

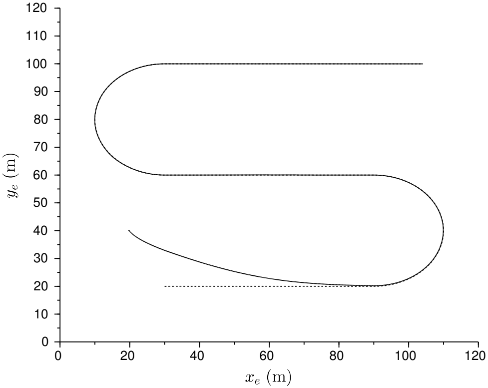


**FIGURE 3** Trajectory of the vessel on the water surface (solid line), and reference trajectory (doted line).

# 8.5 Tracking Control of a Surface Vessel (cont.)

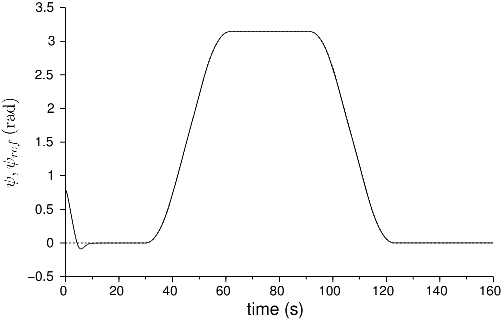


**FIGURE 4** Heading angle of the vessel (solid line), and reference heading angle (doted line).

# 8.5 Tracking Control of a Surface Vessel (cont.)

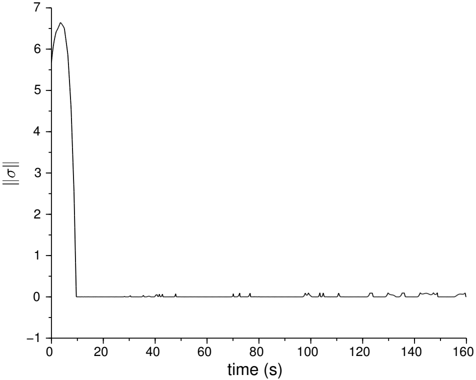


**FIGURE 5** Norm of the sliding variable with $\varepsilon = 0.1$.

# 8.5 Tracking Control of a Surface Vessel (cont.)

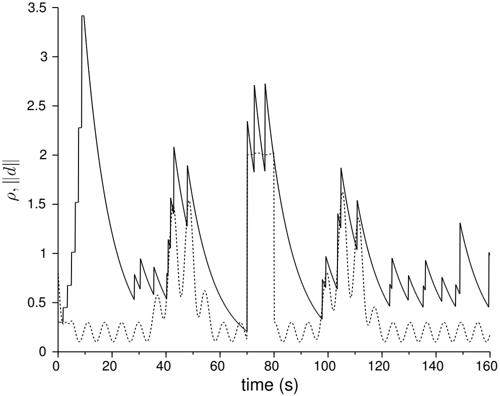


**FIGURE 6** Modulation signal ($\rho$ — solid line), and the norm of the disturbance ($\|d\|$ — doted line) with $\varepsilon = 0.1$.

# 8.6 Future research topics

- Analysis of monitoring function approach with unmodeled dynamics
- Analysis of monitoring function approach with sensor noise
- Numerical implementation issues

# 9. Prediction Error for Chattering Avoidance

# 9.1 SSC Background

- The SSC (Smooth Sliding Control) was first proposed in 1997 in order to alleviate chattering of VS-MRAC in the presence of unmodeled dynamics (Hsu, 1997).

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- Important difference: SSC admits non-exact plant model and unmeasured disturbances.

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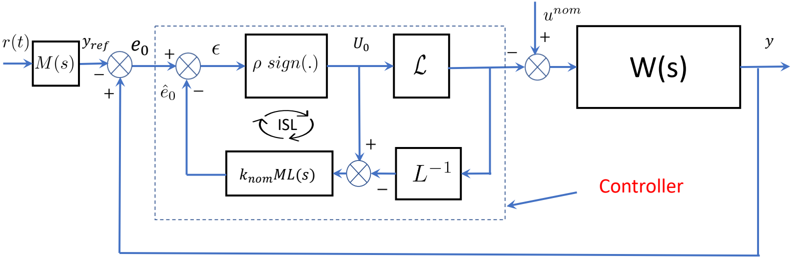
- Both approaches utilize a prediction error.
- Important difference: SSC admits non-exact plant model and unmeasured disturbances.
- New results on SSC with explicit chattering alleviation conditions (Oliveira, Hsu, and Nunes, 2022).

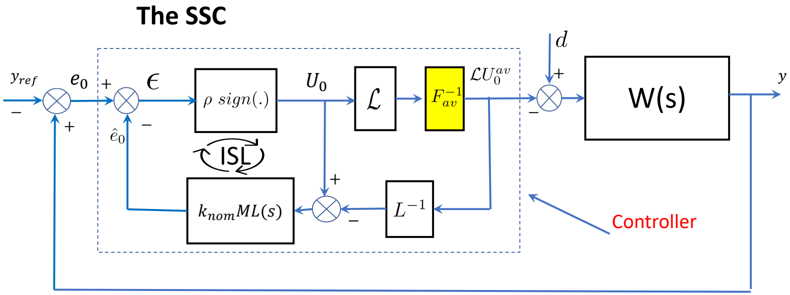
# 9.2 From VS-MRAC to SSC

**The VS-MRAC**



# 9.2 From VS-MRAC to SSC (cont.)

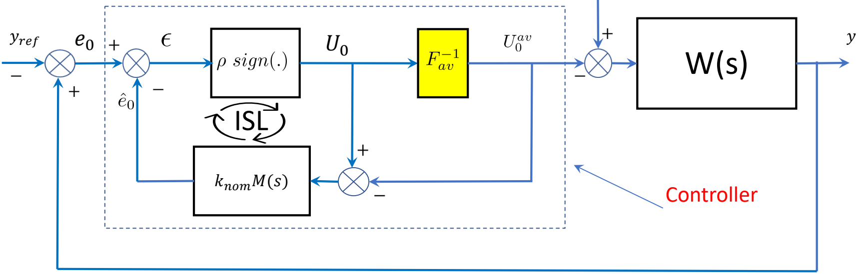
The Smooth Sliding Control: Introduce an averaging filter $F_{av}^{-1}$.




$\epsilon$ is the prediction error.

# 9.2 From VS-MRAC to SSC (cont.)

**The SSC: n*=1;** $\mathcal{L}$ =L = 1;




$$F_{av} = \tau s + 1 \qquad (\tau > 0)$$

# 9.4.1 SSC chattering alleviation example

Consider the plant

$$G(s) = \frac{K_p}{(s+1)} \frac{w_n^2}{(s^2 + 2\zeta w_n s + w_n^2)}$$

where the second rational function is a resonant underdamped parasitics ($\mu := w_n^{-1} << 1$).

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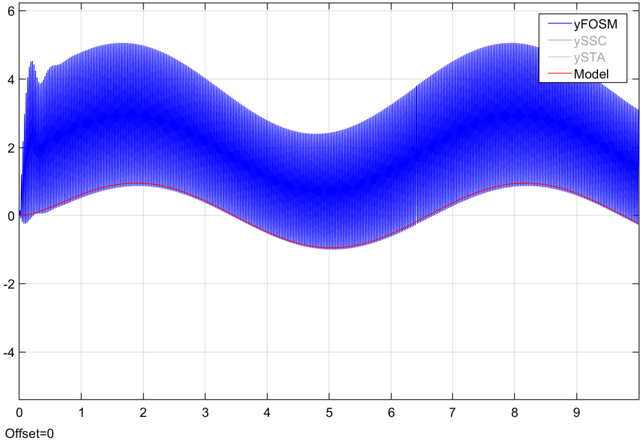
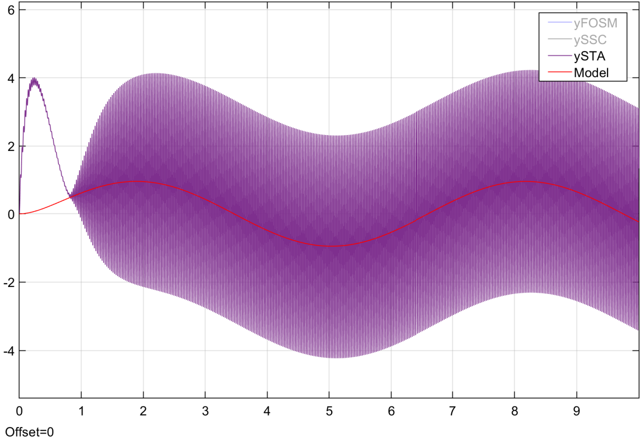
where the second rational function is a resonant underdamped parasitics ($\mu := w_n^{-1} << 1$).

In the simulations: $w \approx 240$ rd/s $\approx 40$ Hz, $\zeta = 0.025$

# .9.4.1 SSC example simulations




yFOSM
ySSC
ySTA
Model
6
4
2
0
-2
-4
0
1
2
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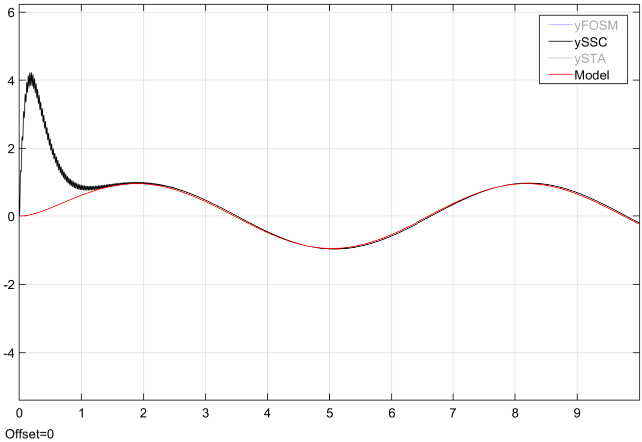
4
5
6
7
8
9
Offset=0


6
4
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4
5
6
7
8
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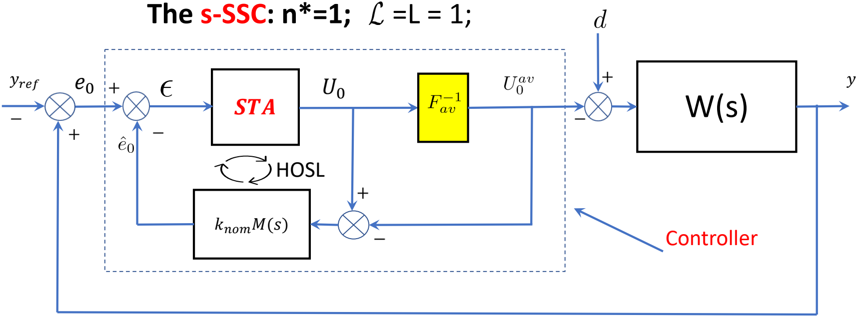
yFOSM
ySSC
ySTA
Model
Offset=0

# 9.5 The super-SSC (s-SSC)

**The s-SSC: n*=1;** $\mathcal{L}$ =L = 1;




$$F_{av} = \tau s + 1 \qquad (\tau > 0)$$

SSC works well with the Super Twisting Algorithm (STA)

# 9.5.1 super-SSC example

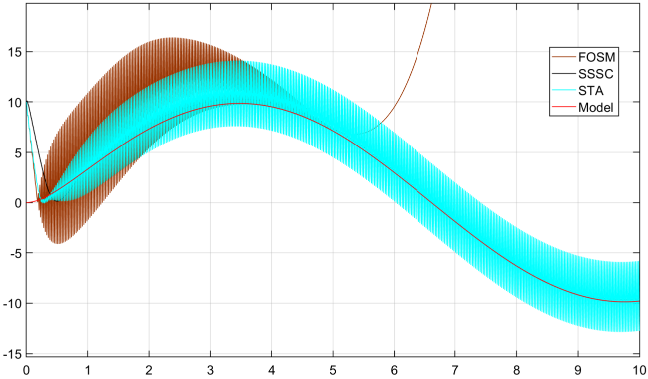
Unstable plant

$$G(s) = \frac{K_p}{(s-1)} \frac{w_n^2}{(s^2 + 2\zeta w_n + w_n^2)}$$

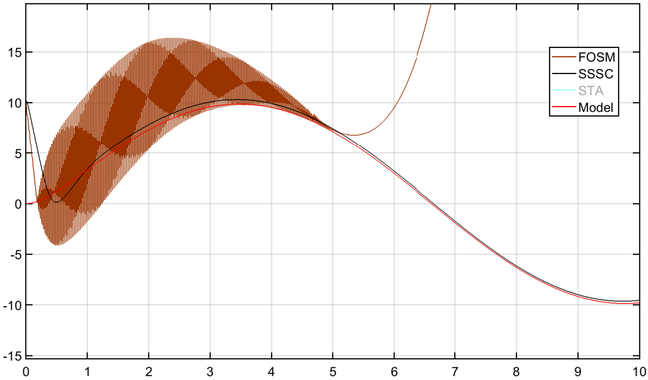
and ramp disturbance.

**Remark:** First Order Sliding Mode with constant gain looses stability.

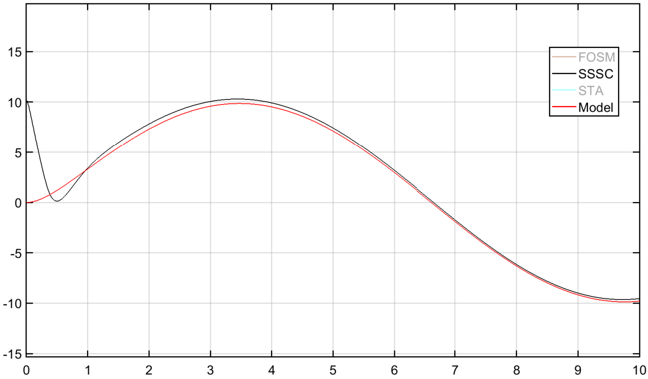
# 9.5.1 super-SSC example (cont.)



# 9.5.1 super-SSC example (cont.)



# 9.5.1 super-SSC example (cont.)



# Acknowledgements


- Funding: CNPq, FAPERJ, CAPES, Petrobras
- Support: COPPE-Federal University of Rio de Janeiro

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E. Kaszkurewicz, J.I. Rego, Maria Chan, P. Resende, L.D. Colvara, R. Costa, R. Guenther, A.D. Araújo, G. Damm, M.S. Netto, F. Lizarralde, J.A. Real, P.L. Aquino, L.J. Min, A.K. Imai, J.P.V.S. Cunha, A.R.L. Zacchi, A.J. Peixoto, E.V.L. Nunes, T.R. Oliveira, A.C. Leite, A.R. Pereira, A.G.H. Battistel, G.M. Freitas, P.V.N.M. Vidal, H.L.C.P. Pinto, N.O. Aminde, V.H.P. Rodrigues

## Colleagues & Collaborators

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