Università degli Studi di Cagliari
Dip. di Ingegneria Elettrica ed Elettronica

VSS Rio 2022 international workshop

Robust distributed control of multi-agent systems with application to electrical networks

Elio USAI
elio.usai@unica.it

All we're saying is
give peace a chance
(J. Lennon 1975)

Summary

- Introduction to Multi-Agent Systems
- Centralised vs Decentralised control
- Consensus in Decentralised control
- Decentralised Leader-following
- Robust Consensus
- Consensus in electrical networks

Introduction to Multi-Agent Systems

Multi-Agents System: A complex system composed by several interconnected agents that exchange information and/or materials

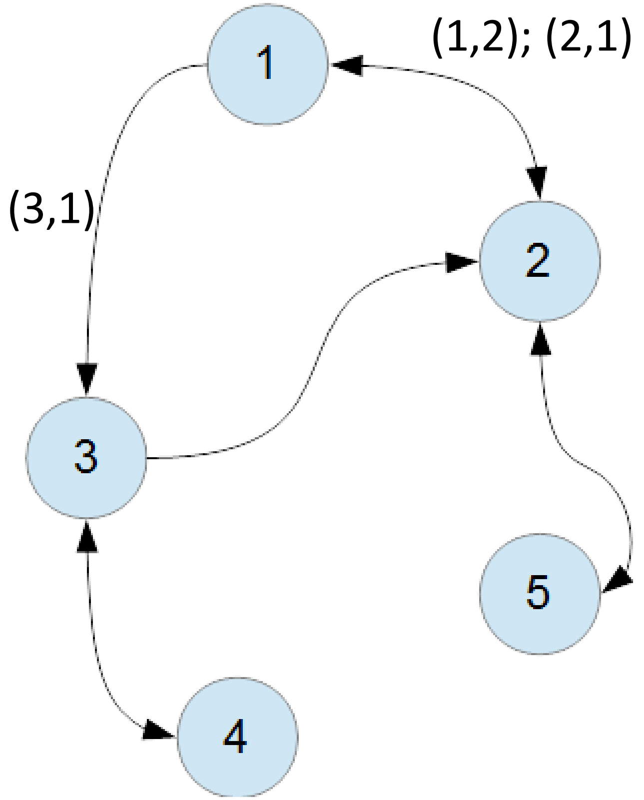
$$G = (V, E, A)$$

$$V = \{1, 2. 3, 4, 5\}$$

$(i,j) \in E \Rightarrow$ agent i takes info from agent j

$$E = \{(1,2),(2,1),(2,3),(2,5),(3,1),(3,4),(4,3),(5,2)\}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

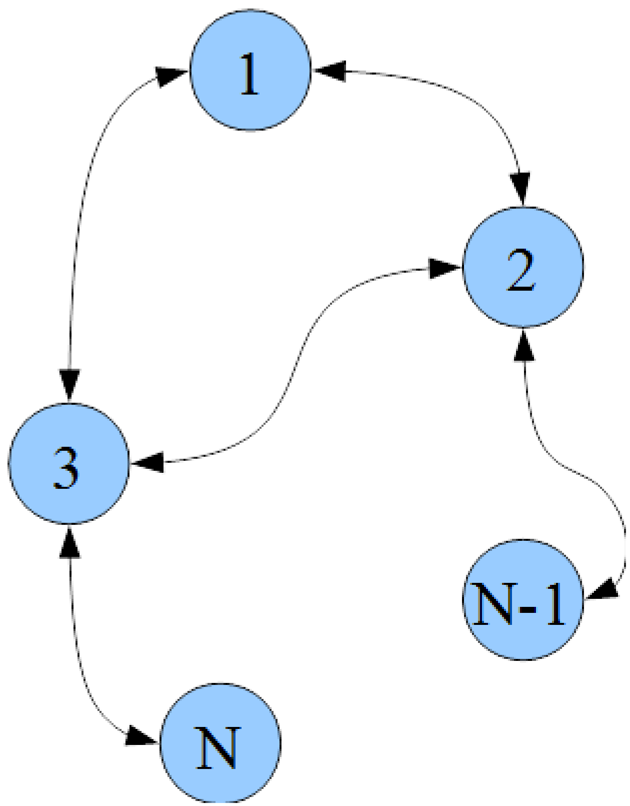


Introduction to Multi-Agent Systems

Multi-Agents System: A complex system composed by several interconnected agents that exchange information and/or materials

$$G = (V, E, A)$$

$V = \{1, 2, \cdots, N\}$	vertex set
$E \subseteq V \times V$	edge set
$A = [a_{i,j}] \in \mathbb{R}_{\geq 0}^{N \times N}$	adjacency matrix
$\mathcal{N}_{in} = \{j \in V : (j, i) \in E\}$	set of in – neighbourhoods of vertex i
$\mathcal{N}_{out} = \{j \in V : (i, j) \in E\}$	set of out – neighbourhoods of vertex i
$D_{in} = diag(A^T \cdot 1_N)$	in – degree matrix
$D_{out} = diag(A \cdot 1_N)$	out – degree matrix
$L = [l_{i,j}] = D_{out} - A \in \mathbb{R}^{N \times N}$	laplacian matrix



Introduction to Multi-Agent Systems

Multi-Agents System: A complex system composed by several interconnected agents that exchange information and/or materials

$$G = (V, E, A)$$

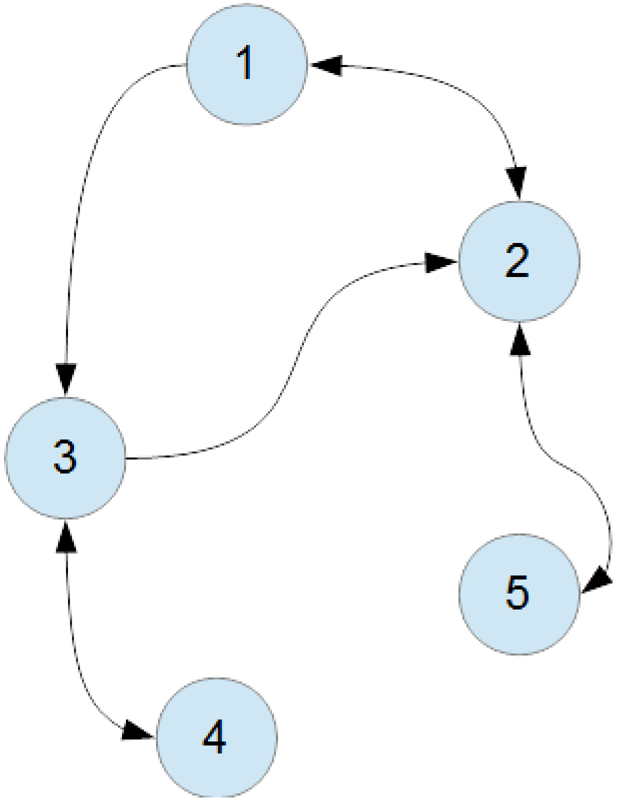
The Laplacian matrix is semi-definite positive

$$eig(L) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$$

$$0 = \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_N$$

λ_2 gives info on the graph connectivity

$\lambda_2 > 0 \Rightarrow$ the graph is connected



Introduction to Multi-Agent Systems

Multi-Agents System: A complex system composed by several interconnected agents that exchange information and/or materials

$$G = (V, E, A)$$

The Laplacian matrix is semi-definite positive

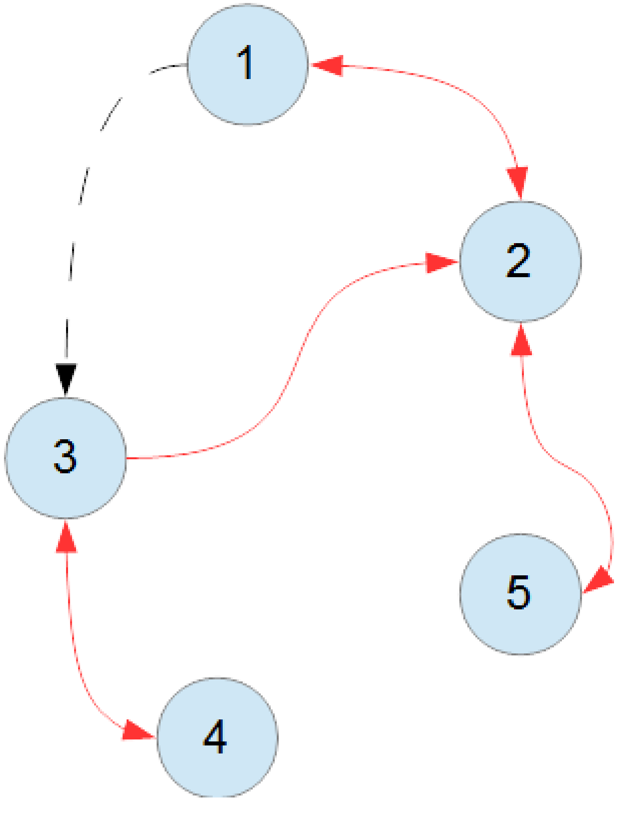
$$eig(L) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$$

$$0 = \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_N$$

λ_2 gives info on the graph connectivity

$\lambda_2 > 0 \Rightarrow$ the graph is connected

The existence of spanning-trees implies connectivity



Introduction to Multi-Agent Systems

Multi-Agents System: A complex system composed by several interconnected agents that exchange information and/or materials

$$G = (V, E, A)$$

A graph is undirected if

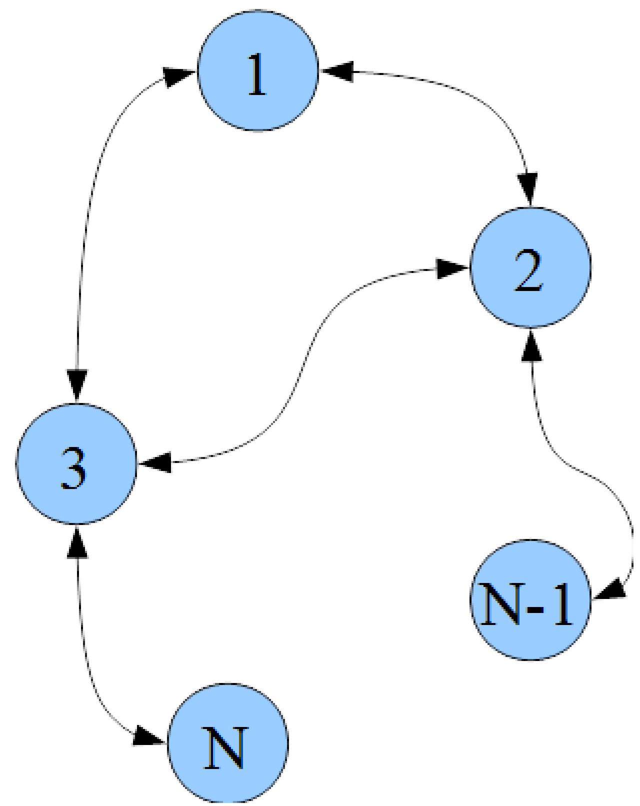
$$(i,j) \in E \Leftrightarrow (i,i) \in E$$

$$\Downarrow$$

matrices A and L are symmetric

if the connected agents have the same "authority"

$$a_{i,j} = a_{j,i}$$



Introduction to Multi-Agent Systems

Agents' dynamics: each agent's dynamics depends on the interaction with its in-neighbourhoods

$$\dot{\mathbf{x}}_i(t) = \boldsymbol{F}_i(\boldsymbol{x}_i(t),t) + \boldsymbol{G}_i(\boldsymbol{x}_i(t),t) \cdot \boldsymbol{u}_i + \sum_{j \in \mathcal{N}_i} a_{j,i} \boldsymbol{\Gamma}_{\mathrm{i,j}}(\boldsymbol{x}_i(t))$$

$\boldsymbol{x}_i$: the n_i-dimensional state of the i^{th} agent

$\boldsymbol{u}_i$: the m_i-dimensional control of the i^{th} agent

$\boldsymbol{F}_i$: the n_i-dimensional vector field of the i^{th} agent's autonomous dynamics

$\boldsymbol{G}_i$: the n_ixm_i-dimensional gain matrix of the i^{th} agent

Γ_i: the n_ixm_j-dimensional vector function of the j^{th} agent's influence on the i^{th} agent

$a_{j,i}$: the weight of the connection between the j^{th} and i^{th} agents

Introduction to Multi-Agent Systems

Agents' dynamics: each agent's dynamics depends on the interaction with its in-neighbourhoods

$$\dot{\mathbf{x}}_i(t) = \boldsymbol{F}_i(\boldsymbol{x}_i(t),t) + \boldsymbol{G}_i(\boldsymbol{x}_i(t),t) \cdot \boldsymbol{u}_i + \sum_{j \in \mathcal{N}_i} a_{j,i} \boldsymbol{\Gamma}_{\mathrm{i,j}}(\boldsymbol{x}_i(t))$$

The state of the in-neighbourhoods generally affects the dynamics of the agent in an unmatched way

$$\boldsymbol{\Gamma}_{i,j}(\boldsymbol{x}_j(t), t) \neq \boldsymbol{G}_i\left(\boldsymbol{x}_i(t), t\right) \cdot \boldsymbol{\lambda}_j\left(\boldsymbol{x}_{\mathrm{j}}(\mathrm{t}), \mathrm{t}\right)$$

The control and estimation problems become complex

Introduction to Multi-Agent Systems

Collective Multi-Agents System: the overall behaviour can have a high-dimensional dynamics

$$\dot{x} = F(x(t), t) + diag\{G_i(x(t), t)\} \cdot u(\mathrm{t}) + A \otimes \Gamma(x(t))$$

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix} \quad u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_N(t) \end{bmatrix} \quad F(x(t), t) = \begin{bmatrix} F_1\left(x_1(t), t\right) \\ \vdots \\ F_N\left(x_N(t), t\right) \end{bmatrix}$$

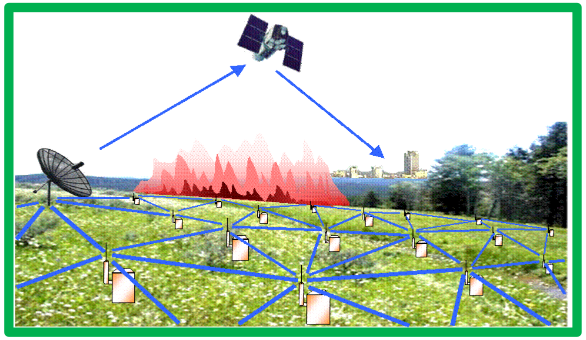
$$\Gamma(x(t)) = \begin{bmatrix} 0 & \Gamma_{1,2}\left(x_2(t)\right) & \dots & \Gamma_{1,N}\left(x_N(t)\right) \\ \Gamma_{2,1}\left(x_1(t)\right) & 0 & & \Gamma_{2,N}\left(x_N(t)\right) \\ \vdots & & \ddots & \vdots \\ \Gamma_{N,1}\left(x_1(t)\right) & \dots & \Gamma_{N,N-1}\left(x_{N-1}(t)\right) & 0 \end{bmatrix}$$

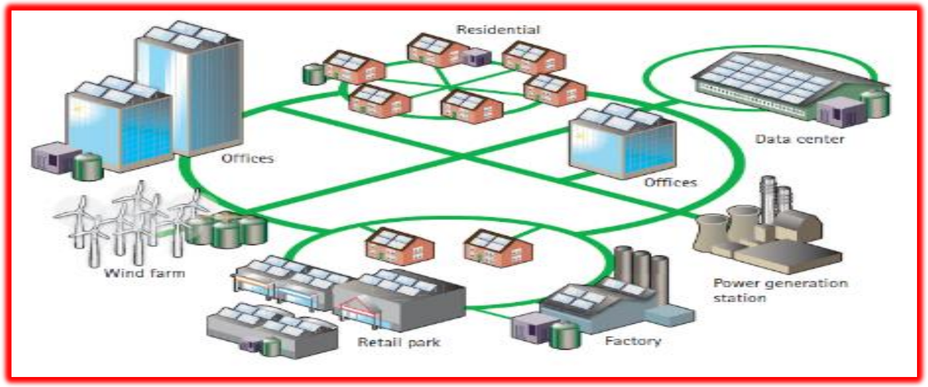
The control and estimation problems become complex

Introduction to Multi-Agent Systems

Multi-Agents System: many examples in practice

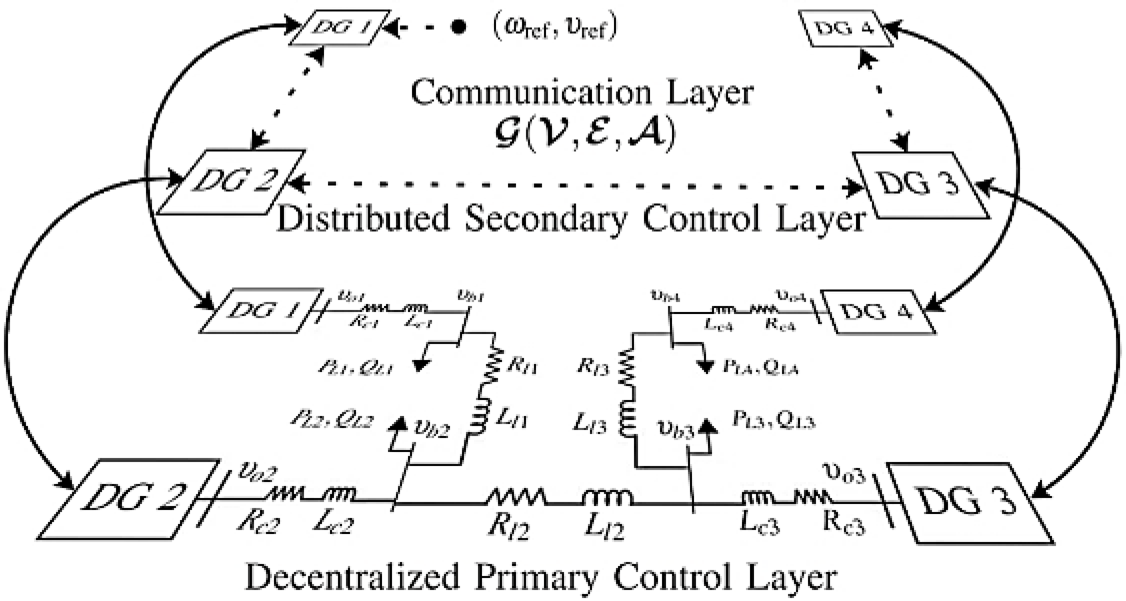
- Power distribution (i.e. **Smart Grids**)
- Coordination of mobile robots (i.e. **UAVs**)
- Sensing in wide areas (i.e. **WSN**)
- Social Networks (**opinion dynamics**)
- Traffic control
- Communication networks (routing)

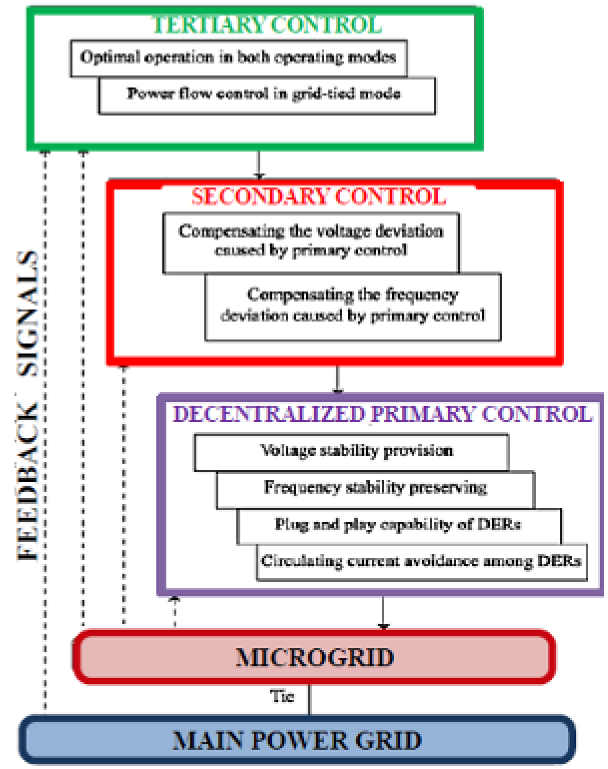




Introduction to Multi-Agent Systems

Multi-Agents System: many examples in practice





Different graphs can represent the physical interaction and the communication among the agents

Introduction to Multi-Agent Systems

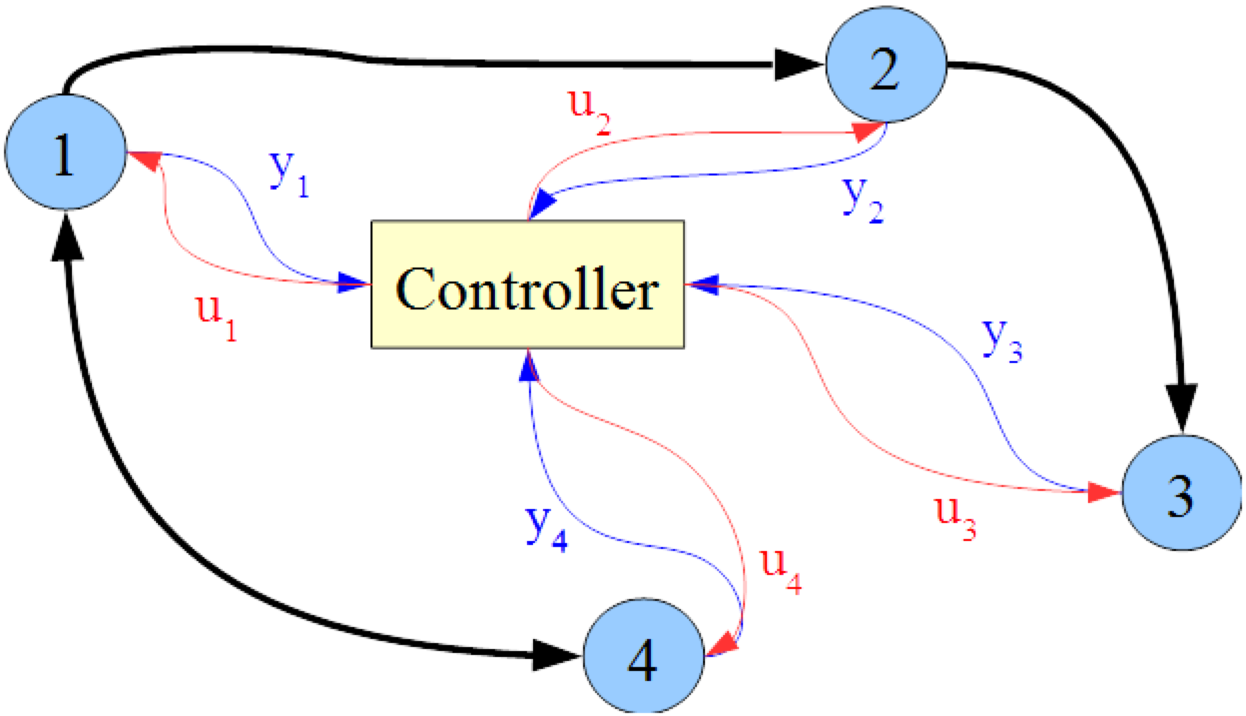
Multi-Agents System: the overall behaviour can have a high-dimensional dynamics

$$\dot{x} = F(x(t),t) + diag\{G_i(x(t),t)\} \cdot u(\mathrm{t}) + A \otimes \Gamma(x(t))$$

- The overall dynamics **IS NOT** a combination of the single agents' dynamics
- Emerging behaviours appear in uncontrolled MAS
- A centralised state-feedback control could solve the problem of imposing a behaviour to the MAS

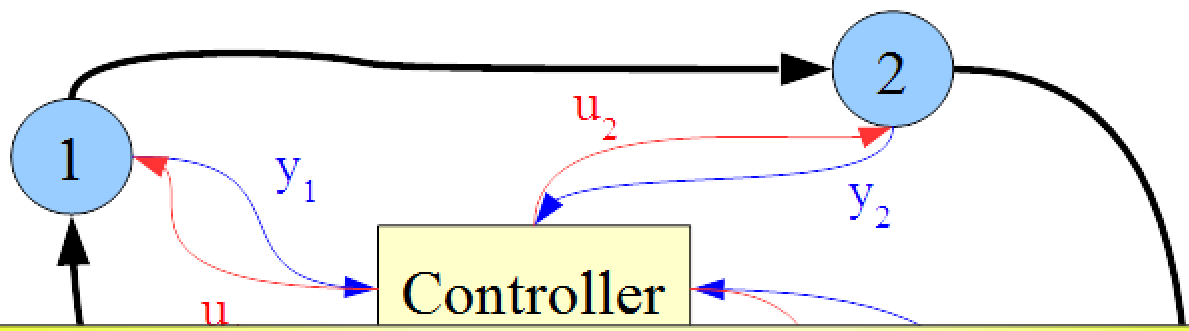
Centralised vs Decentralised control

Centralised control: All measurements are collected by a central agent that defines the control law for all the connected agents considering the overall system condition



Centralised vs Decentralised control

Centralised control: All measurements are collected by a central agent that defines the control law for all the connected agents considering the overall system condition

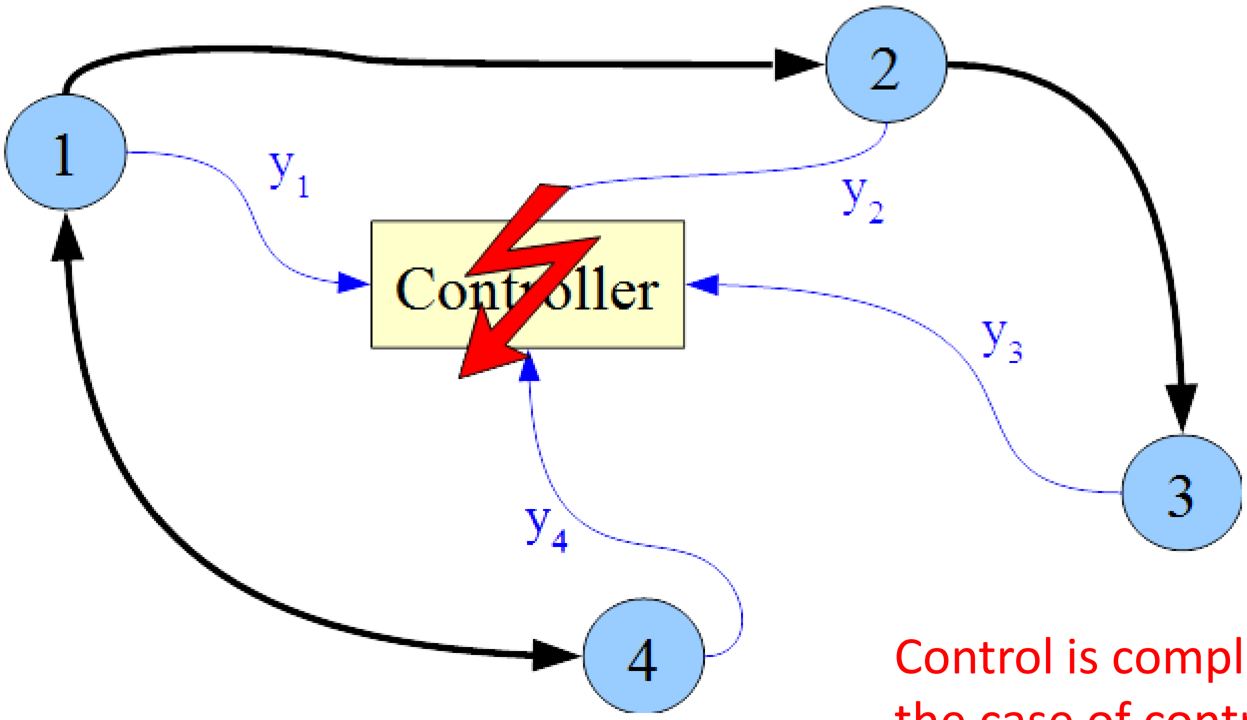


Optimization can be obtained since the controller can consider all the information that are available

$$J(\hat{\boldsymbol{u}}) = \min_{\boldsymbol{u} \in U} \int_{t=t_0}^{t} L(\boldsymbol{y};\boldsymbol{u})\,dt$$

Centralised vs Decentralised control

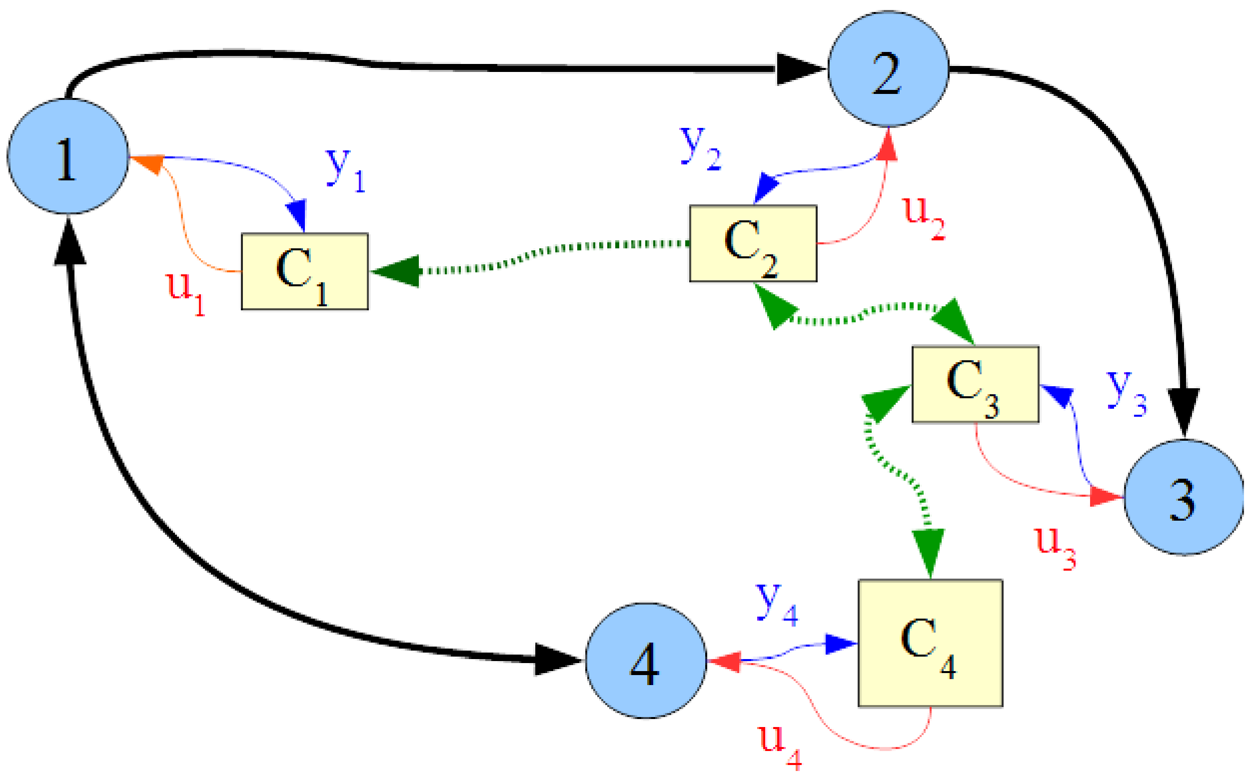
Centralised control: All measurements are collected by a central agent that defines the control law for all the connected agents considering the overall system condition



Control is completely lost in the case of controller's fault

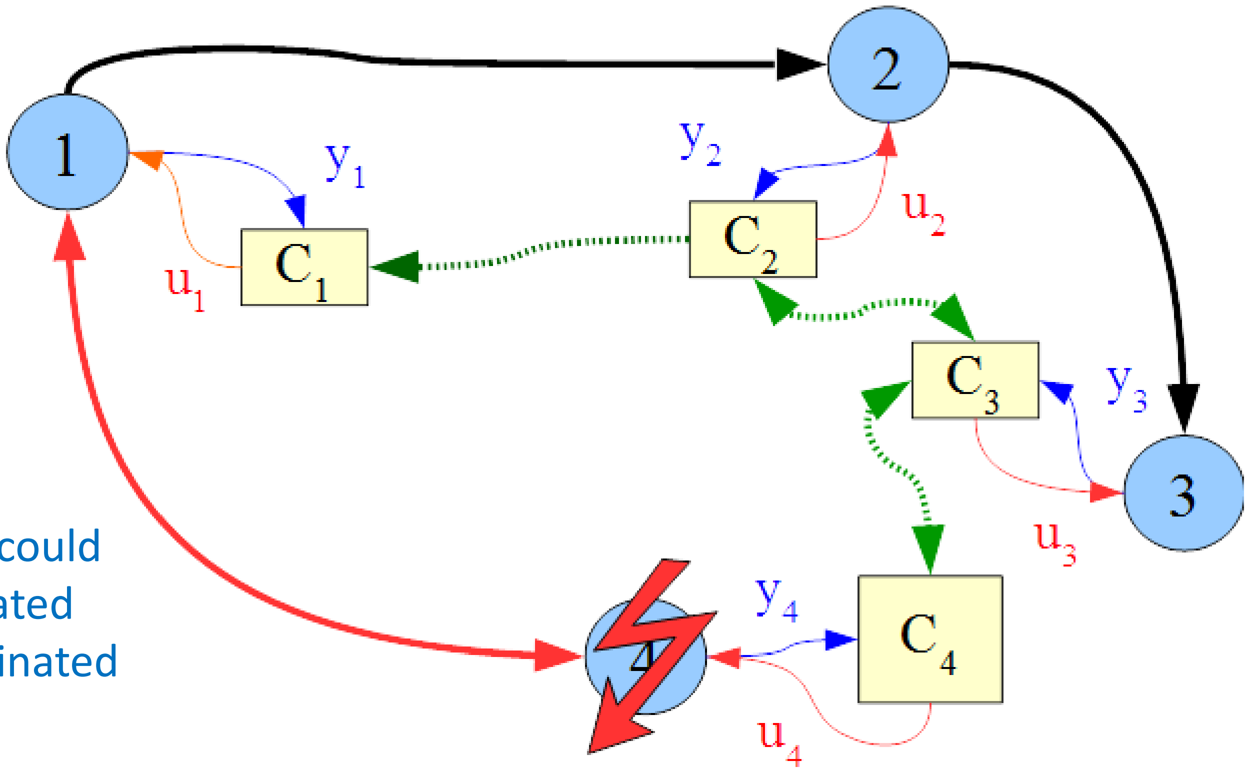
Centralised vs Decentralised control

Decentralised control: Each agent has its own control based on its own measurement and some data from its neighbors; same information are shared



Centralised vs Decentralised control

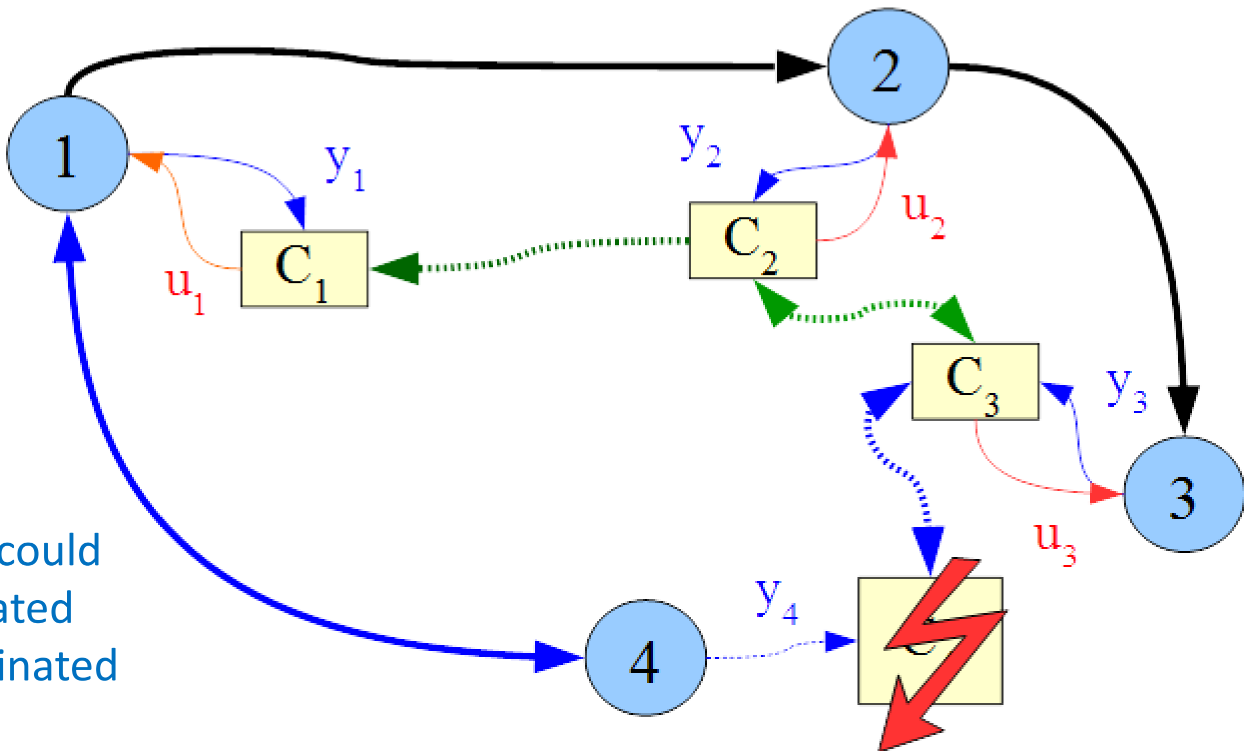
Decentralised control: Each agent has its own control based on its own measurement and some data from its neighbors; same information are shared



Single faults could be compensated by the coordinated actions

Centralised vs Decentralised control

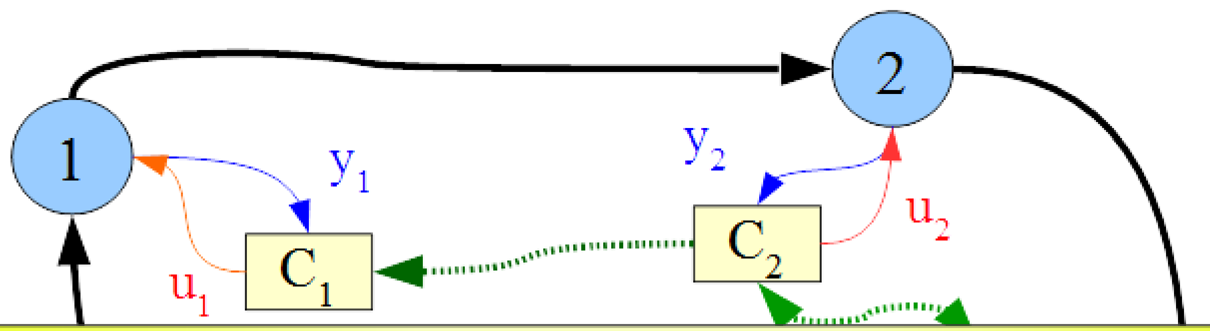
Decentralised control: Each agent has its own control based on its own measurement and some data from its neighbors; same information are shared



Single faults could be compensated by the coordinated actions

Centralised vs Decentralised control

Decentralised control: Each agent has its own control based on its own measurement and some data from its neighbors; same information are shared



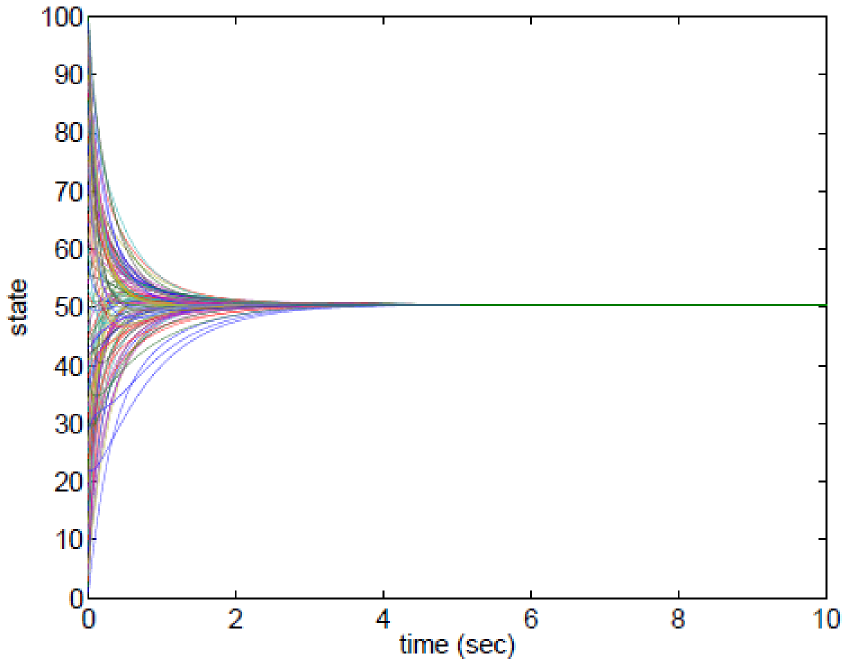
Optimization be can obtained by combining the optimization of a set of locally defined indexes, taking into account neighbors

$$J(\hat{\boldsymbol{u}}) = \sum_{i=1}^{N} \min_{\boldsymbol{u}_i \in U_i} \int_{t=t_0}^{t} L_i(\boldsymbol{y}_i; \boldsymbol{u}_i; \boldsymbol{x}_j)\, dt$$

Consensus in Decentralised control

Consensus: all agents converge to a common behaviour

$$\lim_{t \to \infty} \left\| x_i - x_j \right\| = 0; \qquad \forall i \neq j; \;\; i, j = 1, \ldots, N$$



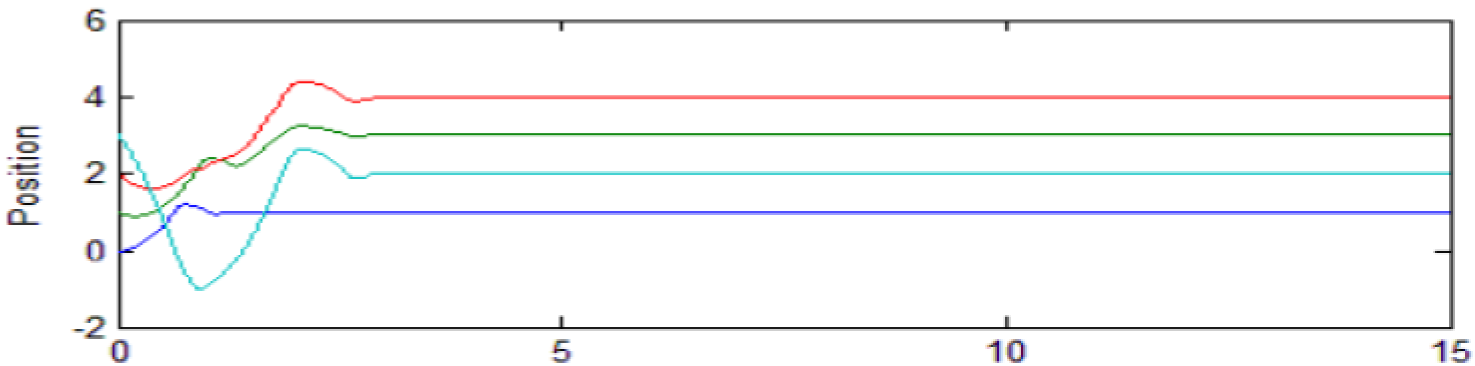
Consensus in Decentralised control

Consensus: all agents converge to a common behaviour

$$\lim_{t \to \infty} \left\| x_i - x_j \right\| = 0; \qquad \forall i \neq j; \;\; i,j = 1, \ldots, N$$

In case of formation control the systems' state don't converge to a common value but a prescribed offset is required

$$\lim_{t \to \infty} \left\| x_i - x_j \right\| = d_{i,j}; \qquad \forall i \neq j; \;\; i,j = 1, \ldots, N$$



Consensus in Decentralised control

Consensus: all agents converge to a common behaviour

$$\lim_{t \to \infty} \left\| x_i - x_j \right\| = 0; \qquad \forall i \neq j;\ \ i,j = 1, \ldots, N$$

In case of formation control the systems' state don't converge to a common value but a prescribed offset is required

$$\lim_{t \to \infty} \left\| x_i - x_j \right\| = d_{i,j}; \qquad \forall i \neq j;\ \ i,j = 1, \ldots, N$$

Wei Ren, Randal W. Beard, Ella M. Atkins. A Survey of Consensus Problems in Multi-agent Coordination, Proceedings of the American Control Conference, Volume 3, Pages 1859 – 1864, 2005.

Reza Olfati-Saber, J. Alex Fax, Richard M. Murray . Consensus and Cooperation in Networked Multi-Agent Systems, Proceedings of the IEEE, Volume: 95, Issue: 1, January 2007.

Consensus in Decentralised control

Some references

- Wei Ren, Yongcan Cao. *Distributed Coordination of Multi-agent Networks. Emergent Problems, Models, and Issues*. Springer, 2011
- M. Mesbahi, M. Egerstedt. *Graph Theoretic Methods for Multiagent Networks*, Princeton University Press, Princeton, NJ, 2010
- R. Olfati-Saber, RM. Murray. Consensus problems in networks of agents with switching topology and time-delays, IEEE Trans Aut Contr, 2004
- R. Olfati-Saber, “Flocking for multi-agent dynamic system: Algorithms and theory,” IEEE Trans. Autom. Control, vol. 51, pp. 401–420, 2006.
- W. Ren, R. W. Beard, and E. M. Atkins, “Information consensus in multivehicle cooperative control: Collective group behavior through local interaction,” IEEE Control Syst. Mag., vol. 27, no. 2, pp. 71–82, 2007.
- Fagnani F., Zampieri S., Randomized consensus algorithms over large scale networks, IEEE Journal on Selected Areas in Communications, 26(4), pp. 634-649, 2008
- Dörfler F., Bullo F., Synchronization in complex networks of phase oscillators: A survey, Automatica, 50(6), pp. 1539-1564, 2014

Consensus in Decentralised control

The most simple consensus decentralised control problem refers to a single or double integrator agent

$$\dot{x}_i = u_i \qquad \text{or} \qquad \ddot{x}_i = u_i \qquad i=1,2,\ldots,N$$

- Single integrator: *velocity driven agent*
- Double integrator: *acceleration driven agent*

In the case of **single integrators**, the control of each agent can be defined to "modify" the strength of the connection between neighbour agents

$$u_i = -\sum_{j \in \mathcal{N}_{out_i}} a_{j,i}\left(x_i - x_j\right) \qquad i = 1,2,\ldots,N$$

The MAS dynamics results into

$$\dot{\boldsymbol{x}} = -L \cdot \boldsymbol{x}$$

A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," IEEE Trans. on Automatic Control, vol. 48, pp. 988–1001, June 2003

Consensus in Decentralised control

Considering single integrator agents

The MAS dynamics results into

$$\dot{x} = -L \cdot x$$

Because of the semi-positive definiteness of the Laplacian matrix L, the state of a **connected MAS** converge to a, generally not zero, globally exponentially stable state

$$x \rightarrow \alpha \cdot \mathbf{1};$$

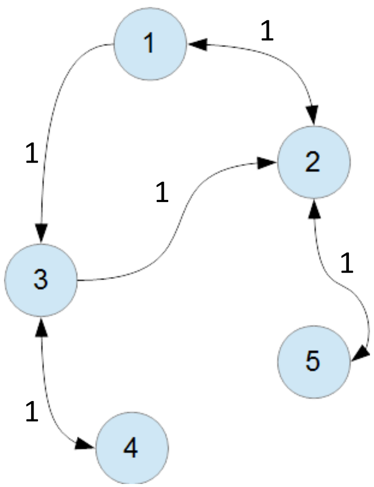
The α value depends on the coefficients of the adjacency matrix A

Consensus in Decentralised control

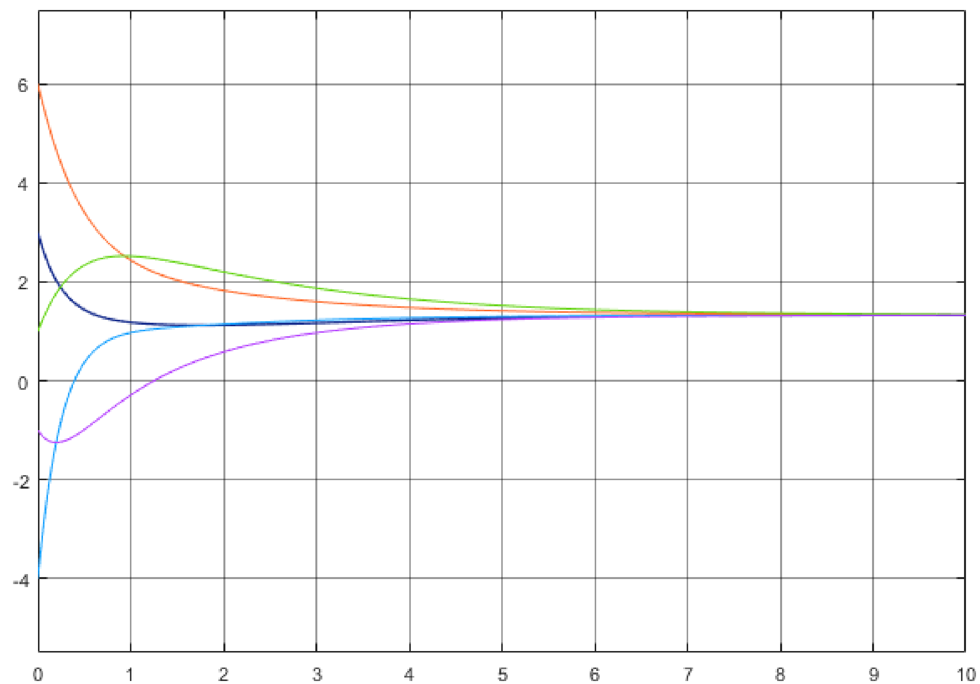
Considering single integrator agents

The MAS dynamics results into

$$\dot{x} = -L \cdot x$$



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

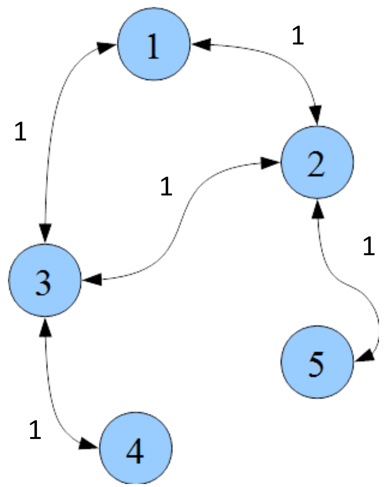


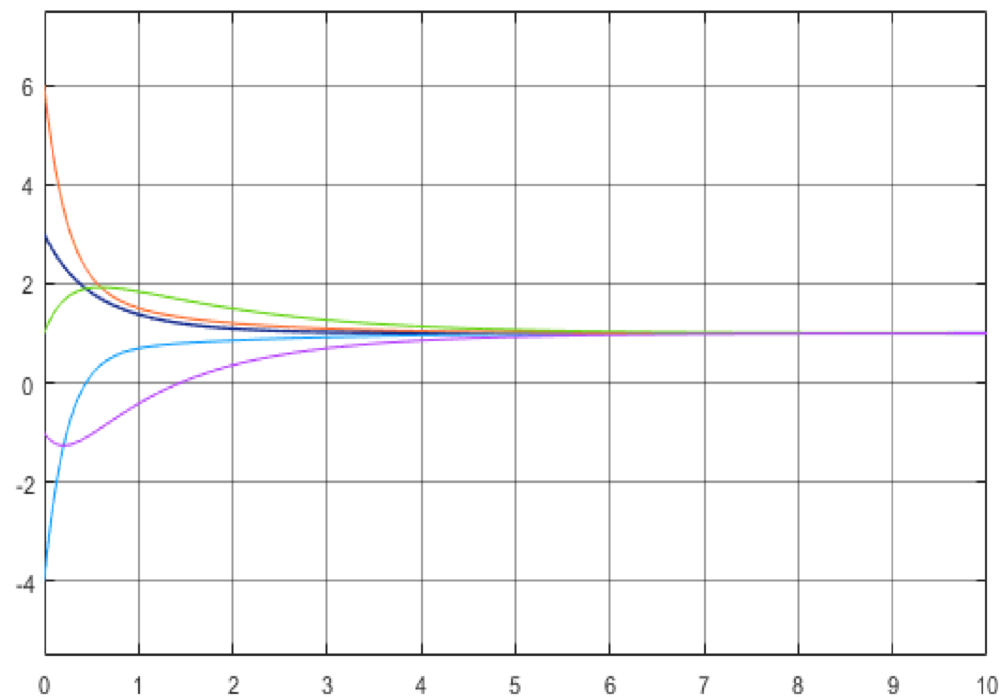
Consensus in Decentralised control

Considering single integrator agents

The MAS dynamics results into

$$\dot{x} = -L \cdot x$$





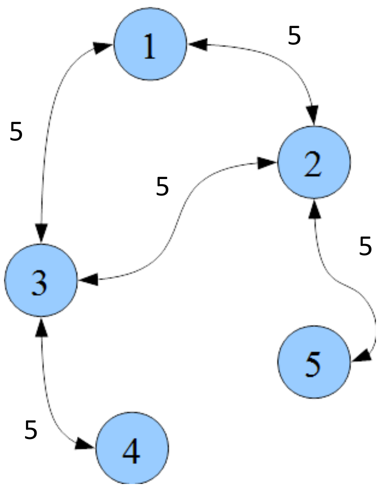
If the graph is *strongly connected* and *doubly stochastics* α is the **mean value** of the agents' initial state

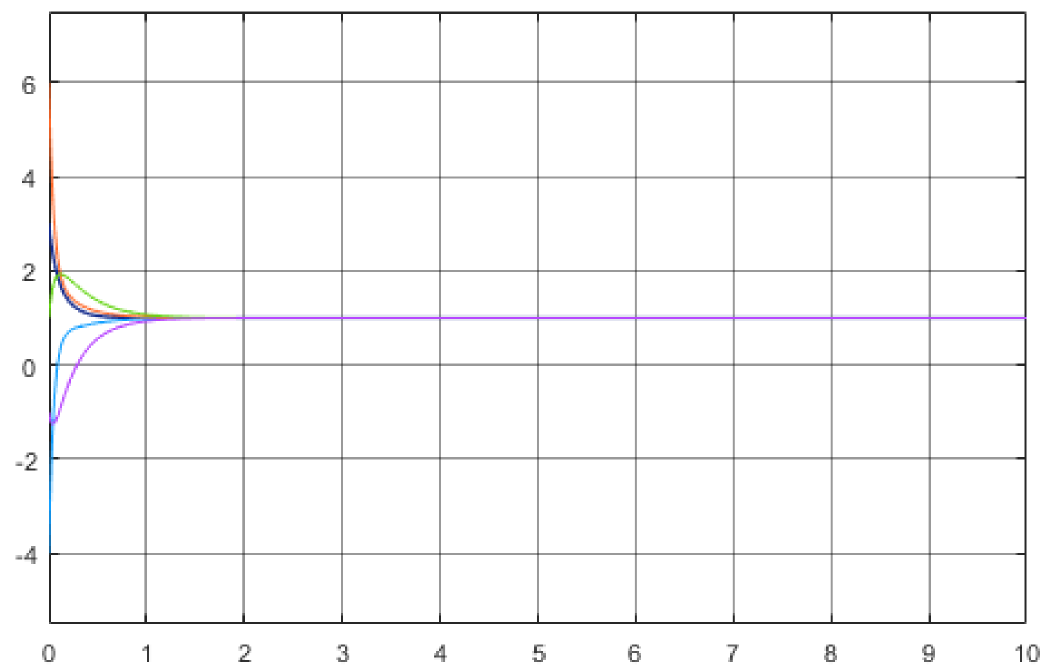
Consensus in Decentralised control

Considering single integrator agents

The MAS dynamics results into

$$\dot{x} = -L \cdot x$$





The larger the coupling coefficients the shorter the convergence, i.e λ_2 increases

Consensus in Decentralised control

Considering single integrator agents

The MAS dynamics results into

$$\dot{\boldsymbol{x}} = -L \cdot \boldsymbol{x}$$

Stability can be proved by a disagreement function

$$\varphi(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T L \boldsymbol{x} = \frac{1}{2}\sum_{(i,j)\in E} a_{i,j}\left(x_j - x_i\right)^2$$

$$\dot{\varphi}(\boldsymbol{x}) = \nabla\varphi(\boldsymbol{x}) \cdot (-L\boldsymbol{x}) = \dot{\mathbf{x}} L \boldsymbol{x}$$

$$\Downarrow$$

$$\dot{\mathbf{x}} = -\nabla\varphi(\mathbf{x})$$

Gradient descent algorithm on a convex definite positive function

J. Cortés, "Finite-time convergent gradient flows with applications to network consensus," *Automatica*, vol. 42, no. 11, pp. 1993–2000, 2006.

Consensus in Decentralised control

Considering double integrator agents

The MAS dynamics results into

$$\dot{x}_{i,1} = x_{i,2} \qquad i = 1, 2, \ldots, N$$
$$\dot{x}_{i,2} = u_i$$

Also in the case of **double integrators**, the control of each agent can be defined to "modify" the strength of the connection between neighbour agents , which exchange both position and velocity variables

$$u_i = -\sum_{i \in \mathcal{N}_{out_i}} a_{j,i}\left(x_{i,1} - x_{j,1}\right) - \lambda \sum_{i \in \mathcal{N}_{out_i}} a_{j,i}\left(x_{i,2} - x_{j,2}\right)$$

W. Ren and E. M. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," Int. J. Robust Nonlinear Contr., vol. 17, no. 10–11, pp. 1002–1033, Jul. 2007

Consensus in Decentralised control

Considering double integrator agents

The MAS dynamics results into

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -\lambda L \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}$$

Since the Laplacian L is a semi-definite positive matrix with a unique zero eigenvalue if the graph is fully connected, the MA dynamics' matrix has a double eigenvalue in zero and all the others have negative real part.

$$\boldsymbol{x}_1 \to \alpha_1 \cdot \mathbf{1} + \alpha_2 \cdot \mathbf{1} t$$

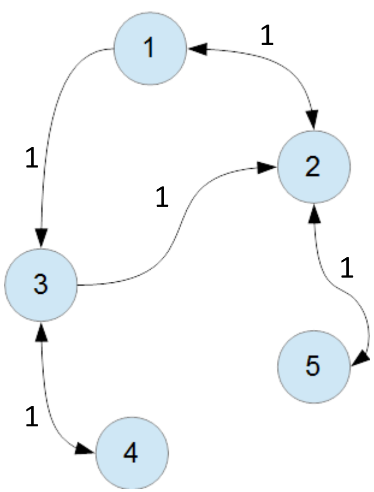
$$\boldsymbol{x}_2 \to \alpha_2 \cdot \mathbf{1}$$

Coefficients α_1 and α_2 are weighted values of the initial position and velocity of the agents, and depend on the entries of matrix *A*

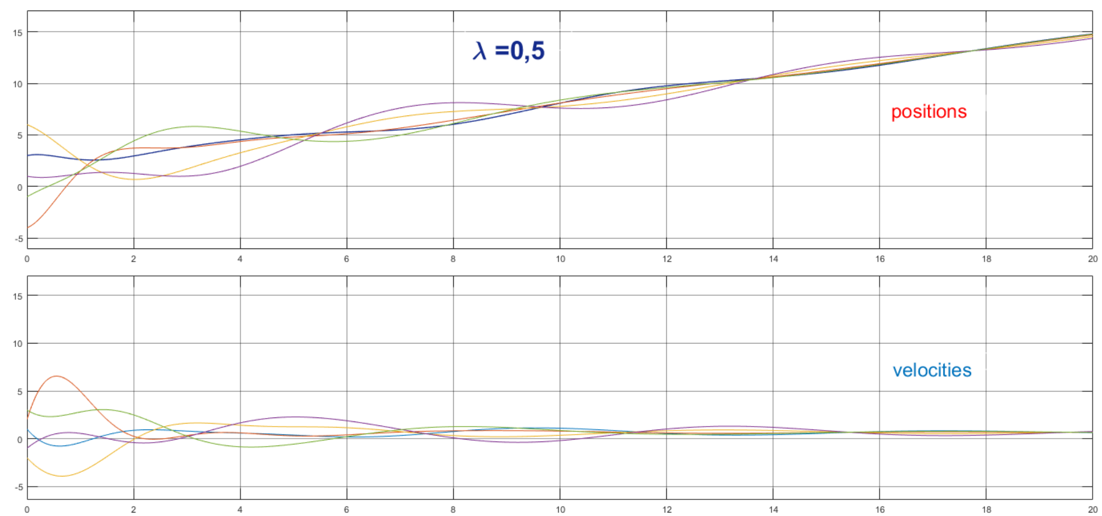
Consensus in Decentralised control

Considering double integrator agents

The MAS dynamics results into



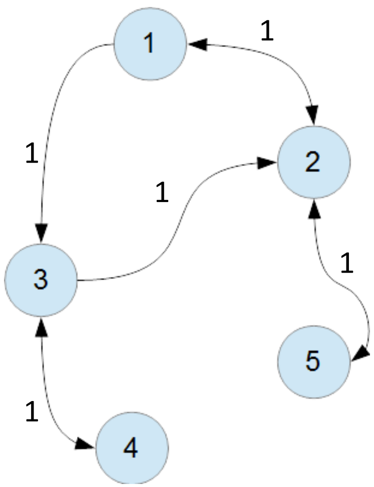
$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -\lambda L \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}$$



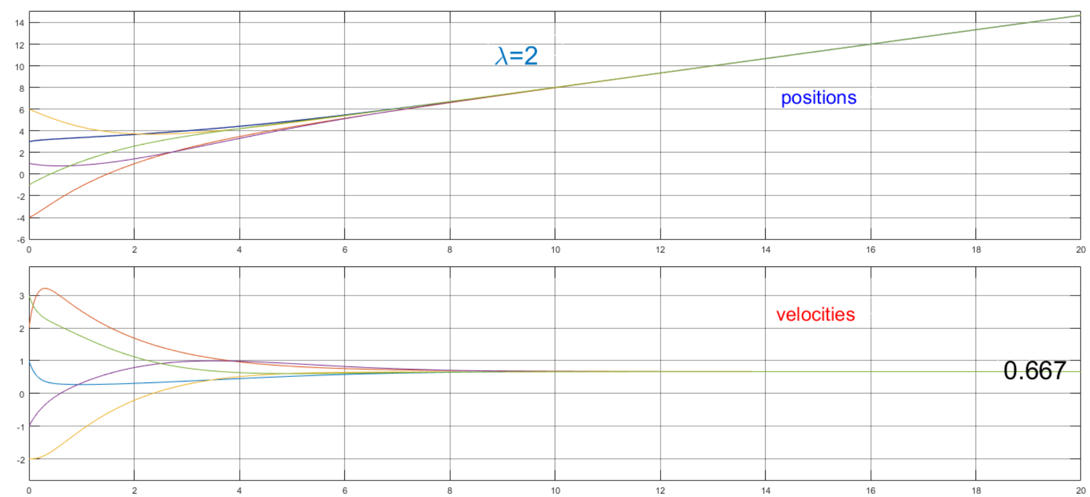
Consensus in Decentralised control

Considering double integrator agents

The MAS dynamics results into



$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -\lambda L \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

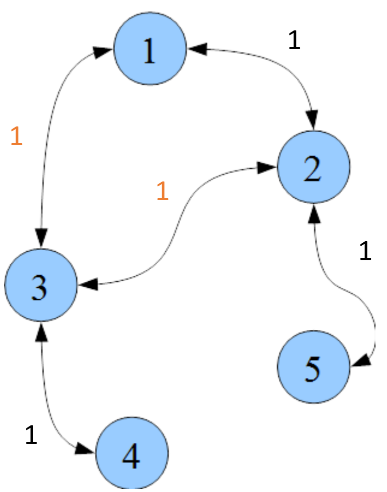


Parameter λ affects the transient behaviour

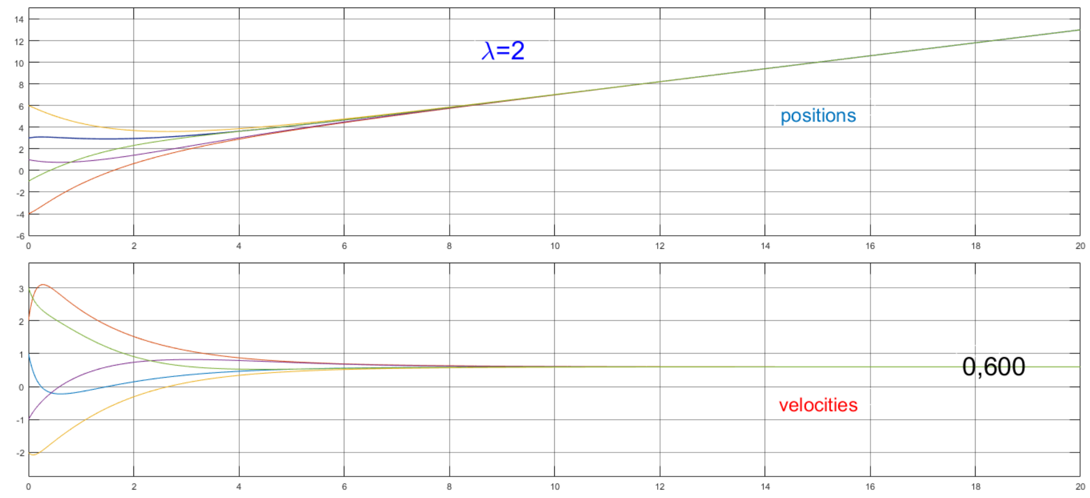
Consensus in Decentralised control

Considering double integrator agents

The MAS dynamics results into



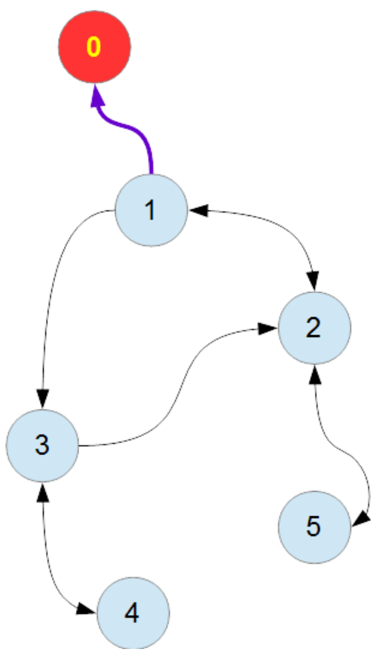
$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -\lambda L \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}$$



The entries of matrix A change the final common state

Decentralised Leader-following

Some of the agents act independently from the others

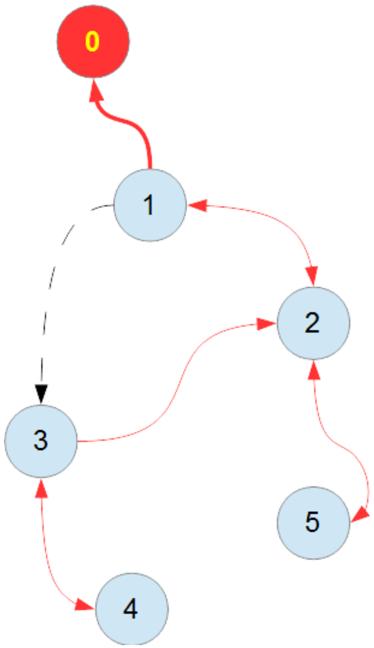


$$\bar{A} = \left[\begin{array}{c|ccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{0} & 1 \quad 0 \quad \cdots \quad 0 \\ \hline \mathbf{0} & A \end{array}\right]$$

Decentralised Leader-following

Some of the agents act independently from the others

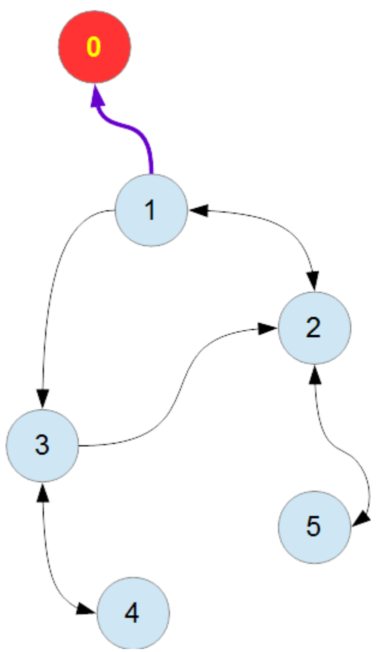
They are the root of spanning trees



$$\bar{A} = \left[\begin{array}{c|ccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{0} & 1 \quad 0 \quad \cdots \quad 0 \\ \hline \mathbf{0} & A \end{array}\right]$$

Decentralised Leader-following

The MA dynamics can be represented by a “classic” interaction dynamics plus an external reference



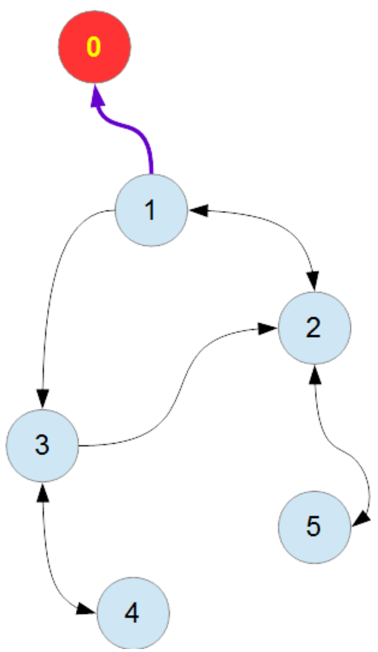
Single integrator case

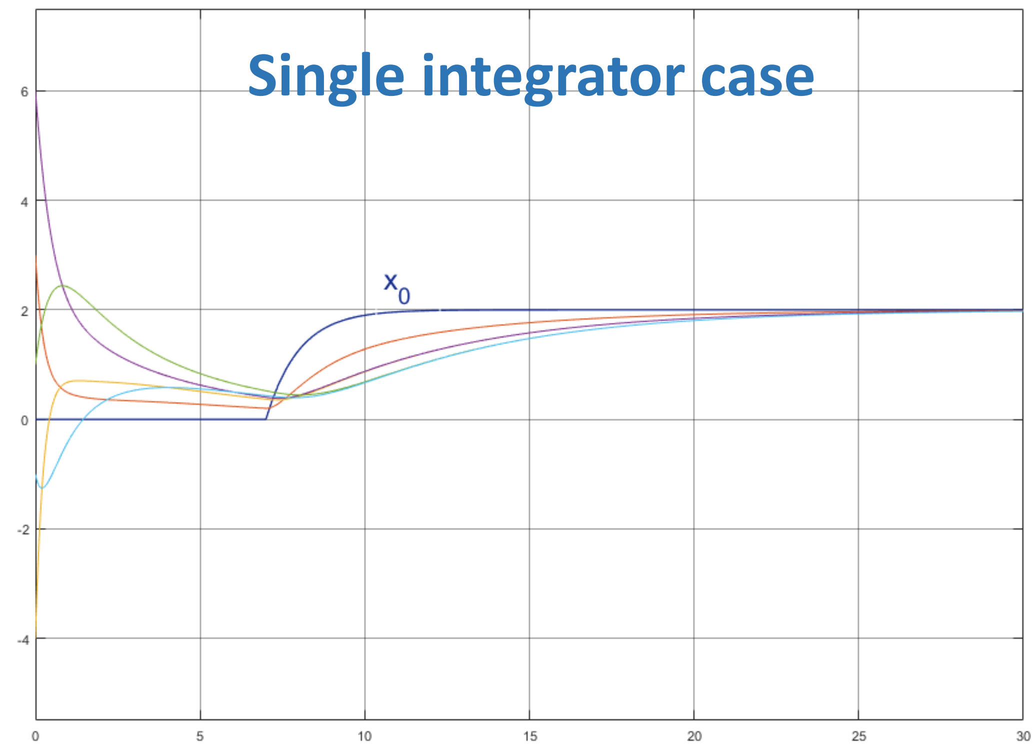
$$\begin{bmatrix} \dot{x}_0 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -a_0 & \mathbf{0} \\ \begin{bmatrix} a_{1,0} \\ \mathbf{0} \end{bmatrix} & -\bar{L} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} u_0$$

$$\bar{L} = \begin{bmatrix} a_{1,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + L$$

Decentralised Leader-following

The MA dynamics can be represented by a "classic" interaction dynamics plus an external reference

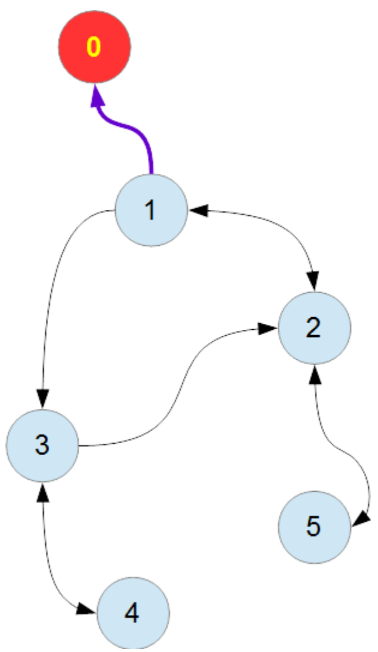




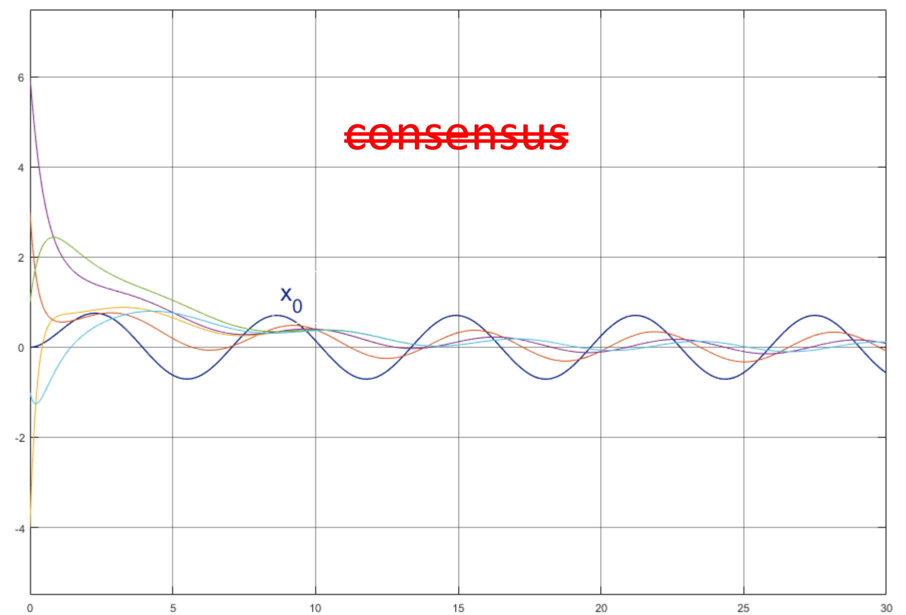
$a_0=1$, $u_0=2\,\delta_{-1}(t-7)$ $a_{1,0}=1$

Decentralised Leader-following

The MA dynamics can be represented by a “classic” interaction dynamics plus an external reference



Single integrator case



a_0=1, u_0=sin(t) $a_{1,0}$=1

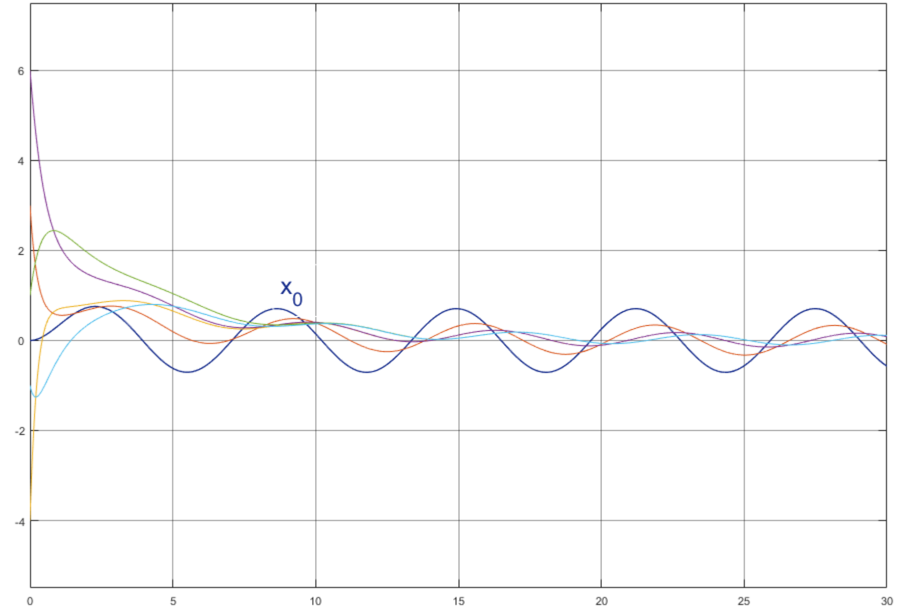
Decentralised Leader-following

The MA dynamics can be represented by a "classic" interaction dynamics plus an external reference

Single integrator case

Leader's velocity propagates as a **disturbances** in agents'dynamics

Even tracking for the agent connected to the leader is lost

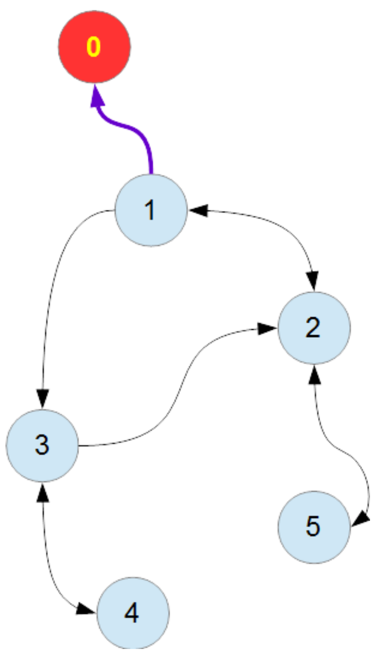


a_0=1, u_0=sin(t)

$a_{1,0}$=1

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference



Single integrator case

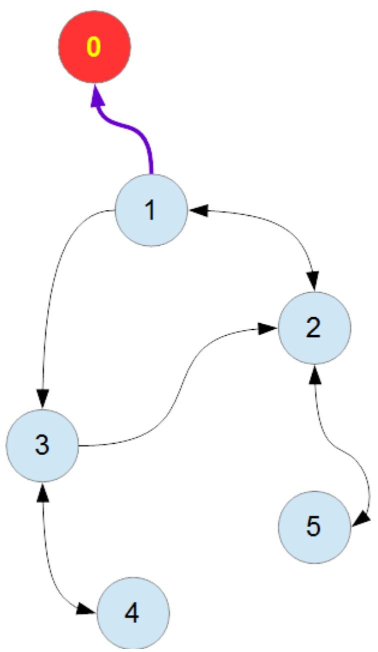
$$\dot{x}_0 = -a_0 x_0 + u_0$$

$$\dot{\mathbf{x}} = sign\left(\left[\begin{bmatrix} a_{1,0} \\ \mathbf{0} \end{bmatrix} - \bar{L}\right] \cdot \begin{bmatrix} x_0 \\ \boldsymbol{x} \end{bmatrix}\right)$$

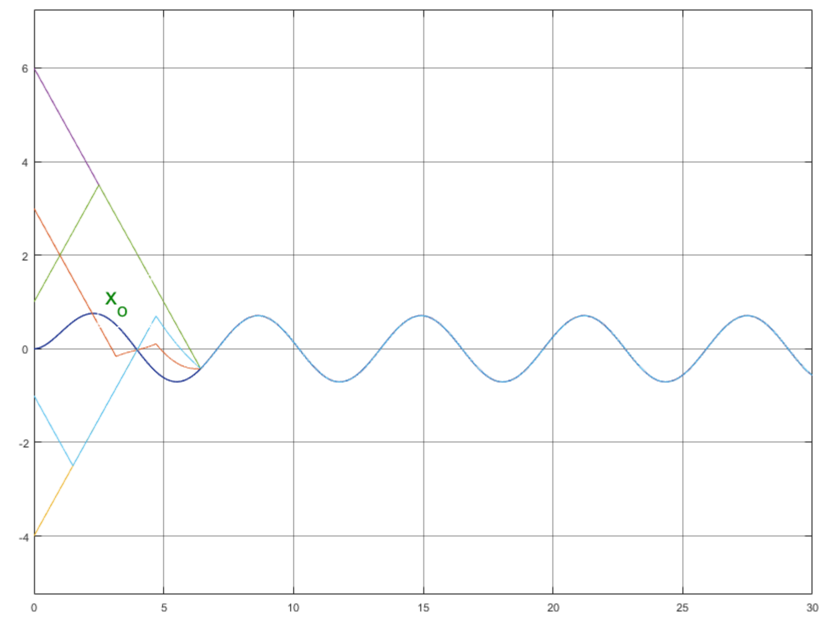
$$\bar{L} = \begin{bmatrix} a_{1,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + L$$

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference



Single integrator case



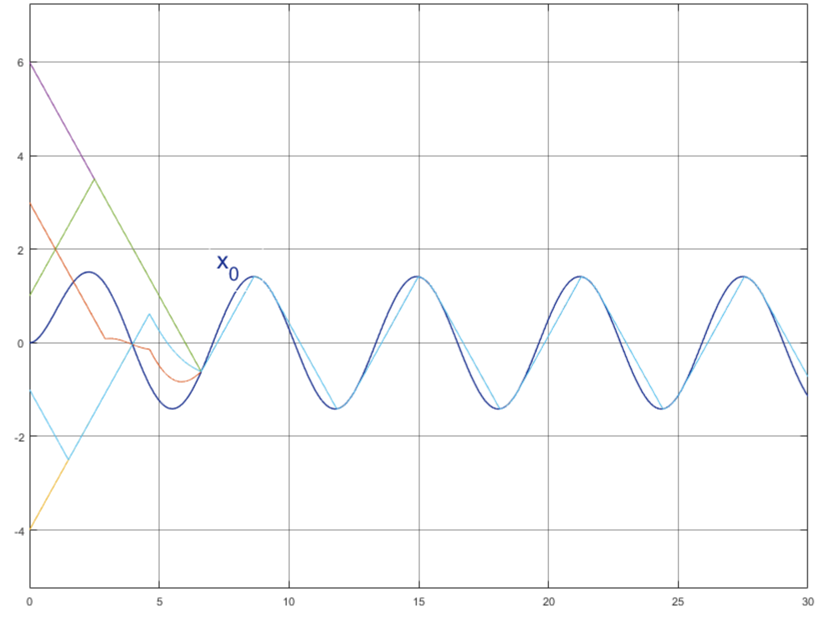
a_0=1, u_0=sin(t) $a_{1,0}$=1

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference

The gain of the switching interactions must be large enough

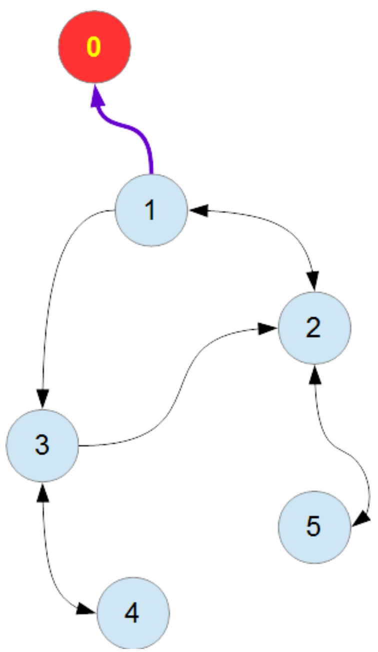
Single integrator case



a_0=1, u_0=2sin(t) $a_{1,0}$=1

Decentralised Leader-following

The MA dynamics can be represented by a "classic" interaction dynamics plus an external reference



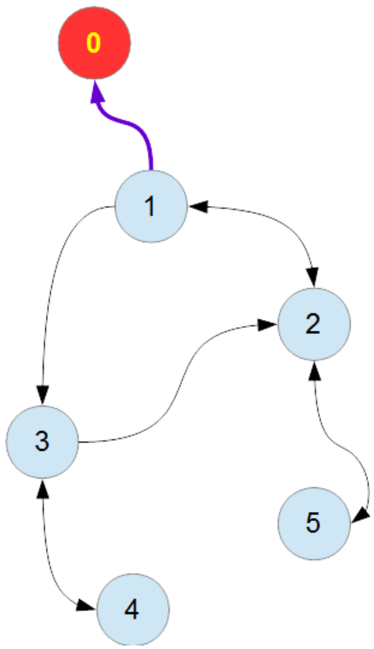
Double integrator case

$$\begin{bmatrix} \dot{x}_{0_1} \\ \dot{\mathbf{x}}_1 \\ \dot{x}_{0_2} \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & I \\ -a_0 & 0 & -a_1 & 0 \\ a_{1,0} & -\bar{L} & a_{1,0}\lambda & -\lambda\bar{L} \end{bmatrix} \begin{bmatrix} x_{0_1} \\ \boldsymbol{x}_1 \\ \mathbf{x}_{0_2} \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_0$$

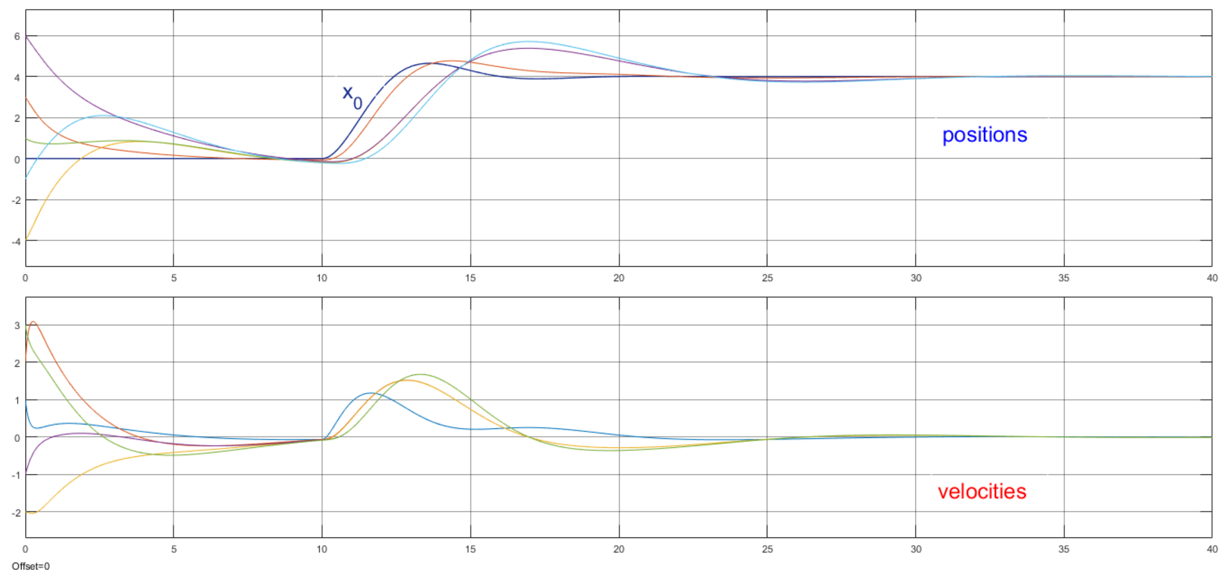
$$\bar{L} = L + \begin{bmatrix} -a_{1,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Decentralised Leader-following

The MA dynamics can be represented by a “classic” interaction dynamics plus an external reference



Double integrator case



a_0=1, a_1=1, u_0=4 δ_{-1}(t-10) $a_{1,0}$=1 λ=2

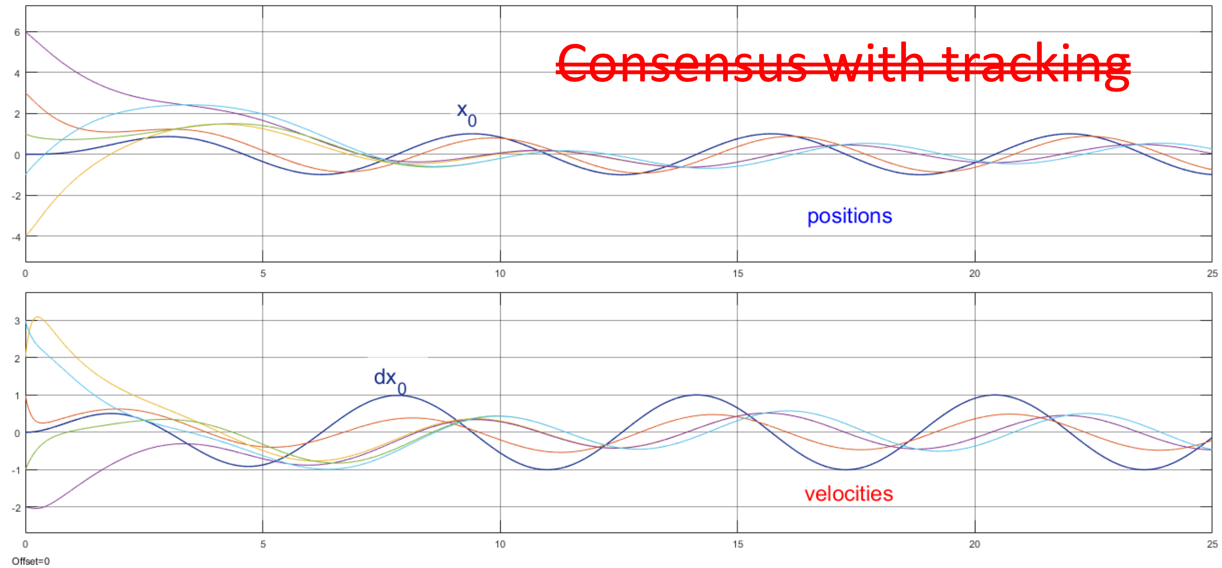
Decentralised Leader-following

The MA dynamics can be represented by a "classic" interaction dynamics plus an external reference

Double integrator case

Leader's velocity and control propagates as a **disturbances** in agents'dynamics

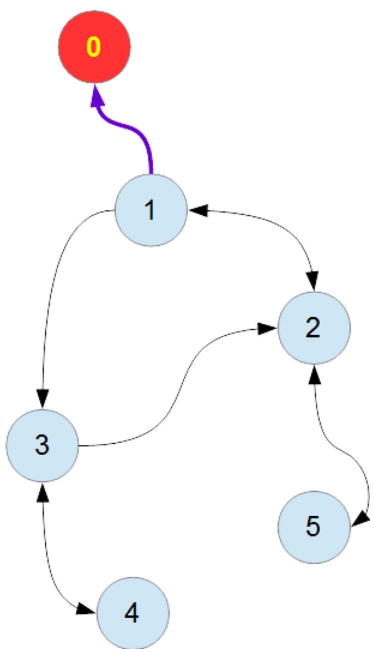
Even tracking for the agent connected to the leader is lost



a_0=1, a_1=1, u_0=sin(t) $a_{1,0}$=1 λ=2

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference



Double integrator case

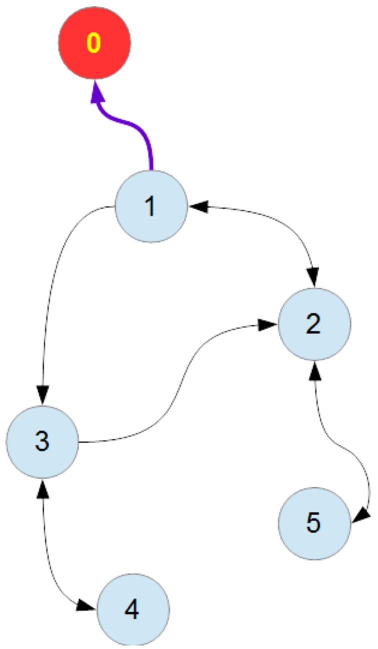
$$\begin{cases} \dot{x}_{0_1} = x_{0_2} \\ \dot{x}_{0_2} = -a_0 x_{0_1} - a_1 x_{0_2} + u_0 \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \alpha \cdot sign\left(\left[\begin{bmatrix} a_{1,0} \\ 0 \end{bmatrix} - \bar{L}\right] \cdot \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}\right) + \lambda \cdot sign\left(\left[\begin{bmatrix} a_{1,0} \\ 0 \end{bmatrix} - \bar{L}\right] \cdot \begin{bmatrix} \dot{x}_0 \\ x_2 \end{bmatrix}\right) \end{cases}$$

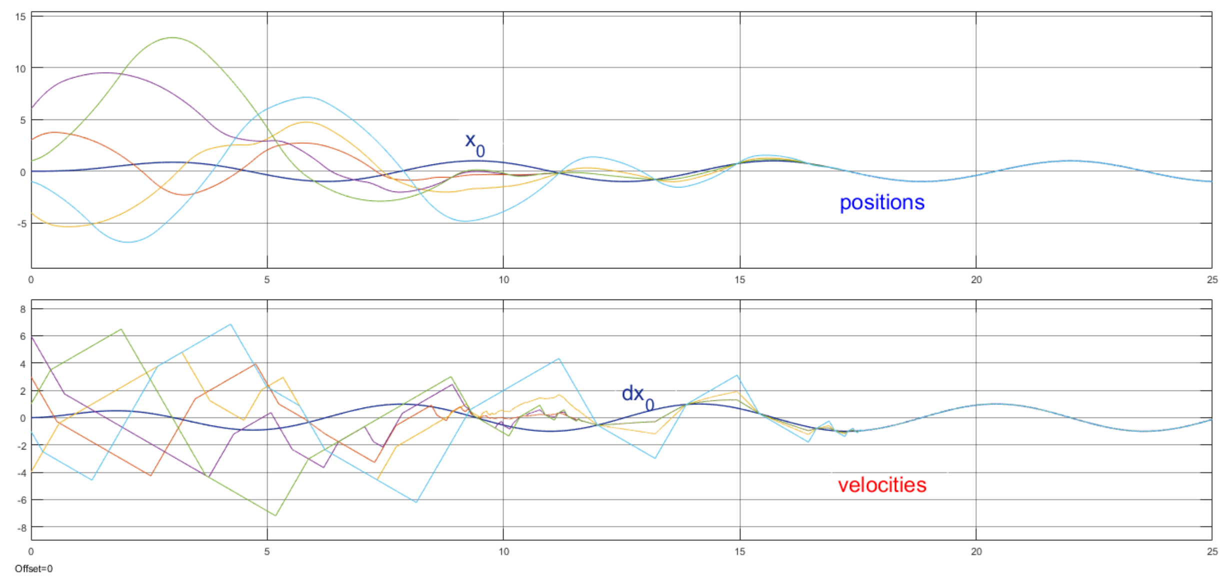
$$\bar{L} = \begin{bmatrix} a_{1,0} & 0 \\ 0 & 0 \end{bmatrix} + L$$

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference



Double integrator case



a_0=1, a_1=1, u_0=sin(t) $a_{1,0}$=1 α=4, λ=2

Decentralised Leader-following

The MA dynamics should be represented by a switching interaction dynamics plus an external reference

$$\begin{cases} \dot{x}_{0_1} = x_{0_2} \\ \dot{x}_{0_2} = -a_0 x_{0_1} - a_1 x_{0_2} + u_0 \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \alpha \cdot sign\left(\left[\begin{bmatrix} a_{1,0} \\ 0 \end{bmatrix} - \bar{L} \right] \cdot \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \right) + \lambda \cdot sign\left(\left[\begin{bmatrix} a_{1,0} \\ 0 \end{bmatrix} - \bar{L} \right] \cdot \begin{bmatrix} \dot{x}_0 \\ x_2 \end{bmatrix} \right) \end{cases}$$

$$\bar{L} = \begin{bmatrix} a_{1,0} & 0 \\ 0 & 0 \end{bmatrix} + L$$

- The leader's trajectory becomes an equilibrium movement of the MAS
- Consensus with tracking is reached in a finite time
- The drift due to the integrative effect between velocity and position is eliminated
- Controller tuning depends on the characteristics of the movement of the leader

Decentralised Leader-following

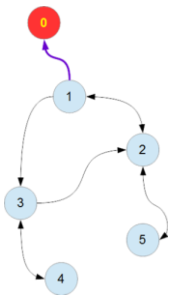
Single integrators case with formation control

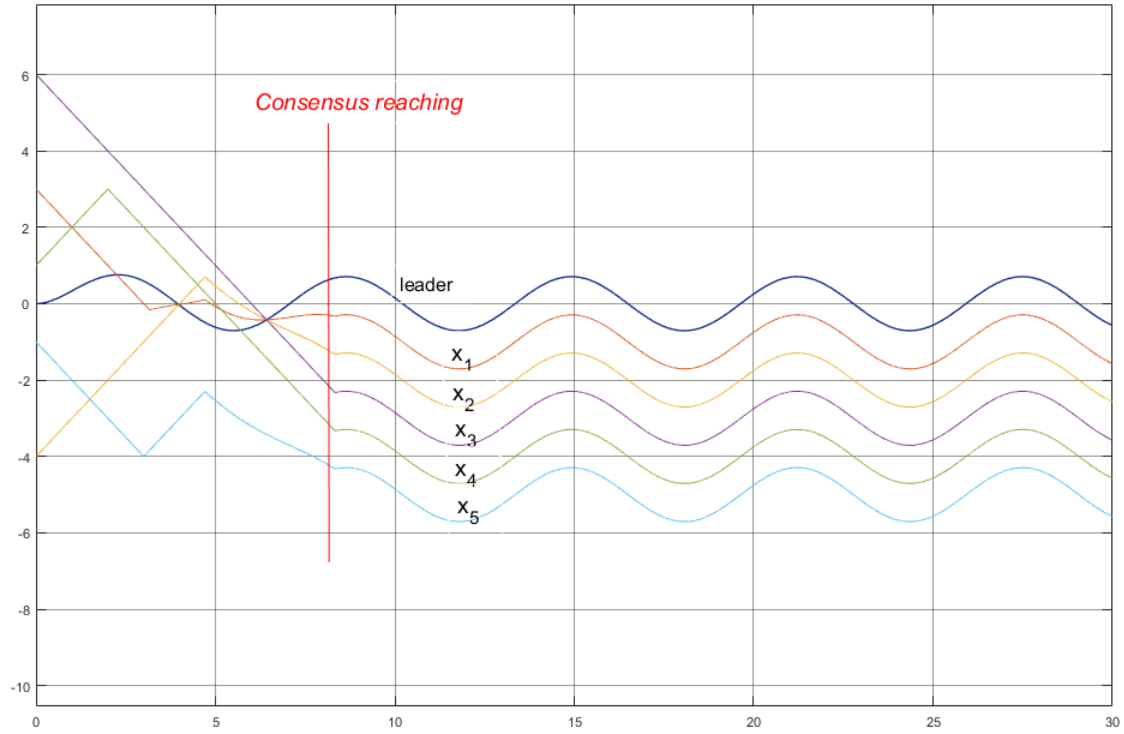
The MA dynamics should be represented by a switching interaction dynamics plus an external reference and a offset vector

$$\dot{x}_0 = -a_0 x_0 + u_0$$

$$\dot{\mathbf{x}} = sign\left(\left[\begin{bmatrix} a_{1,0} \\ \mathbf{0} \end{bmatrix} - \bar{L}\right] \cdot \begin{bmatrix} x_0 \\ x \end{bmatrix} + D\right)$$

$$\bar{L} = \begin{bmatrix} a_{1,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + L$$





Robust Consensus

Agents' dynamics is affected by uncertainty

- Uncertain drift term
- Disturbances
- Measurement noise
- Unmodelled dynamics
- Switching topology of the net
- Communication delays
- Packets' jittering
- Packets' loss

In order to deal with such a number of "*accidents*"
assumptions should be made to find solutions

Robust Consensus

Robustness is an issue in MAS control and VSC could help!

- Uncertain drift term
- Disturbances
- Measurement noise
- Unmodelled dynamics
- Switching topology of the net
- Communication delays
- Packets' jittering
- Packets' loss
- Adverse agents

Just some of the "accidents" will be considered now on

Robust Consensus

Agents' dynamics is affected by uncertainty

Agents are 1[th] order systems

$$\dot{x}_i = f_i(x_i, t) + u_i$$

Agents are 2[nd]order systems

$$\begin{cases} \dot{x}_{i_1} = x_{i_2} \\ \dot{x}_{i_2} = f_i(\mathbf{x}_i, t) + u_i \end{cases}$$

The uncertain drift should fulfil some requirements, e.g., boundedness, continuity, lipshitzness, etc

Robust Consensus

Agents are 1th order systems

$$\dot{x}_i = f_i(x_i, t) + u_i$$

$$u_i = -\alpha \cdot sign\left(\sum_{j \in N_{out_i}} \left(x_i - x_j\right)\right)$$

$$\alpha > \Phi \geq \left| F(x, t) \right|_\infty$$

$f_1 = \sin(t)$
$f_2 = 0{,}5 \sin(3t)$
$f_3 = \sin((0.2+0.6t)t)$
$f_4 = 0$
$f_5 = \delta_{-1}(t-10)$

$\alpha = 3$

A control depending on the bounds of the uncertainty could help and compensate for the uncertainty

Robust Consensus

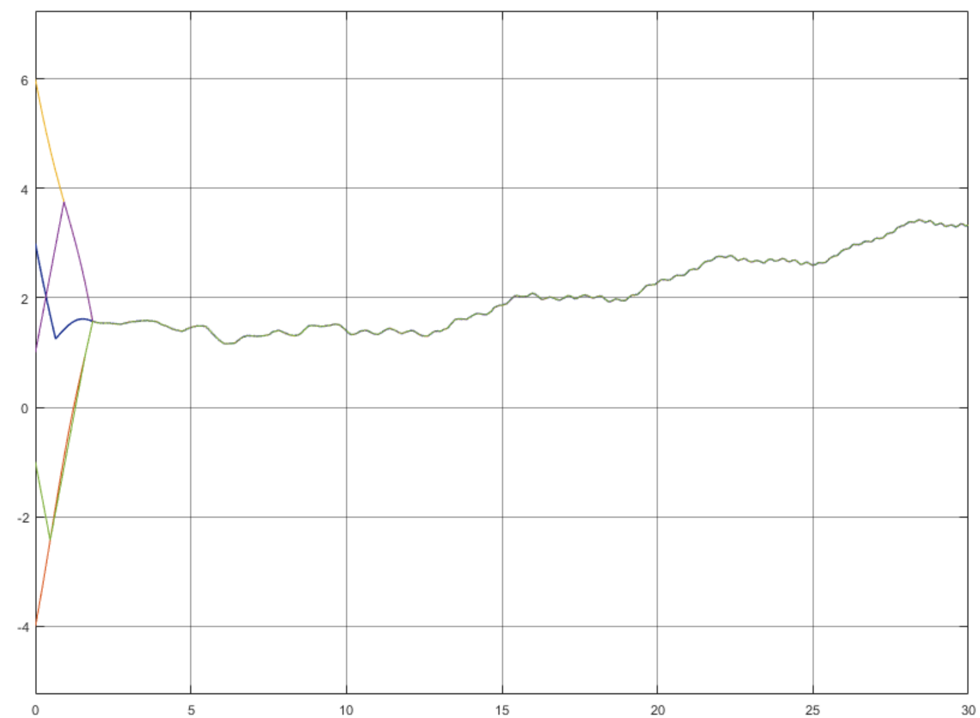
Agents are 1th order systems

$$\dot{x}_i = f_i(x_i, t) + u_i$$

$$u_i = -\alpha \cdot sign\left(\sum_{j \in N_{out_i}} (x_i - x_j)\right)$$

$$\alpha > \Pi \geq \left| F(x,t) \right|_\infty$$

f_1=sin(t)
f_2=0,5 sin(3t)
f_3= sin((0.2+0.6t)t)
f_4= 0
f_5=δ_{-1}(t-10)



Consensus to an instantaneous value depending on the disturbance

Robust Consensus

Agents are 1th order systems $\qquad \dot{x}_i = f_i(x_i, t) + u_i$

$$u_i(t) = -\alpha \cdot \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)) - \beta \cdot \mathrm{SIGN}(x_i(t) + z_i(t))$$

$$\dot{z}_i(t) = \alpha \cdot \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)), \qquad z_i(0) = -x_i(0)$$

Defining the "sliding variable" $\qquad \boldsymbol{\sigma}(t) = \boldsymbol{x}(t) + \boldsymbol{z}(t)$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{w}(t) - \alpha \cdot \boldsymbol{\mathcal{L}}\,\boldsymbol{x}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t))$$

$$\dot{\boldsymbol{\sigma}}(t) = \boldsymbol{w}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t)), \qquad \boldsymbol{\sigma}(0) = \mathbf{0}_n$$

A. Pilloni, M. Franceschelli, A. Pisano, E. Usai., Sliding Mode-Based Robustification of Consensus and Distributed Optimization Control Protocols, IEEE TAC, VOL. 66, NO. 3, pp. 1207-1214, 2021

Robust Consensus

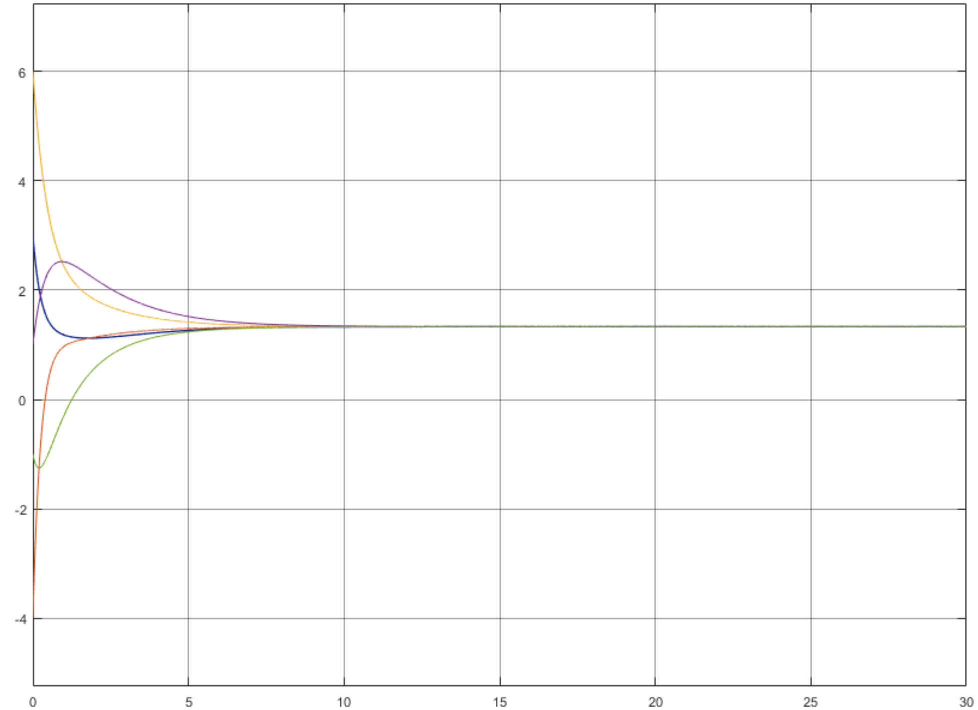
Agents are 1th order systems $\dot{x}_i = f_i(x_i, t) + u_i$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{w}(t) - \alpha \cdot \boldsymbol{\mathcal{L}}\,\boldsymbol{x}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t))$$

$$\dot{\boldsymbol{\sigma}}(t) = \boldsymbol{w}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t)), \qquad \boldsymbol{\sigma}(0) = \mathbf{0}_n$$

$\beta > \Pi$

A sliding mode on s=0 is established since the beginning



Robust Consensus

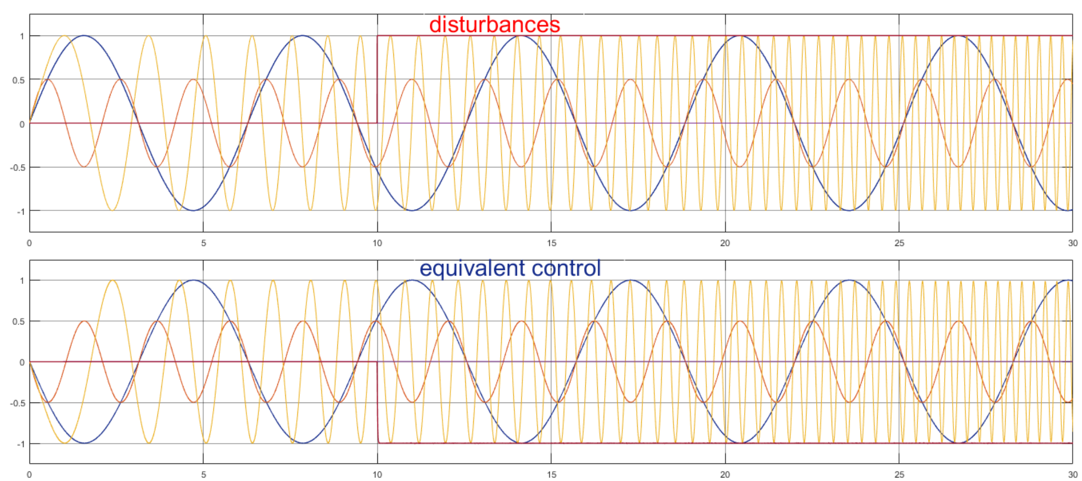
Agents are 1th order systems $\dot{x}_i = f_i(x_i, t) + u_i$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{w}(t) - \alpha \cdot \boldsymbol{\mathcal{L}}\boldsymbol{x}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t))$$
$$\dot{\boldsymbol{\sigma}}(t) = \boldsymbol{w}(t) - \beta \cdot \mathrm{SIGN}(\boldsymbol{\sigma}(t)), \qquad \boldsymbol{\sigma}(0) = \boldsymbol{0}_n$$

$$\beta > \Pi$$

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{0}_n \quad \Rightarrow \quad \mathrm{SIGN}_{eq}(\boldsymbol{\sigma}(t)) = \frac{1}{\beta}\boldsymbol{w}(t)$$

The discontinuous control compensate for uncertainty

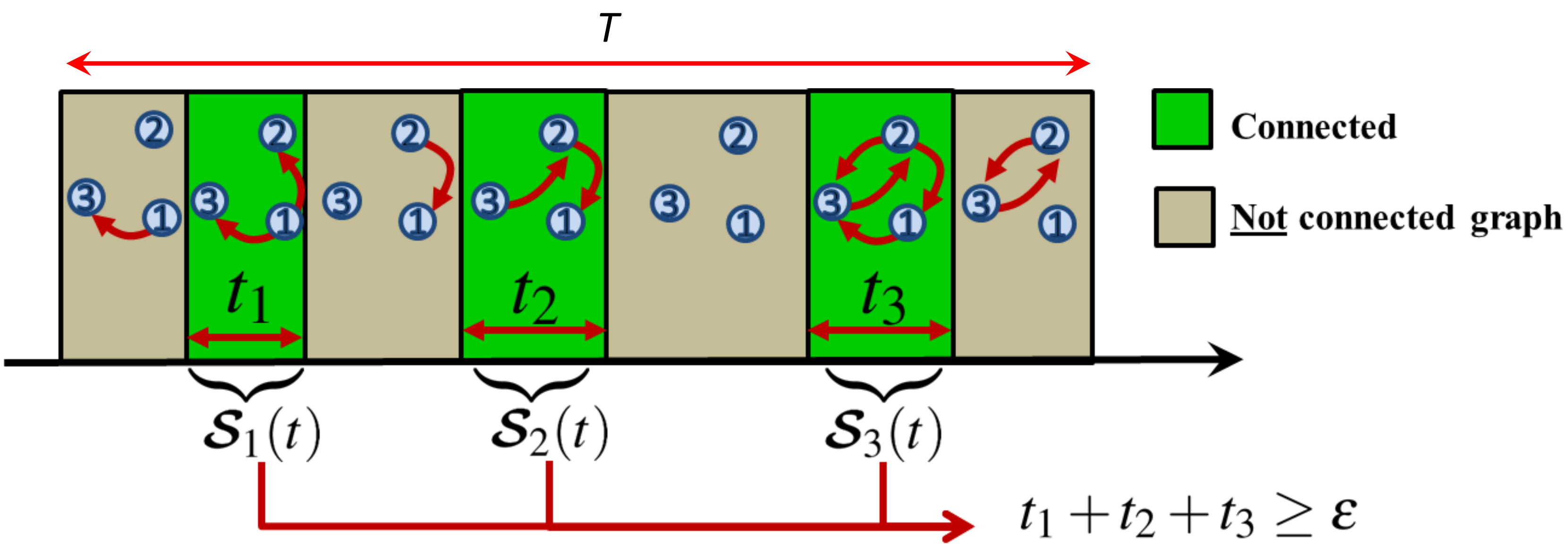


Robust Consensus

Networks of first order agents with switching connections

$$\dot{x}_i = \alpha + f_i(t) + u_i \qquad \left| f_i(t) \right| \le \pi_i \le \max_{i \in V}\{\pi_i\} = \Pi$$

$$G(t) = \{V, E(t), A(t)\} \qquad A(t) = A^T(t)\,;\ \forall t$$



α : desired "velocity" $f_i(t)$: bounded uncertainty

Robust Consensus

Networks of first order agents with switching connections

$$\dot{x}_i = \alpha + f_i(t) + u_i \qquad \left|f_i(t)\right| \leq \pi_i \leq \max_{i \in V}\{\pi_i\} = \Pi$$

$$G(t) = \{V, E(t), A(t)\} \qquad A(t) = A^T(t)\,;\ \forall t$$

$$u_i = -\lambda \sum_{j \in \mathcal{N}_i(t)} a_{i,j}(t)\ sign\left(x_i - x_j\right) \qquad \lambda \geq \frac{|E|\Pi T + \mu^2}{\varepsilon};\mu \neq 0$$

$|E|$ is the cardinality of the set containing all possible edge of the time-varying graph

$$\Downarrow$$

$$\left|x_i - x_j\right| \leq 2|E|\Pi(T - \varepsilon)\,; \quad \forall t \geq \frac{T}{2\mu^2} \sum_{(i,j) \in E} \left|x_{i_0} - x_{j_0}\right|$$

Franceschelli M., Giua A., Pisano A., Usai E., Finite-Time Consensus for Switching Network Topologies with Disturbances, Nonlinear Analysis: Hybrid Systems, vol. 10, pp. 83-93, 2014.

Robust Consensus

Networks of first order agents with switching connections

$$\dot{x}_i = \alpha + f_i(t) - \lambda \sum_{j \in \mathcal{N}_i(t)} a_{i,j}(t)\ sign\left(x_i - x_j\right)$$

$$G(t) = \{V, E(t), A(t)\} \qquad A(t) = A^T(t);\ \forall t \qquad E(t) \subset E\ \forall t$$

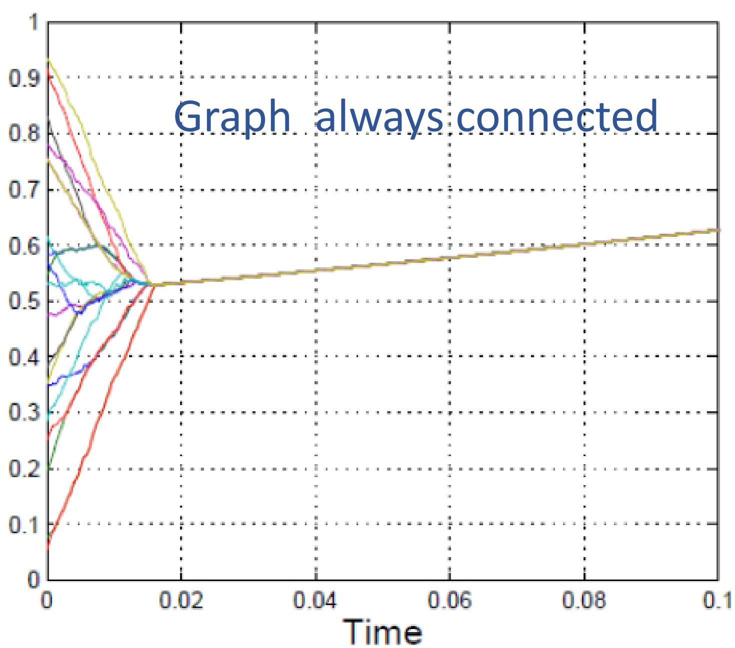
- Number of clocks: 20.
- Topology: a random graph.
- Sampling step: 0.1 milliseconds.
- Desired clock speed: $\alpha = 1$
- Initial clocks bias chosen uniformly at random within $[0, 1]$.
- Disturbance model: $\nu_i(t) = \frac{5}{2}(n_i(t) + sin(3|n_i(t)|t))$
- $n_i(t) \in [-1, 1]$ is a uniformly distributed random variable for any t.
- $\lambda = 30$

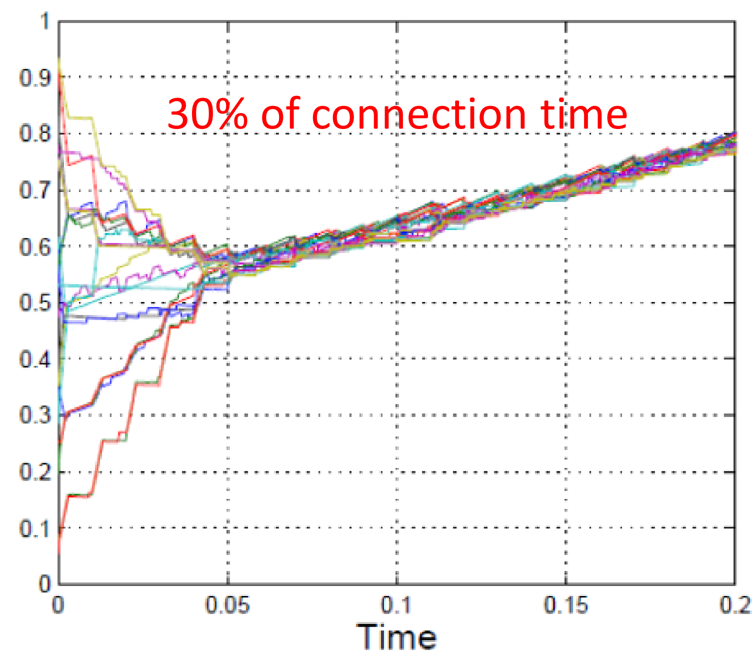
Robust Consensus

Networks of first order agents with switching connections

$$\dot{x}_i = \alpha + f_i(t) - \lambda \sum_{j \in \mathcal{N}_i(t)} a_{i,j}(t)\ sign\left(x_i - x_j\right)$$

$$G(t) = \{V, E(t), A(t)\} \qquad A(t) = A^T(t)\,;\ \forall t \qquad E(t) \subset E\ \forall t$$





Robust Consensus

Convergence to the median value is more robust in the presence of adverse agents

$$\dot{x}_i = u_i$$

Agent	value	Attack 1	Attack 2	Attack 3	Attack 4	Attack 5	Attack 6
1	4	8	4	4	4	4	4
2	0	0	0	0	0	0	0
3	3	3	3	3	3	3	3
4	2	2	2	2	2	2	2
5	-1	-1	-4	-1	-1	-1	-1
6	1	1	1	4	1	1	1
7	1	1	1	1	1	1	1
8	---	---	---	---	5	2	-3
mean	1,43	2,00	1,00	1,86	1,88	1,50	0,88
median	1,00	**1,00**	**1,00**	2,00	1,50	1,50	**1,00**

Robust Consensus

Convergence to the median value is more robust in the presence of adverse agents

$$\dot{x}_i = \boxed{-\alpha_i \, sign\left(x_i - x_{i_0}\right) - \sum_{j \in \mathcal{N}_i} \lambda_{i,j} \, sign\left(x_i - x_j\right)} \qquad \lambda_{i,j} = \lambda_{j,i} \;\; \forall\, (i,j) \in E$$

$$\lambda_{i,j} > 0 \quad 0 < \max_{i \in V} \{\alpha_i\} < \frac{2k \min_{(i,j) \in E} \{\lambda_{i,j}\}}{N}$$

$$\left| x_i - x_{i_0} \right| \xrightarrow[t \to T]{} 0 \qquad T \leq \frac{\max_{i \in V}\left\{x_{i_0}\right\} - \min_{i \in V}\left\{x_{i_0}\right\}}{2\left(\frac{2k \min_{(i,j) \in E} \{\lambda_{i,j}\}}{N} - \max_{i \in V}\{\alpha_i\}\right)}$$

k is the maximum number of vertex that can be eliminated maintain a connected graph

M. Franceschelli, A. Giua, A. Pisano, "Finite-Time Consensus on the Median Value with Robustness Properties," IEEE Trans. on Automatic Control, Vol. 62, No. 4, pp. 1652-1667, 2017

Robust Consensus

Convergence to the median value is more robust in the presence of adverse agents

Once the consensus is reached the states are driven to the median value provided that

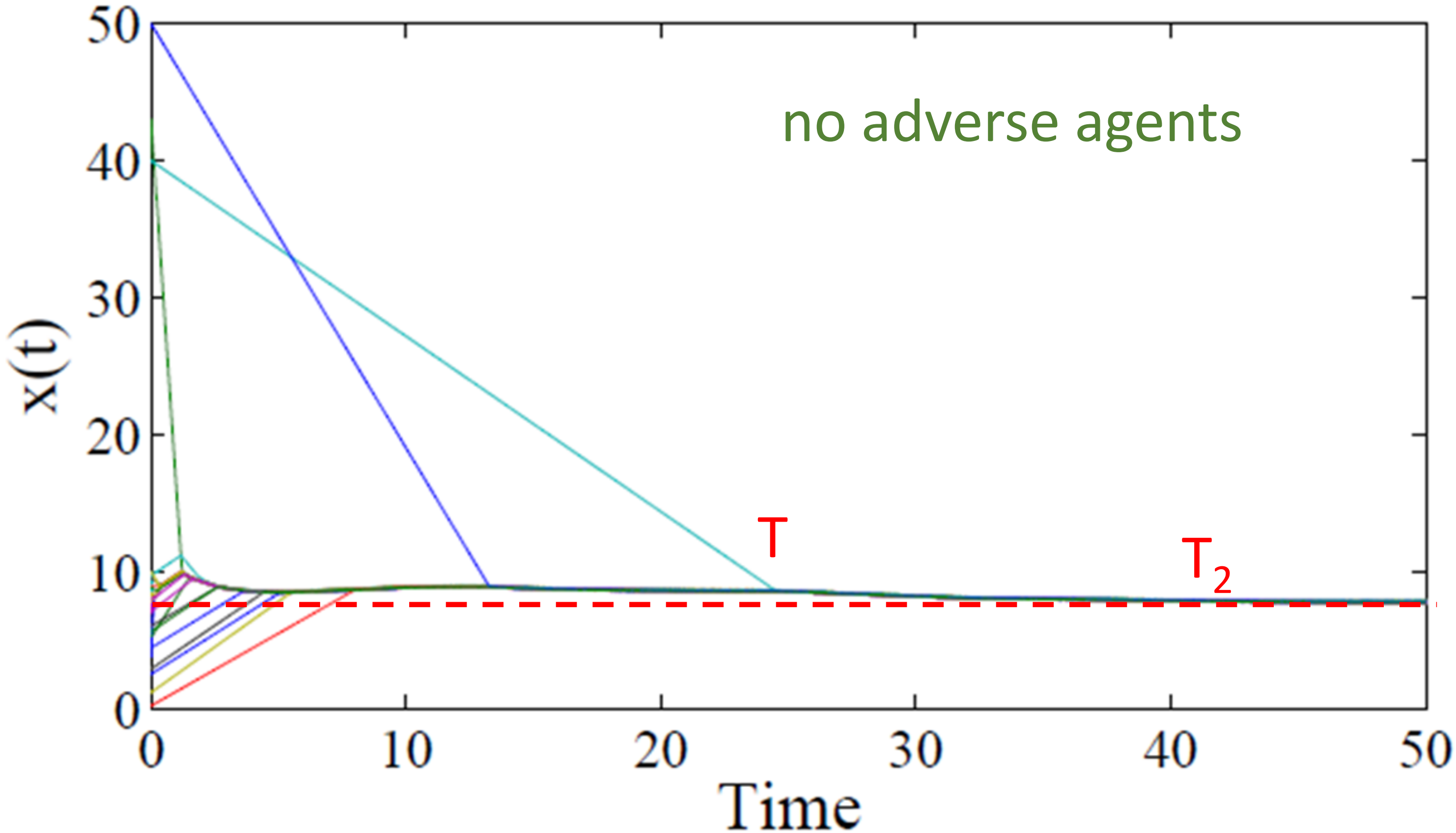
$$\frac{\max_{i\in V}\{\alpha_i\} - \min_{i\in V}\{\alpha_i\}}{\min_{i\in V}\{\alpha_i\}} < \frac{1}{N}$$

$$x_i \xrightarrow[t\to T_2]{} m(x_0) = \begin{cases} z_{N/2+1} & \text{if } N \text{ is odd} \\ \left[z_{N/2}; z_{N/2+1}\right] & \text{if } N \text{ is even} \end{cases} \qquad T_2 \le T + 2N \frac{\left|x_i(T) - m(x_0)\right|}{\max_{i\in V}\{\alpha_i\}}$$

M. Franceschelli, A. Giua, A. Pisano, "Finite-Time Consensus on the Median Value with Robustness Properties," IEEE Trans. on Automatic Control, Vol. 62, No. 4, pp. 1652-1667, 2017

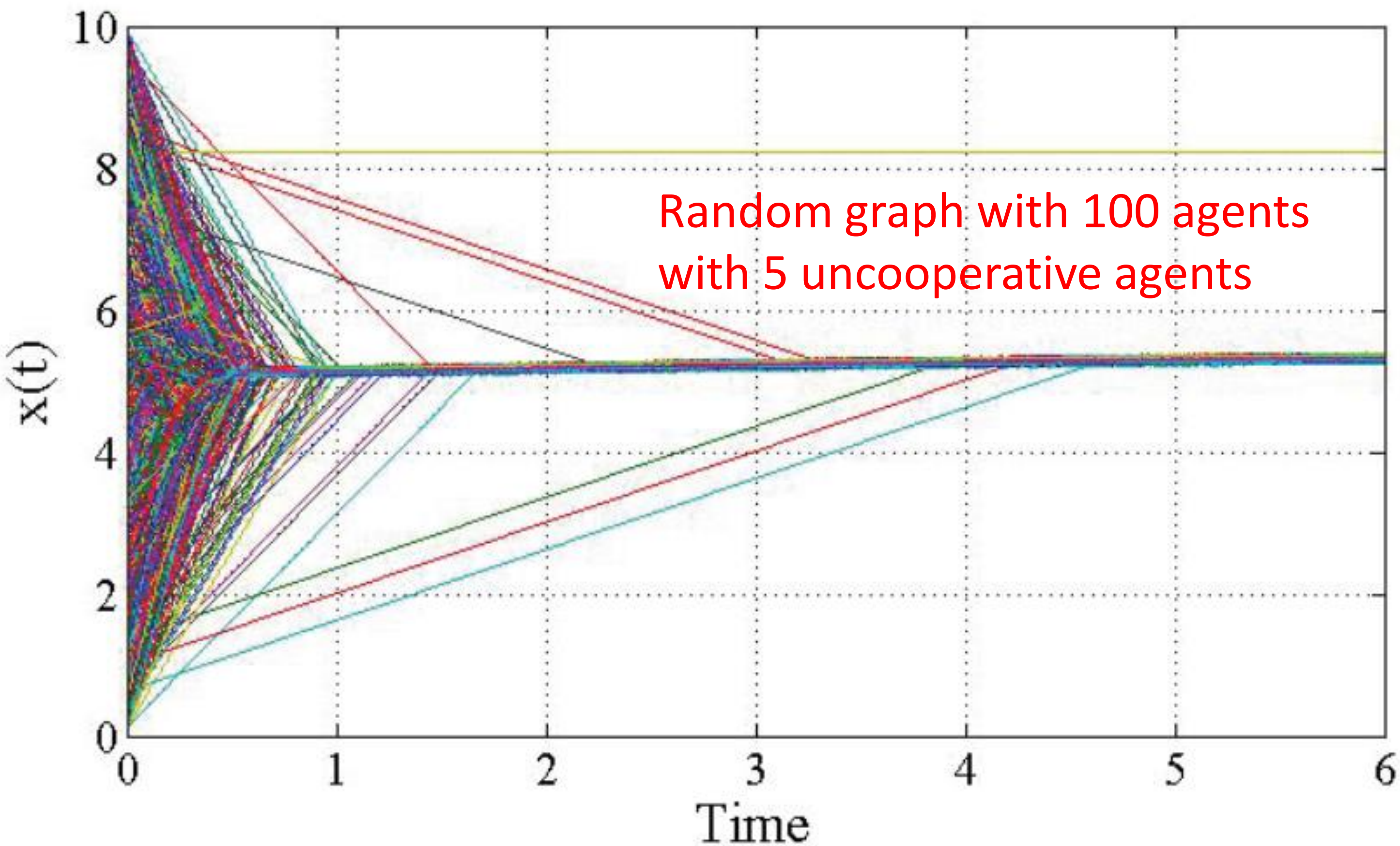
Robust Consensus

Convergence to the median value is more robust in the presence of adverse agents

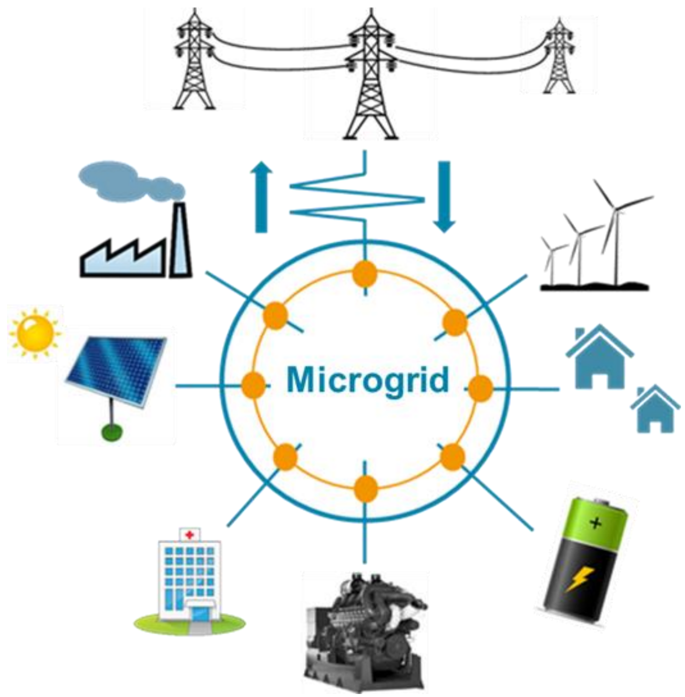


Robust Consensus

Convergence to the median value is more robust in the presence of adverse agents



Consensus in electrical networks



A physical network connecting:

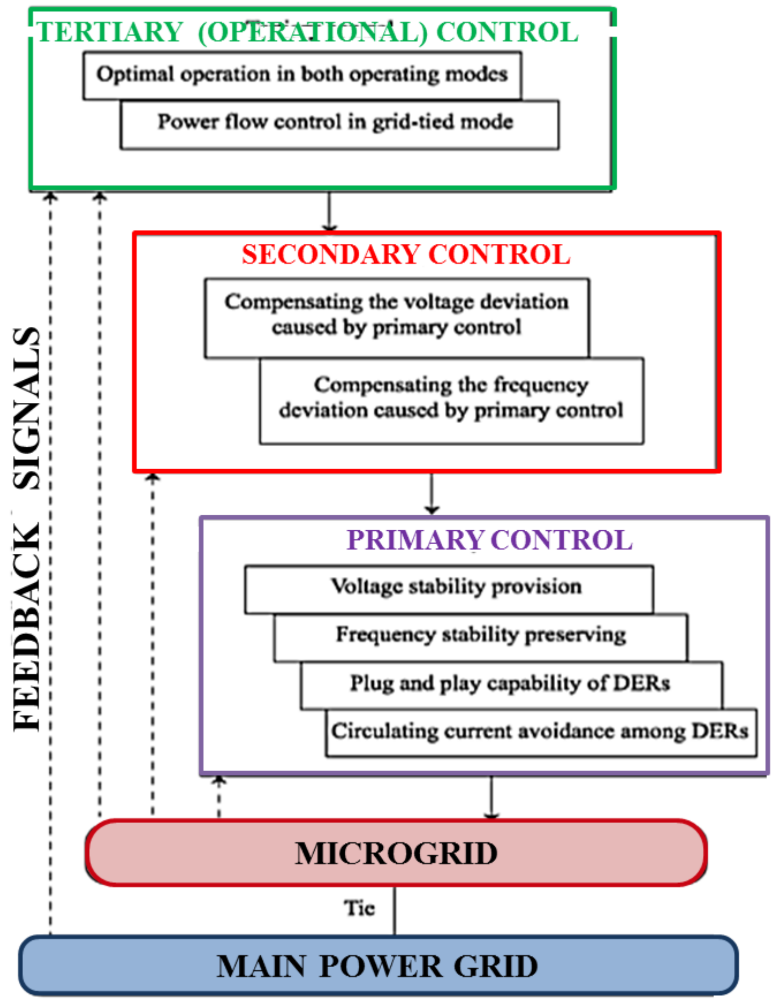
- Generation systems
- Loads
- Storages

Increasing the renewable energies production introduces strong variability in generation systems

Loads are uncertain

Generation and loads must be balanced in a very short time

Consensus in electrical networks



Main control issues

- Stability and Power sharing
- Frequency and voltage restoration
- Dispatching of power flows

Consensus in electrical networks

Dispatching of power flows

Usually, but not only, based on centralised heuristic optimization methods

Gan, D., Thomas, R.J., Zimmerman, R.D., Stability-constrained optimal power flow, IEEE Transactions on Power Systems, 15(2), pp. 535-540, 2000

Borges, C.L.T., Falcão, D.M., Optimal distributed generation allocation for reliability, losses, and voltage improvement, International Journal of Electrical Power and Energy Systems, 28(6), pp. 413-420, 2006

Georgilakis, P.S., Hatziargyriou, N.D., Optimal distributed generation placement in power distribution networks: Models, methods, and future research, IEEE Transactions on Power Systems, 28(3), pp. 3420-3428, 2013

Parisio, A., Rikos, E., Glielmo, L., A model predictive control approach to microgrid operation optimization, IEEE Transactions on Control Systems Technology, 22(5), pp. 1813-1827, 2014

Biswas, P.P., Suganthan, P.N., Amaratunga, G.A.J., Optimal power flow solutions incorporating stochastic wind and solar power, Energy Conversion and Management, 148, pp. 1194-1207, 2017

Rinaldi, G., Menon, P.P., Edwards, C., Ferrara, A., Sliding Mode Observer-Based Finite Time Control Scheme for Frequency Regulation and Economic Dispatch in Power Grids, IEEE Transactions on Control Systems Technology, 30(3), pp. 1296-1303, 2022

Consensus in electrical networks

Frequency and voltage restoration

Nowadays most approaches are based on MAS theory

Shafiee Q., Guerrero J.M., Vasquez J.C., Distributed secondary control for islanded microgrids-a novel approach, IEEE Transactions on Power Electronics, 29(2), pp. 1018-1031, 2014

Guo F., Wen C., Mao J., Song Y.-D., Distributed Secondary Voltage and Frequency Restoration Control of Droop-Controlled Inverter-Based Microgrids, IEEE Transactions on Industrial Electronics, 62(7), pp. 4355-4364, 2015

J. Schiffer, T. Seel, J. Raisch, T. Sezi, Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control, IEEE Transactions on Control Systems Technology, 24(1), pp. 96 - 109, 2016

Dorfler F., Simpson-Porco J.W., Bullo F., Breaking the hierarchy: Distributed control and economic optimality in Microgrids, IEEE Transactions on Control of Network Systems, 3(3), pp. 241-253, 2016

Dorfler F., Simpson-Porco J.W., Bullo F., Electrical Networks and Algebraic Graph Theory: Models, Properties, and Applications, Proceedings of the IEEE, 106(5), pp. 977-1005, 2018

Y. Han, K. Zhang, H. Li, E.A. Alves Coelho, J.M. Guerrero, MAS-Based Distributed Coordinated Control and Optimization in Microgrid and Microgrid Clusters: A Comprehensive Overview, IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 33, NO. 8, pp. 6488-6508, 2018

A. Pilloni, A. Pisano, E. Usai, Robust Finite Time Frequency and Voltage Restoration of Inverter-based Microgrids via Sliding Mode Cooperative Control, IEEE Trans. on Industrial Electronics, vol. 65, n. 1, pp. 907-917, 2018

Cucuzzella M., Trip, ., De Persis C., Cheng X., Ferrara A., Van Der Schaft A., A Robust Consensus Algorithm for Current Sharing and Voltage Regulation in DC Microgrids, IEEE Transactions on Control Systems Technology, 27(4), pp. 1583-1595, 2019

Consensus in electrical networks

Stability and Power sharing

Most approaches are based on droop control

Guerrero J.M., de Vicuna L.G., Matas J., Castilla M., Miret J., A wireless controller to enhance dynamic performance of parallel inverters in distributed generation systems, IEEE Transactions on Power Electronics, 19(5), pp. 1205-1213, 2004

Yao W., Chen M., Matas J., Guerrero J.M., Qian Z.-M., Design and analysis of the droop control method for parallel inverters considering the impact of the complex impedance on the power sharing, IEEE Transactions on Industrial Electronics, 58(2), pp. 576-588, 2011

Schiffer J., Ortega R., Astolfi A., Raisch J., Sezi T., Conditions for stability of droop-controlled inverter-based microgrids, Automatica, 50(10), pp. 2457-2469, 2014

F. Guo, C. Wen, J. Mao, and Y.-D. Song, "Distributed secondary voltage and frequency restoration control of droop-controlled inverter-based microgrids," IEEE Trans. Ind. Electron., 62(7), 4355–4364, 2015.

Han Y., Li H., Shen P., Coelho E.A.A., Guerrero J.M., Review of Active and Reactive Power Sharing Strategies in Hierarchical Controlled Microgrids, IEEE Transactions on Power Electronics, 32(3), pp. 2427-2451, 2017

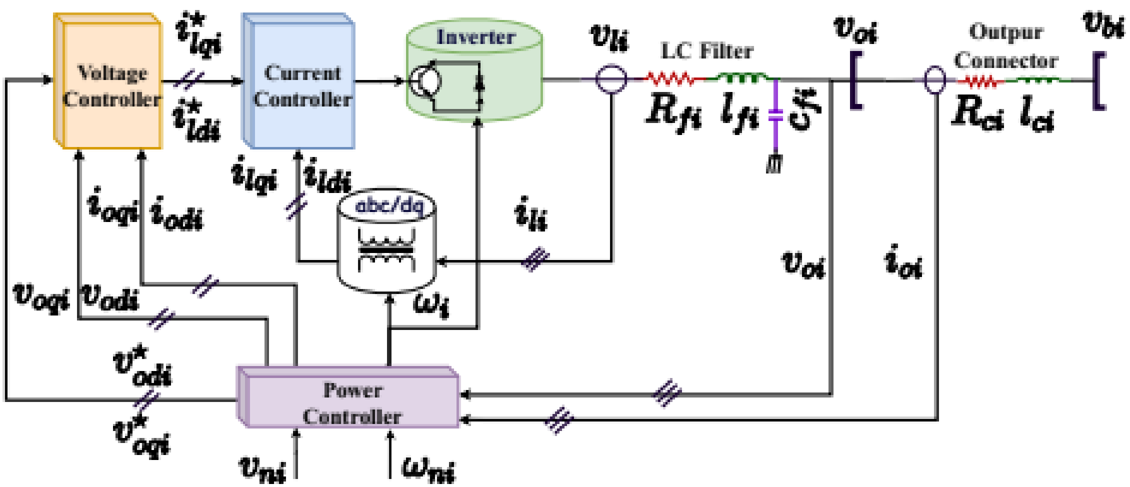
J. W. Simpson-Porco, F. Dorfler, and F. Bullo, "Voltage stabilization in microgrids via quadratic droop control," IEEE Trans. Automatic Control, 62(3), 1239–1253, 2017.

Minetti M., Rosini A., Denegri G.B., Bonfiglio A., Procopio R., An Advanced Droop Control Strategy for Reactive Power Assessment in Islanded Microgrids, IEEE Transactions on Power Systems, 37(4), pp. 3014-3025, 2022

Consensus in electrical networks

Stability and Power sharing

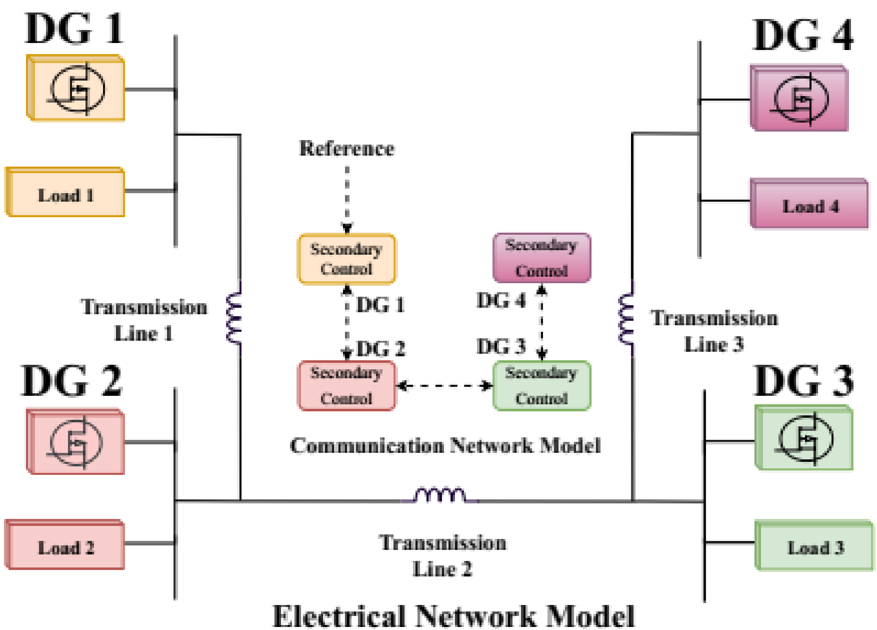
Most approaches are based on droop control



The presence of **converter** based connections between generators and the network allows for minimizing the load effects on the system dynamics

Consensus in electrical networks

Modelling the network



The presence of interacting physical and communication networks makes an electrical network a multi-agent cyber-physical system

Consensus in electrical networks

Modelling the network

ω_{com}	Speed of the common rotating ref. frame
ω_i	Local rotating ref. frame's speed of the i-th DG
δ_i	Angle between the local and the common rotating ref. frame, i.e., $\dot{\delta}_i = \omega_i - \omega_{com}$
$\omega_{ni}, \upsilon_{ni}$	Frequency and voltage droop-power setpoints
υ_{ki}, i_{ki}	3-ph voltages, currents at node k of the i-th DG
$\upsilon_{kdi}, \upsilon_{kqi}$	d-q voltages of the i-th DG at node k
i_{kdi}, i_{kqi}	d-q currents of the i-th DG at node k
$k = l, o, b$	input, output and branch local node of a DG
$\bar{\omega}, \bar{\upsilon}$	Rated values of MG's frequency and voltage
$\omega_{ref}, \upsilon_{ref}$	Desired values of frequency and voltage
P_i, Q_i	Active and reactive powers dc-components at the output node of the i-th DG
$\upsilon^*_{odi}, \upsilon^*_{oqi}$	d-q voltage setpoints of the i-th DG
ψ_{di}, ψ_{qi}	d-q voltage error's integral of the i-th DG
i^*_{ldi}, i^*_{lqi}	d-q current setpoints of the i-th DG
ϕ_{di}, ϕ_{qi}	d-q current error's integral of the i-th DG

Consensus in electrical networks

Modelling the network

Frequency dynamics

$$\frac{d\delta_i}{dt} = \omega_i - \omega_{\text{com}}$$

Power droop control

$$\omega_i = \omega_{ni} - m_i \cdot P_i$$
$$\upsilon_{odi}^* = \upsilon_{ni} - n_i \cdot Q_i$$
$$\upsilon_{oqi}^* = 0$$

Active and reactive power (measured)

$$\frac{dP_i}{dt} = -\omega_{ci} P_i + \omega_{ci}\left(\upsilon_{odi} \cdot i_{odi} + \upsilon_{oqi} \cdot i_{oqi}\right)$$
$$\frac{dQ_i}{dt} = -\omega_{ci} Q_i + \omega_{ci}\left(\upsilon_{oqi} \cdot i_{odi} - \upsilon_{odi} \cdot i_{oqi}\right)$$

Voltage primary PI control

$$\frac{d\psi_{di}}{dt} = k_{ivi}(\upsilon_{odi}^* - \upsilon_{odi})$$
$$\frac{d\psi_{qi}}{dt} = k_{ivi}(\upsilon_{oqi}^* - \upsilon_{oqi})$$
$$i_{ldi}^* = \psi_{di} + k_{pvi}(\upsilon_{odi}^* - \upsilon_{odi}) + k_{fvi} i_{odi} - \bar{\omega} C_{fi} \upsilon_{oqi}$$
$$i_{lqi}^* = \psi_{qi} + k_{pvi}(\upsilon_{oqi}^* - \upsilon_{oqi}) + k_{fvi} i_{oqi} + \bar{\omega} C_{fi} \upsilon_{odi}$$

Consensus in electrical networks

Modelling the network

Current primary PI control

$$\frac{d\phi_{di}}{dt} = k_{ici}(i^*_{ldi} - i_{ldi})$$
$$\frac{d\phi_{qi}}{dt} = k_{ici}(i^*_{lqi} - i_{lqi})$$
$$\upsilon^*_{ldi} = \phi_{di} + k_{pci}(i^*_{ldi} - i_{ldi}) - \bar{\omega} L_{fi} i_{lqi}$$
$$\upsilon^*_{lqi} = \phi_{qi} + k_{pci}(i^*_{lqi} - i_{lqi}) + \bar{\omega} L_{fi} i_{ldi}$$

LC filter and output connector

$$\frac{di_{ldi}}{dt} = -\frac{R_{fi}}{L_{fi}} i_{ldi} + \frac{1}{L_{fi}}(\upsilon_{ldi} - \upsilon_{odi}) + \omega_i i_{lqi}$$
$$\frac{di_{lqi}}{dt} = -\frac{R_{fi}}{L_{fi}} i_{lqi} + \frac{1}{L_{fi}}(\upsilon_{lqi} - \upsilon_{oqi}) - \omega_i i_{ldi}$$
$$\frac{d\upsilon_{odi}}{dt} = \frac{1}{C_{fi}}(i_{ldi} - i_{odi}) + \omega_i \upsilon_{lqi}$$
$$\frac{d\upsilon_{oqi}}{dt} = \frac{1}{C_{fi}}(i_{lqi} - i_{oqi}) - \omega_i \upsilon_{ldi}$$
$$\frac{di_{odi}}{dt} = -\frac{R_{ci}}{L_{ci}} i_{odi} + \frac{1}{L_{ci}}(\upsilon_{odi} - \upsilon_{bdi}) + \omega_i i_{oqi}$$
$$\frac{di_{oqi}}{dt} = -\frac{R_{ci}}{L_{ci}} i_{oqi} + \frac{1}{L_{ci}}(\upsilon_{oqi} - \upsilon_{bqi}) - \omega_i i_{odi}$$

Consensus in electrical networks

Modelling the network

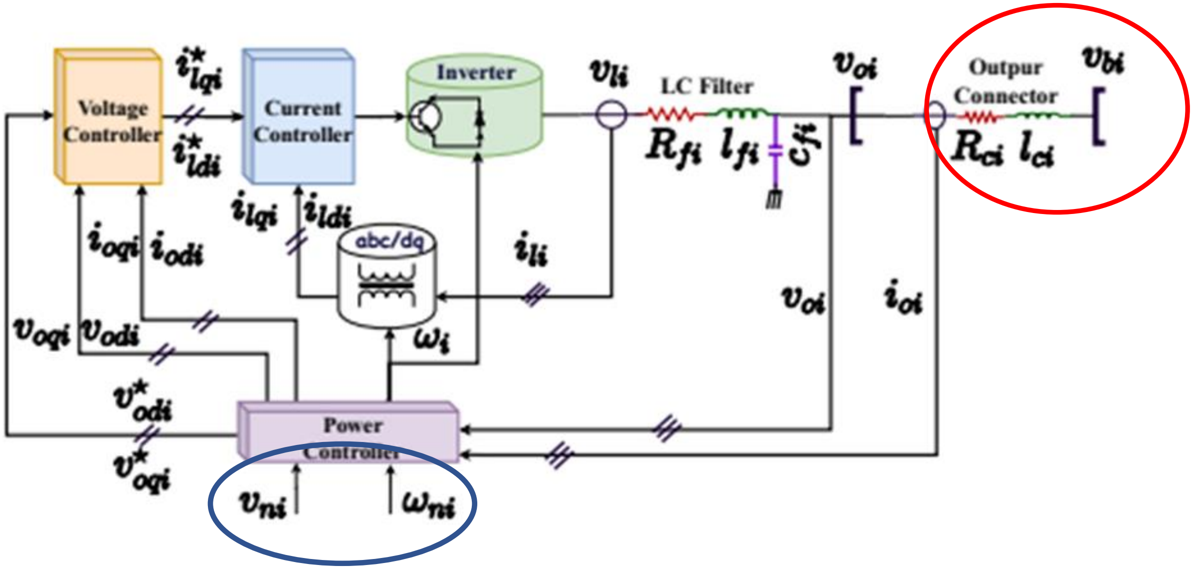
$$\dot{x}_i = f_i(x_i) + g_i(x_i) \cdot u_i + w_i(x_i) d_i$$

$$x_i = [\delta_i, P_i, Q_i, \phi_{di}, \phi_{qi}, \psi_{di}, \psi_{qi}, i_{ldi}, i_{lqi}, \upsilon_{odi}, \upsilon_{oqi}, i_{odi}, i_{oqi}]$$

$$w_i = ((\omega_i - \bar{\omega})\upsilon_{oqi} + \psi_{di} - k_{pvi}\upsilon_{odi} + (k_{fvi} - 1)i_{odi}) \qquad |\dot{w}_i| \leqslant \bar{\Omega}_i, \quad \bar{\Omega}_M = \max_i \bar{\Omega}_i$$

$$\boxed{u_i = [\omega_{ni}, \upsilon_{ni}]}$$

$$\boxed{d_i = [\upsilon_{bdi}, \upsilon_{bqi}]}$$



Consensus in electrical networks

Control goals

1 - Compensate for frequency and voltage variation

$$\left.\begin{aligned} \omega_i &= \omega_{ref} \\ v_{od_i} &= v_{ref} \end{aligned}\right. \qquad \forall\, i \in V$$

2 – Guarantee the desired power sharing between generators

$$\frac{P_i}{P_k} = \frac{m_k}{m_i} \qquad \forall\,(i,k) \in E$$

Since $\omega_i = \omega_{ni} - m_i P_i$ frequency restoration $\Rightarrow$ Power sharing by droop control

Consensus in electrical networks

The control problems can be formulated as two consensus problems

Frequency restoration

$$\omega_{ni} = \tilde{\omega}_i + \bar{\omega}$$
$$\dot{\tilde{\omega}}_i = -\alpha \cdot \operatorname{sign}\left(\sum_{j \in \mathcal{N}_i} (\tilde{\omega}_i - \tilde{\omega}_j) + \sum_{j \in \mathcal{N}_i} (\omega_i - \omega_j) + g_i (\omega_i - \omega_{ref}) \right)$$

$\bar{\omega}$ is the rated nominal MG's frequency

The MAS dynamics results into

$$\dot{\delta} = \omega - \omega_{com} = \tilde{\omega} + 1_n \otimes (\bar{\omega} - \omega_{com}) - m \cdot P$$
$$\dot{\tilde{\omega}} = -\alpha \cdot \operatorname{Sign}\left(\mathcal{L} (\omega + \tilde{\omega}) + G (\omega - 1_n \otimes \omega_{ref}) \right)$$

A. Pilloni, A. Pisano, E. Usai, Robust Finite Time Frequency and Voltage Restoration of Inverter-based Microgrids via Sliding Mode Cooperative Control, IEEE Trans. on Industrial Electronics, vol. 65, n. 1, pp. 907-917, 2018

Consensus in electrical networks

The control problems can be formulated as two consensus problems

Frequency restoration

Once the sliding mode is established the MAS dynamics moves along the sliding manifold

$$\sigma_\omega = [\sigma_{\omega.i}] = \mathcal{L}\left(\omega + \tilde{\omega}\right) + G\left(\omega - 1_n \otimes \omega_{ref}\right)$$

and the resulting dynamics is

$$\dot{\delta} = \omega - \omega_{com} = \tilde{\omega} + 1_n \otimes \left(\bar{\omega} - \omega_{com}\right) - m \cdot P$$
$$\dot{\tilde{\omega}} = -\alpha \cdot \mathrm{Sign}\left(\mathcal{L}\left(\omega + \tilde{\omega}\right) + G\left(\omega - 1_n \otimes \omega_{ref}\right)\right)$$

A. Pilloni, A. Pisano, E. Usai, Robust Finite Time Frequency and Voltage Restoration of Inverter-based Microgrids via Sliding Mode Cooperative Control, IEEE Trans. on Industrial Electronics, vol. 65, n. 1, pp. 907-917, 2018

Consensus in electrical networks

The control problems can be formulated as two consensus problems

Voltage restoration

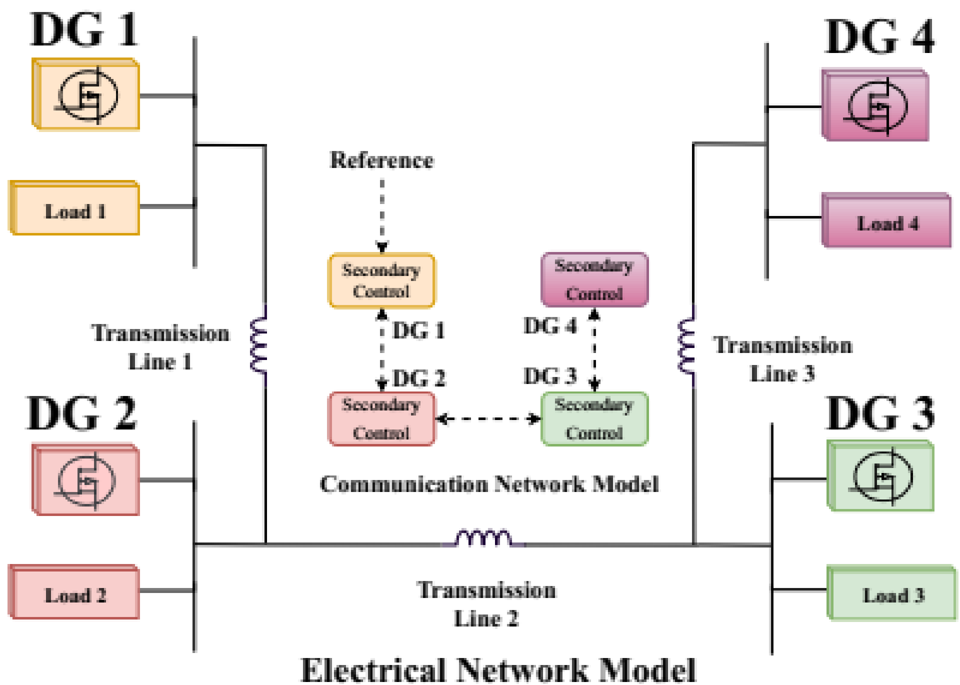
The agent's dynamics is $$\frac{dv_{\mathrm{odi}}}{dt} = w_i(\omega_i, \psi_{di}, v_{\mathrm{odi}}, v_{\mathrm{oqi}}, i_{\mathrm{odi}}) + \bar{w}_i \cdot v_{ni}$$

v_{ni} is the control variable for the *i-th* agent, and should be continuous

$$\dot{v}_{ni} = -\frac{C_{fi}}{k_{\mathrm{pvi}}} \left[\varsigma_1 \cdot \mathrm{sign}\left(\sum_{j\in\mathcal{N}_i} (v_{\mathrm{odi}} - v_{\mathrm{odj}}) + g_i\,(v_{\mathrm{odi}} - v_{\mathrm{ref}}) \right) \right. \\ \left. + \varsigma_2 \cdot \mathrm{sign}\left(\sum_{j\in\mathcal{N}_i} (\dot{v}_{\mathrm{odi}} - \dot{v}_{\mathrm{odj}}) + g_i\,(\dot{v}_{\mathrm{odi}} - \dot{v}_{\mathrm{ref}}) \right) \right]$$

A. Pilloni, A. Pisano, E. Usai, Robust Finite Time Frequency and Voltage Restoration of Inverter-based Microgrids via Sliding Mode Cooperative Control, IEEE Trans. on Industrial Electronics, vol. 65, n. 1, pp. 907-917, 2018

Consensus in electrical networks



- MG with 4 generators and four local loads
- Realistic Noisy Measurement with SNR=90dB
- Load changes and faults

Consensus in electrical networks

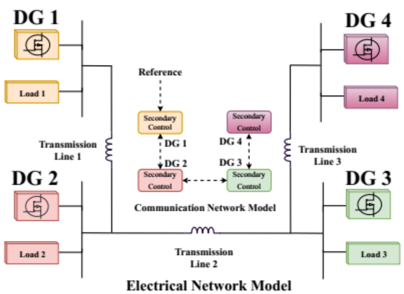


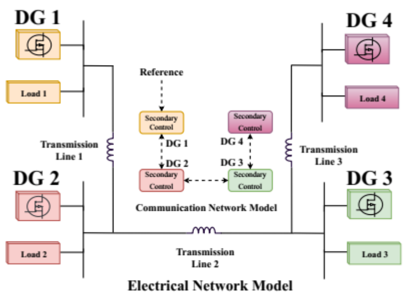
TABLE I
SPECIFICATION OF THE MICROGRID TEST SYSTEM

DG's Parameters		DG 1	DG 2	DG 3	DG 4
Droop Control	m_p	10×10^{-5}	6×10^{-5}	4×10^{-5}	3×10^{-5}
	n_Q	1×10^{-2}	1×10^{-2}	1×10^{-2}	1×10^{-2}
Voltage Control	k_{pv}	0.4	0.4	0.4	0.4
	k_{iv}	500	500	500	500
	k_{fv}	0.5	0.5	0.5	0.5
Current Control	k_{pc}	0.4	0.4	0.4	0.4
	k_{ic}	700	700	700	700
LC Filter [Ω],[mH],[μ F]	R_f	0.1	0.1	0.1	0.1
	L_f	1.35	1.35	1.35	1.35
	C_f	50	50	50	50
Connector [Ω],[mH]	R_c	0.03	0.03	0.03	0.03
	L_c	0.35	0.35	0.35	0.35

Lines [Ω],[μ H]	Line 1		Line 2		Line 3	
	R_{l1}	0.23	R_{l2}	0.23	R_{l3}	0.23
	L_{l1}	318	L_{l2}	324	L_{l3}	324

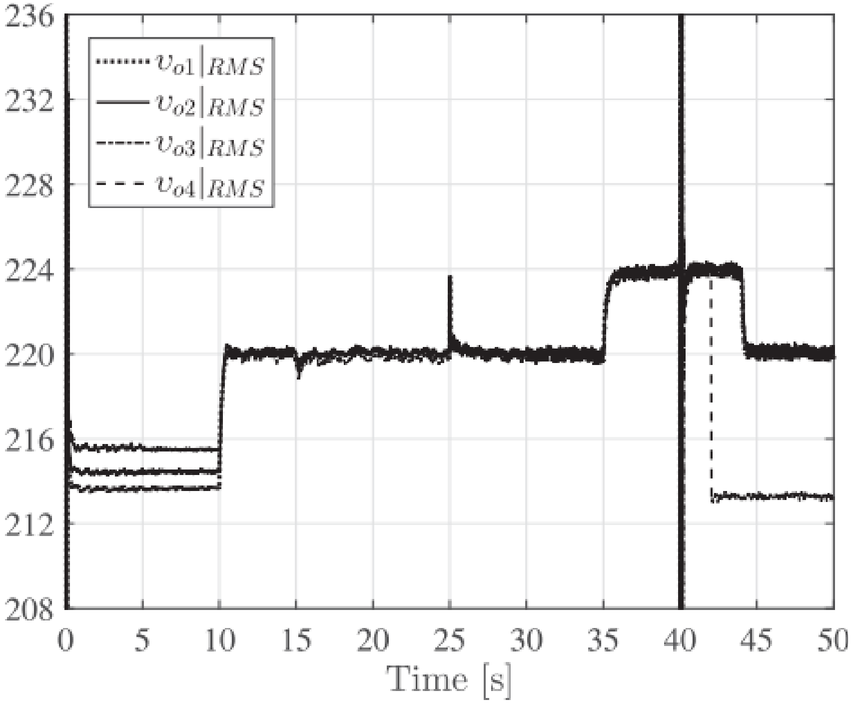
Loads [kW],[kVar]	Load 1		Load 2		Load 3		Load 4	
	P_{L1}	3	P_{L2}	3	P_{L3}	2	P_{L4}	3
	Q_{L1}	1.5	Q_{L2}	1.5	Q_{L3}	1.3	Q_{L4}	1.5

Consensus in electrical networks

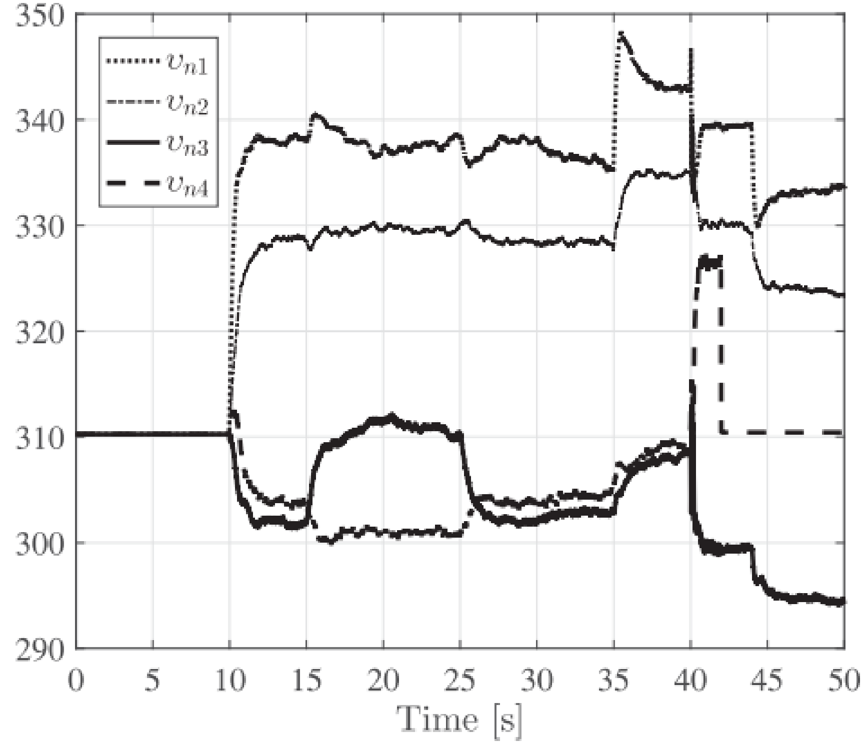


1) At the startup ($t = 0$ s), only the PC is active with PC set-points $\omega_{ni} = 2\pi \cdot 50$ Hz and $v_{ni} = 220\ \mathrm{V_{RMS}} \approx 380\ \mathrm{V_{ph-ph}}$.
2) At $t = 5$ s the frequency restoration SC (30)–(31) is activated with $\omega_{\mathrm{ref}} = 2\pi \cdot 50$ Hz.
3) At $t = 10$ s the voltage restoration SC (39) is activated with $v_{\mathrm{ref}} = 220\ \mathrm{V_{RMS}} \approx 380\ \mathrm{V_{ph-ph}}$.
4) By using a three-phase breaker, Load 3, i.e., (P_{L3}, Q_{L3}), is connected at $t = 15$ s, and removed at $t = 25$ s.
5) At $t = 30$ s the SC frequency setpoint is changed to $\omega_{\mathrm{ref}} = 2\pi \cdot 50.2$ rad/s.
6) At $t = 35$ s the SC voltage setpoint is changed to $v_{\mathrm{ref}} = 224\ \mathrm{V_{RMS}} \approx 388\ \mathrm{V_{ph-ph}}$.
7) At $t = 40$ s a three-phase to ground fault occurs on the Line 3.
8) At $t = 40.01$ s overcurrent protection devices isolate the Line 3, and thus DG 4 and Load 4, from the MG.
9) At $t = 42$ s the SC is reconfigured to take into account the changes occurred at the physical layer, i.e., $a_{34} = a_{43} = 0$.
10) At $t = 43$ s the SC frequency setpoint is changed to $\omega_{\mathrm{ref}} = 2\pi \cdot 50.2$ rad/s.
11) At $t = 44$ s the SC voltage setpoint gets back to $v_{\mathrm{ref}} = 220\ \mathrm{V_{RMS}} \approx 380\ \mathrm{V_{ph-ph}}$.

Consensus in electrical networks

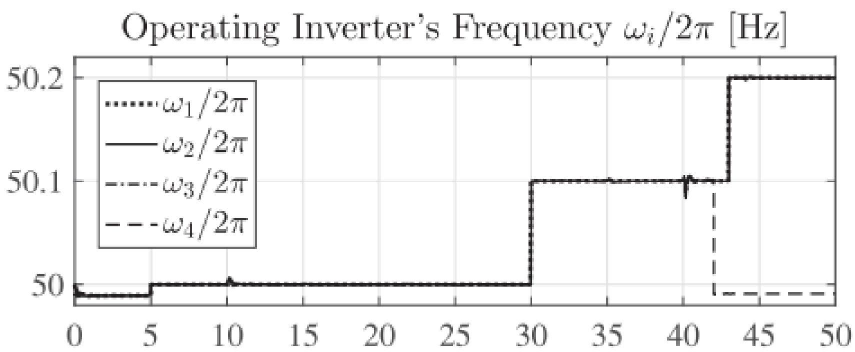


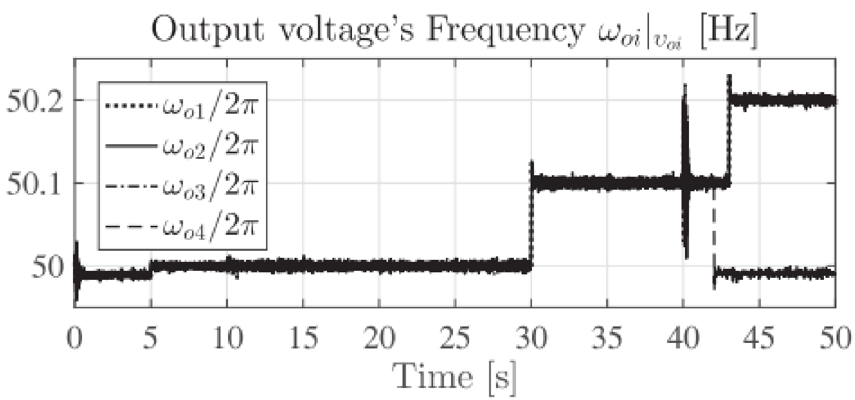
Output voltages are well regulated at desired values



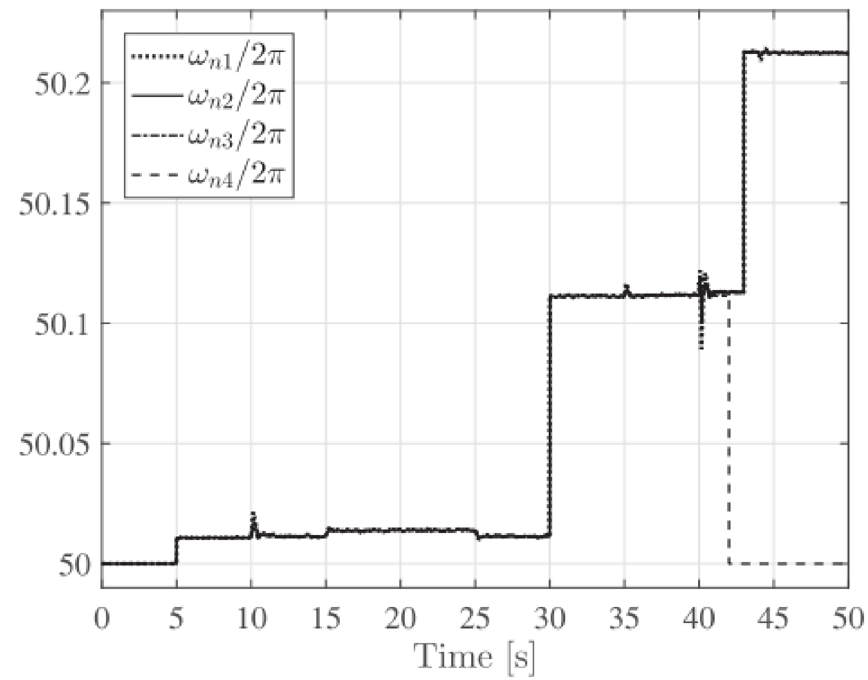
The voltage commands are defined to compensate for uncertainty

Consensus in electrical networks



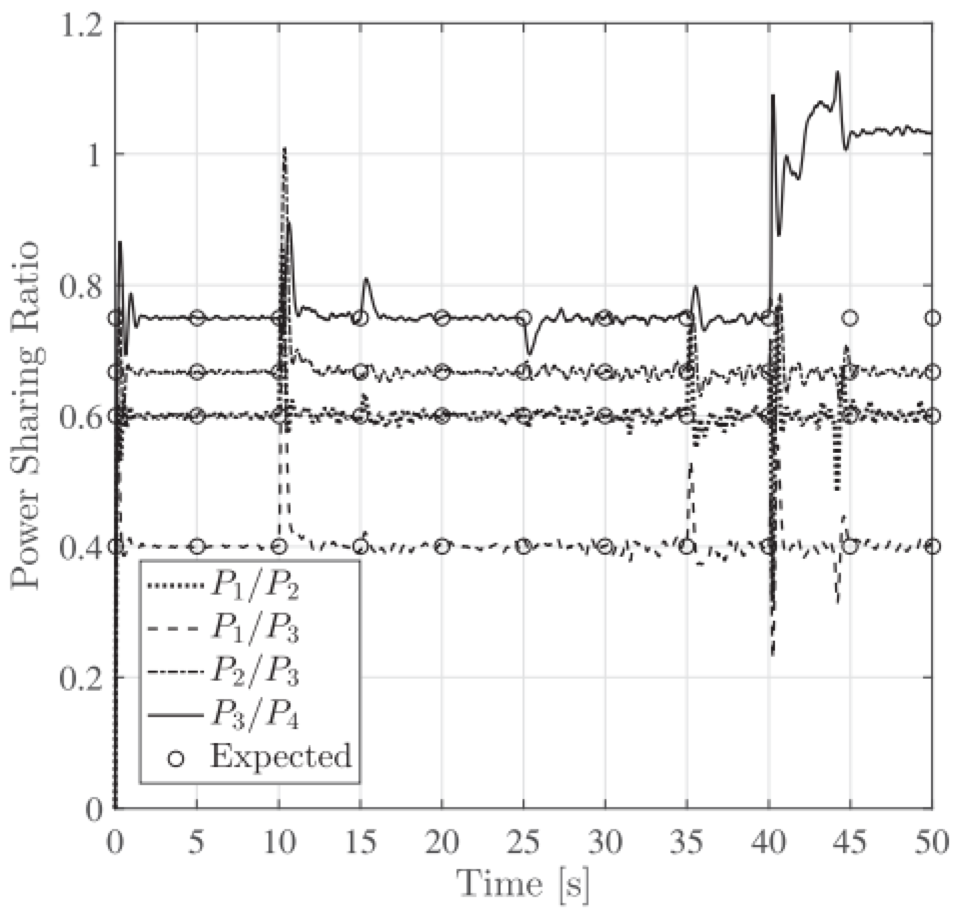


Output voltages' frequencies are well regulated at desired values



The reference frequencies are defined to compensate for uncertainty

Consensus in electrical networks



Power sharing is obtained as a side-by effect of the frequency restoration

Consensus in electrical networks

The control in the presence of communication delays among the generators has been recently considered and a solution is presented in

Gholami M.; Pilloni A.; Pisano A.; Usai E., Robust Distributed Secondary Voltage Restoration Control of AC Microgrids under Multiple Communication Delays. *Energies* **2021**, *14*, 1165. https://doi.org/10.3390/en14041165

Preliminary (but not well proved) results on the transition from islanded to grid-connected microgrid by consensus of a MAS with a leader

Summing up

Variable structure control with sliding mode seams to be able to contribute to the development of robust decentralised control of multi agent systems

Peculiar Lyapunov function could be taken into account to prove stability properties of the controlled MAS

Good Work to all!

Acknowledgements

Co-authors in MAS control

Alessandro PISANO

Alessandro PILLONI

Mauro FRANCESCHELLI

Alessandro GIUA

PhD students & Post doc

Diego DEPLANO, Milad GHOLAMI, Mujtaba KAHENI

Many others colleagues on various topics

Università degli Studi di Cagliari

Dip. di Ingegneria Elettrica ed Elettronica

VSS Rio 2022 international workshop

Thank you for you attention

Elio USAI

elio.usai@unica.it

All we're saying is
give peace a chance
(J. Lennon 1975)